Appendix A. Further Discussion and Results of the Shorted Turn Fault Test

A.1. Current Harmonics Induced in the Rotor Winding

The air-gap MMF which is produced by the stator winding of a squirrel cage three-phase 2p-pole induction motor can be expressed as in (Eq. A.1), assuming that the three-phase supply is balanced [19].

\[
F_s(t, x) = \frac{3N_s I_s}{\pi} \sum_{v_s=1}^{\infty} \left( \frac{1}{v_s} \sum_{b=1}^{c} \sin \left( v_s \frac{\tau_{cb} \pi}{\tau_p} \right) \right) \times \sin \left( \omega_s t \pm \frac{v_s \pi \tau_{cb} x}{\tau_p} \right) \]  \quad \text{(Eq. A.1)}

where \(v_s\) is the stator MMF space harmonic rank, \(I_s\) is the maximum value of the stator phase current, \(\omega_s\) is the angular supply frequency, \(N_s\) is the number of turns of each stator coil, \(c\) is the number of coils of each stator phase, \(\tau_{cb}\) is the \(b^{th}\) coil pitch, \(\tau_p\) is the pole step, and \(x\) is the circumferential angular position.
This stator MMF in the air-gap causes a flux to be induced in the rotor. The flux induced in the $\alpha^{th}$ mesh of the rotor is given by (Eq. A.2) [19].

$$\phi_{sr,\alpha}(t) = \int_{x_1}^{x_2} \frac{\mu_0 F_s(t,x)}{\delta} l_c \, dx$$  \hspace{1cm} (Eq. A.2)

where $\mu_0$ is the magnetic permeability of air, $l_c$ is the core length, $\delta$ is the air gap length, and $x_1, x_2$ are the defined angular positions of the $\alpha^{th}$ mesh.

Hence,

$$\phi_{sr,\alpha}(t) = \sum_{v_s=1}^{\infty} \phi_{\text{max}v_s} \sin \left( \left[ 1 \pm v_s (1 - s) \right] \times \omega_s t \right)$$

$$\pm \frac{2 v_s \pi (k - 1)}{pR}$$  \hspace{1cm} (Eq. A.3)

where

$$\phi_{\text{max}v_s} = \frac{3 \mu_0 N_s l_s l_c D_r}{p^2 \pi \delta} \sum_{b=1}^{c} \sin \left( v_s \frac{\tau_{ch} \pi}{\tau_p} \right) \times \sin \left( v_s p \frac{\pi}{R} \right)$$  \hspace{1cm} (Eq. A.4)

and $D_r$ is the external rotor diameter.

The presence of shorted turn faults in the stator windings will affect the distribution of the stator MMF in the air-gap because a degree of the stator current will now flow through the short-circuited turns. Therefore by considering the relationship of the induced flux in (Eq. A.3) and the affected stator MMF, the presence of shorted turn faults will be expected to introduce new frequency components (Eq. A.5) in the rotor currents [19, 36].

A generalisation of these fault frequency components which include the stator time harmonics is shown in (Eq. 5.1).

$$f_{\text{rotor st}} = f \left[ k \frac{1 - s}{p} \pm 1 \right]$$  \hspace{1cm} (Eq. A.5)

where $k = 1, 2, 3, \ldots$
A.2. Current Harmonics Induced in the Stator Winding

The air-gap MMF which is produced by the rotor winding is shown in (Eq. A.6) [19].

\[
F_r(t, x) = \sum_{v_r=1}^{\infty} \frac{Rl_{rv \ max}}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin \left( \frac{k \tau_{cr}}{2} \right) \times \sin \left( kRx \pm s\omega_s t \right) \quad \text{(Eq. A.6)}
\]

where \( I_{rv \ max} \) is the maximum value of the \( v_r^{th} \) harmonic rotor current and \( \tau_{cr} \) is the rotor equivalent coil pitch.

