THE BEHAVIOUR OF STOCHASTIC RUMOURS

By

Selma BELEN

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Abstract

This thesis presents results concerning the limiting behaviour of stochastic rumour processes.

The first result involves our published analysis of the evolution for the general initial conditions of the (common) deterministic limiting version of the classical Daley-Kendall and Maki-Thompson stochastic rumour models, [14].

The second result being also part of the general analysis in [14] involves a new approach to stiflers in the rumour process. This approach aims at distinguishing two main types of stiflers. The analytical and stochastic numerical results of two types of stiflers in [14] are presented in this thesis.

The third result is that the formulae to find the total number of transitions of a stochastic rumour process with a general case of the Daley-Kendall and Maki-Thompson classical models are developed and presented here, as already presented in [16].

The fourth result is that the problem is taken into account as an optimal control
problem and an impulsive control element is introduced to minimize the number of final ignorants in the stochastic rumour process by repeating the process. Our published results are presented in this thesis as appeared in [15] and [86].

Numerical results produced by our algorithm developed for the extended [MT] model and [DK] model are demonstrated by tables in all details of numerical values in the appendices.
This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

Signed.................................................. Date.............

"Sevgi sabırdır, sevgi şeykatlidir.
Sevgi kıskanmaz, övensiz, bımbırlenmez.
Sevgi kaba davranmaz, kendi çıkarmı aramaz. Sevgi haksızlığa sevinmez,
gerçek olanla sevinir. Sevgi
her şeye katlamır, her şeye inanır,
her şeyi umut eder, her şeye dayanır."

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