

BOUNDARY ELEMENT METHODS FOR THE SOLUTION OF A CLASS OF INFILTRATION PROBLEMS

Maria Lobo

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Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

SIGNED: DATE:

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Dedication

I dedicate this thesis to my beloved son Julio who passed away on the Christmas Eve 2007.

Summary

This thesis is concerned with a mathematical study of several problems involving infiltration from irrigation channels into an unsaturated homogeneous soil. All the problems considered are two dimensional and are solved numerically by employing boundary integral equation techniques.

In the first chapter I introduce some of the literature and ideas surrounding my thesis. Some background information is stated followed by an outline of the thesis and a list of author's published works that support the material in the thesis. Full descriptions of the fundamental equations used throughout the thesis are provided in chapter 2.

Chapter 3 contains the first problem considered in this thesis which is infiltration from various shapes of single and periodic irrigation channels. Specifically strip, semi-circular, rectangular and v shaped channels. The solutions are obtained using the boundary element technique. The solutions are then compared with the results obtained by Batu [14] for single and periodic strip sources.

In chapter 4 a boundary integral equation method is adopted for the solution of flow from single and periodic semi-circular channels into a soil containing impermeable inclusions. The impermeable inclusions considered are of rectangular, circular and square shapes. The aim is to observe how the various shapes of inclusions can affect the direction of the flow particularly in

the region adjacent to the zone where plant roots would be located.

Chapter 5 solves the problem of infiltration from single and periodic semi-circular irrigation channels into a soil containing impermeable layers. A modification is made to the boundary integral equation in order to include the impermeable layers with the integration over the layers involving Hadamard finite-part integrals. The objective of the work is to investigate how the number and the depth of the impermeable layers affects the flow.

Chapter 6 employs a particular Green's function in the boundary integral equation. The Green's function is useful for flow from a single channel since it removes the need to evaluate the boundary integral along the soil surface outside the irrigation channel.

A time dependent infiltration problem is considered in chapter 7. The Laplace transform is applied to the governing equations and the boundary integral equation technique is used to solve the resulting partial differential equation. The Laplace transform is then inverted numerically to obtain the time dependent values of the matric flux potential.