This rotor MMF in the air-gap causes a flux to be induced in the stator winding. The final expression of the flux induced by the \( k^{th} \) harmonics in the \( b^{th} \) stator coil is given in (Eq. A.7) [19].

\[
\Phi_{rs,b,k}(t) = \Phi_{max,b,k} \sin \left[ kR \frac{\omega_r}{p} t \pm v\omega_s t + k \frac{2\pi(b - 1)}{3} \right] \quad \text{(Eq. A.7)}
\]

where \( \omega_r \) is the angular rotor frequency and

\[
\Phi_{max,b,k} = \frac{\mu_0 I_{rv \ max} Rl_c D_r N_s}{\pi \delta k^2} \sin \left( \frac{k \tau_{cr}}{2} \right) \sin \left( k \frac{\pi \tau_{cb}}{2 \tau_p} \right) \quad \text{(Eq. A.8)}
\]

The presence of shorted turn faults in the stator windings is expected to affect the frequency components that exist in the stator current. By considering the induced flux relation in (Eq. A.7), the shorted turn faults will be expected to vary the frequency components in (Eq. A.9) in the stator current [19, 37]. A generalisation of these fault frequency components is the frequency components in (Eq. 5.3).

\[
f_{stator \ st} = f \left[ kR \frac{1 - s}{p} \pm v \right] \quad \text{(Eq. A.9)}
\]
A.3. Experimental Results for Turn to Turn Fault Analysis
Using Fundamental Sidebands of Rotor Frequency Harmonics

**Components at \( k = 2 \) and \( v = \pm 1 \)**

Figure A.1 shows how the frequency components in (Eq. 5.1) at \( k = 2 \) and \( v = \pm 1 \) in the current and the leakage flux signals vary under different loading and turn to turn conditions. The figure shows that both the upper \((v = +1)\) and the lower \((v = -1)\) sidebands in the current and the flux signals do not display consistent and significant magnitude variation between the different turn to turn severities. As a consequence, these frequency components may not be suitable for turn to turn features.
A.3. Experimental Results for Turn to Turn Fault Analysis Using Fundamental Sidebands of Rotor Frequency Harmonics

Figure A.1 – Magnitude of the frequency components in (Eq. 5.1) at $k = 2$ and $v = \pm 1$ in the current and the leakage flux signals as a function of load.

Components at $k = 3$ and $v = \pm 1$

Figure A.2 shows that these frequency components cannot be considered as turn to turn features because they may have low signal to noise ratio (hence difficult to detect) and they do not show significant and consistent magnitude variation between the healthy and the faulty conditions.

Based on the analysis so far, it seems that the effectiveness of the frequency components in (Eq. 5.1) to detect turn to turn faults reduces as the variable $k$ is increased. As the variable $k$ is increased, the magnitude of the frequency components in (Eq. 5.1) seems to get weaker and weaker. Therefore, it is desirable to keep the variable $k$ in (Eq.
5.1) to 1, which is the value that the shorted turn investigation in this thesis will concentrate on.

![Graphs showing magnitude of frequency components in (Eq. 5.1) at k = 3 and v = ±1 in the current and leakage flux signals as a function of load.]

**Components at k = 1 and v = ±3**

Figure A.3 shows that the frequency components in the current signal mostly show no significant magnitude variation (i.e., less than 5 dB) among the different fault severities. On the other hand, the frequency components in the flux signal show significant magnitude variation between the healthy and the faulty motors (i.e., about 5 - 15 dB difference between the healthy and the 7.1% fault), where the magnitude consistently decreases as the...
A.3. EXPERIMENTAL RESULTS FOR TURN TO TURN FAULT ANALYSIS USING FUNDAMENTAL SIDEBANDS OF ROTOR FREQUENCY HARMONICS

severity of the fault increases. Therefore, the frequency components in the flux signal can be considered as turn to turn features.

Figure A.3 - Magnitude of the frequency components in (Eq. 5.1) at \( k = 1 \) and \( v = \pm 3 \) from the current and leakage flux signals as a function of load.

**Components at \( k = 1 \) and \( v = \pm 5 \)**

Figure A.4 shows that the frequency components in the current signal cannot be considered as features because they have low signal to noise ratio and their magnitude variation between the different fault severities is not significant and not consistent.
APPENDIX A. FURTHER DISCUSSION AND RESULTS OF THE SHORTED TURN FAULT TEST

Figure A.4 - Magnitude of the frequency components in (Eq. 5.1) at \( k = 1 \) and \( v = \pm 5 \) from the current and the leakage flux signals as a function of load.

Similarly, the upper sideband in the leakage flux signal does not show consistent magnitude variation between the different fault severities but it can distinguish the healthy and certain faulty (i.e. greater than 5.3%) conditions when the load is greater than 60%. The magnitude of this sideband is generally found to be weaker (by about 5 – 10 dB) than the magnitude of the component counterparts at \( v = +1 \) and +3. On the other hand, the lower sideband in the flux signal shows a more consistent magnitude variation between the different fault severities, where the magnitude tends to decrease as the severity of the fault increases. However, the magnitude variation is found to be smaller and less consistent than the magnitude variation of the flux components at \( v = -1 \) and \( v = -3 \).
A.4. Experimental Results for Turn to Turn Fault Analysis Using Twice Supply Frequency

Figure A.5 - Figure A.7 show how the magnitudes of the twice supply frequency component in the vibration sensor signals vary under the different loading and turn to turn fault conditions. Figure A.5 shows that the frequency component from the DEH vibration signal can only distinguish between the healthy and the 7.1% fault (20 shorted turns) effectively, where the magnitude of the healthy condition is about 5 dB lower than the magnitude of the faulty condition. Similarly, the frequency component from the NDEH vibration signal (Figure A.6) can distinguish between the healthy and only the 7.1% fault condition when the load is less than 60% but when the load is greater than 60%, the component can distinguish the healthy, 5.3% fault, and 7.1% fault conditions. On the other hand, the twice supply frequency component in the DEV vibration signal (Figure A.7) does not show observable magnitude variation among the different fault severities, except at the no load condition. Hence, it cannot be considered to be a useful feature.

Figure A.5 – Magnitude of the twice supply frequency in the DEH vibration signal as a function of load.
Figure A.6 - Magnitude of the twice supply frequency in the NDEH vibration signal as a function of load.

Figure A.7 - Magnitude of the twice supply frequency in the DEV vibration signal as a function of load.

A.5. Experimental Results for Turn to Turn Fault Analysis Using Rotor Slot Harmonics

Components at $n_g = 0$, $k = 1$, and $\nu = \pm 1$

Figure A.8 shows how the magnitude of the frequency components varies under the different loading conditions and the different turn to turn fault severities. The figure shows that the lower sideband component in the current signal can distinguish between the healthy, 5.3% fault, and 7.1% fault conditions, where the magnitude tends to increase as the severity of the fault increases, although the magnitude variation is not great (about 5 dB between the healthy and the 7.1% fault). As a result, this frequency component may only
be used as a feature to detect turn to turn faults if the fault severity is greater than or equal to 5.3%. On the other hand, the upper sideband component in the current signal shows no useful magnitude variation among the different fault severities.

Figure A.8 also demonstrates that the frequency components in the flux signal show some magnitude variation between the healthy and the faulty conditions (about 5 to 10 dB between the healthy and the 7.1% fault). However, the magnitude variations are not consistent enough for these frequency components to be considered as good turn to turn features.

Figure A.8 - Magnitude of the frequency components in (Eq. 5.3) at $n_d=0$, $k=1$, $v=-1$ (top), $n_d=0$, $k=1$, $v=+1$ (bottom), from the current and leakage flux signals as a function of load.
Components at $n_d = 0$, $k = 2$, and $v = \pm 1$

Figure A.9 shows the magnitude variations of the rotor slot harmonics (at $n_d = 0$, $k = 2$, and $v = \pm 1$) from the current and the leakage flux signals under different loading and turn to turn conditions. The figure shows that the magnitude of these components are weaker (by about 45 dB in the current signal and about 15 dB in the flux signal) than the counterpart components at $k = 1$. Unlike the components at $k = 1$ in the flux signal, the magnitude of these frequency components in both current and flux signals do not show any consistent variation among the different fault severities.
Components at $n_d = 0, k = 3$, and $v = \pm 1$

Figure A.10 shows that the magnitudes of the sidebands in the current signal fluctuate between -90 and -95 dB under the different loading and turn to turn conditions. These magnitudes are weaker than the counterpart components at $k = 1$ and $k = 2$, and they can be difficult to detect. In addition, the magnitude variations among the different fault severities show no useful pattern.

![Figure A.10 - Magnitude of the frequency components in (Eq. 5.3) at $n_d = 0, k = 3$ and $v = \pm 1$ from the current and leakage flux signals as a function of load.](image)

Similarly, the magnitudes of the sidebands from the leakage flux signal, which vary between -60 dB and -95 dB, are also weaker than the counterpart components at $k = 1$ and...
$k = 2$. The magnitudes tend to decrease as the severity of the fault increases. However, the magnitude variations among the different fault severities are not better than the counterpart components at $k = 1$ and they are not consistent enough for these sidebands to be considered as turn to turn features.

Examining the behaviour of the frequency components in (Eq. 5.3) so far, it seems that the magnitude of the frequency components decreases as the variable $k$ increases. This reduction in the magnitude also reduces the effectiveness of the frequency components to detect turn to turn faults. Therefore, it is desirable to keep the variable $k$ to 1, which is the value that the shorted turn investigation in this thesis will concentrate on.

**Components at $n_d = 0$, $k = 1$, and $v = ±3$**

Figure A.11 shows that the magnitudes of the sidebands in the current signal vary between -45 and -75 dB under the different loading and turn to turn conditions. These magnitudes are weaker than the counterpart components at $k = 1$ and $v = ±1$. Although the magnitudes tend to decrease as the fault severity increases, the magnitude variations among the different fault severities are not significant enough (i.e. less than 5 dB). As a consequence, it is difficult to consider these sidebands as turn to turn features.

On the other hand, the sidebands in the leakage flux signal show significant magnitude variation between the healthy and the faulty conditions (i.e. greater than 5 dB), where the magnitude tends to decrease as the severity of the fault increases, especially when the load is less than 80%. However, the magnitudes of these sidebands may not always be able to distinguish the different fault severities.
A.5. Experimental Results for Turn to Turn Fault Analysis Using Rotor Slot Harmonics

Figure A.11 - Magnitude of the frequency components in (Eq. 5.3) at $n_d = 0$, $k = 1$, and $v = \pm 3$ from the current and leakage flux signals as a function of load.

Components at $n_d = 0$, $k = 1$, and $v = \pm 5$

Figure A.12 shows that the lower sidebands in both current and flux signals cannot be considered as turn to turn features because they do not show significant and consistent magnitude variation (i.e. less than 5 dB) between the healthy and the faulty conditions.

On the other hand, the upper sidebands in both current and flux signals may be used to distinguish between the healthy and some faulty conditions but not to separate the different fault severities. The current sideband shows about 5 - 10 dB increase between the healthy and the 7.1% fault, while the flux sideband shows about 5 - 10 dB decrease between the healthy and the 7.1% fault.
APPENDIX A. FURTHER DISCUSSION AND RESULTS OF THE SHORTED TURN FAULT TEST

A.6. Experimental Results for Turn to Turn Fault Analysis
Using Third Harmonic of the Fundamental

Figure A.12 - Magnitude of the frequency components in (Eq. 5.3) at $n_d = 0$, $k = 1$, and $v = \pm 5$ from the current and leakage flux signals as a function of load.

Figure A.13 and Figure A.14 show how the magnitude of the frequency component at $3f$ in the stator current and the axial leakage flux signals varies under different loading conditions and different turn to turn fault severities.
A.7. Experimental Results for Phase to Phase Turn Fault Analysis Using Fundamental Sidebands of Rotor Frequency Harmonics

Figure A.13 – Magnitude of the third fundamental harmonic in the current signal as a function of load.

Figure A.14 - Magnitude of the third fundamental harmonic in the leakage flux signal as a function of load.

A.7. Experimental Results for Phase to Phase Turn Fault Analysis Using Fundamental Sidebands of Rotor Frequency Harmonics

Figure A.15 show how the magnitude of the frequency component in (Eq. 5.1) at \( k = 1 \) and \( \nu = \pm 3 \) from the leakage flux signal varies under different loading conditions and different phase to phase turn fault severities.
APPENDIX A. FURTHER DISCUSSION AND RESULTS OF THE SHORTED TURN FAULT TEST

A.8. Experimental Results for Phase to Phase Turn Fault Analysis Using Twice Supply Frequency

Figure A.16 and Figure A.17 show how the magnitude of the twice supply frequency component in the motor vibration signals varies under different loading conditions and different phase to phase turn fault severities.

Figure A.16 - Magnitude of the frequency component in (Eq. 5.1) at $k = 1$ and $v = \pm 3$ from the leakage flux signal as a function of load.

Figure A.15 - Magnitude of the frequency component in (Eq. 5.1) at $k = 1$ and $v = \pm 3$ from the leakage flux signal as a function of load.

Figure A.16 - Magnitude of the twice supply frequency in the DEH vibration signal as a function of load.
A.9. Experimental Results for Phase to Phase Turn Fault Analysis Using Rotor Slot Harmonics

Figure A.17 - Magnitude of the twice supply frequency in the NDEH vibration signal as a function of load.

A.9. Experimental Results for Phase to Phase Turn Fault Analysis Using Rotor Slot Harmonics

Figure A.18 and Figure A.19 show how the magnitude of the frequency component in (Eq. 5.3) at \( n_d = 0, k = 1 \) and \( v = \pm 1, \pm 3 \) from the leakage flux signal varies as functions of loading condition and phase to phase turn fault severity.

Figure A.18 - Magnitude of the frequency component in (Eq. 5.3) at \( n_d = 0, k = 1 \) and \( v = \pm 1 \) from the leakage flux signal as a function of load.
Figure A.19 - Magnitude of the frequency component in (Eq. 5.3) at \( n_d = 0, k = 1 \) and \( v = \pm 3 \) from the leakage flux signal as a function of load.

A.10. Experimental Results for Phase to Phase Turn Fault
Analysis Using Third Harmonic of the Fundamental

Figure A.20 and Figure A.21 show how the magnitude of the frequency component at 3\( f \) in the stator current and the leakage flux signals varies as functions of loading condition and phase to phase turn fault severity.

Figure A.20 - Magnitude of the third supply harmonic in the current signal as a function of load.
A.10. Experimental Results for Phase to Phase Turn Fault Analysis Using Third Harmonic of the Fundamental

Figure A.21 - Magnitude of the third supply harmonic in the leakage flux signal as a function of load.
Appendix B. Further Results of the Static Eccentricity Fault Test

B.1. Experimental Results for Static Eccentricity Fault Analysis Using Rotor Slot Harmonics

Figure B.1 and Figure B.2 show how the magnitude of the dynamic eccentricity components of the rotor slot harmonics in (Eq. 6.1) from the stator current and axial leakage flux signals varies under different loading conditions and different static eccentricity severities.
Figure B.1 – Magnitude of the lower sideband of the dynamic eccentricity components in (Eq. 6.1) from the current and the leakage flux signals as a function of load (top) and as a function of static eccentricity level (bottom).
B.2. Experimental Results for Static Eccentricity Fault Analysis Using Rotor Frequency Sidebands of the Fundamental

Figure B.2 – Magnitude of the upper sideband of the dynamic eccentricity components in (Eq. 6.1) from the current and the leakage flux signals as a function of load (top) and as a function of static eccentricity level (bottom).

B.2. Experimental Results for Static Eccentricity Fault Analysis Using Rotor Frequency Sidebands of the Fundamental

Figure B.3 and Figure B.4 show how the magnitude of the frequency components in (Eq. 6.2) from the current and leakage flux signals varies under different loading conditions and different static eccentricity severities.
Figure B.3 – Magnitude of the lower sideband of the frequency components in (Eq. 6.2) from the current and the leakage flux signals as a function of load (top) and as a function of static eccentricity level (bottom).
Figure B.4 – Magnitude of the upper sideband of the frequency components in (Eq. 6.2) from the current and the leakage flux signals as a function of load (top) and as a function of static eccentricity level (bottom).

### B.3. Experimental Results for Static Eccentricity Fault Analysis Using Rotor Frequency Sidebands of Twice the Fundamental

Figure B.5 shows how the magnitude of the frequency components in (Eq. 6.3) from the motor vibration signal varies under different loading conditions and different static eccentricity severities.
Figure B.5 – Magnitude of the rotor frequency sidebands of the twice fundamental (Eq. 6.3) from the vibration signal as a function of load (top) and as a function of static eccentricity level (bottom).

B.4. Experimental Results for Static Eccentricity Fault Analysis Using Third Harmonic of Rotor Frequency

Figure B.6 shows how the magnitude of the frequency component at $3f_r$ in the motor vibration signal varies under different loading conditions and different static eccentricity severities.
B.5. Experimental Results for Static Eccentricity Fault Analysis Using Twice Supply Frequency

Figure B.6 – Magnitude of the third harmonic of the rotor frequency (Eq. 6.4) from the vibration signal as a function of load (left) and as a function of static eccentricity level (right).

B.5. Experimental Results for Static Eccentricity Fault Analysis Using Twice Supply Frequency

Figure B.7 shows how the magnitude of the frequency component at $2f$ in the motor vibration signal varies under different loading conditions and different static eccentricity severities.

Figure B.7 – Magnitude of the twice supply frequency (Eq. 6.5) in the vibration signal as a function of load (left) and as a function of static eccentricity level (right).
B.6. Experimental Results for Static Eccentricity Fault Analysis Using Rotor Frequency

Figure B.8 shows how the magnitude of the frequency component at $f_r$ in the motor vibration signal varies under different loading conditions and different static eccentricity severities.

![Figure B.8 – Magnitude of the rotor frequency (Eq. 6.6) in the vibration signal as a function of load (left) and as a function of static eccentricity level (right).](image)

B.7. Experimental Results for Static Eccentricity Fault Analysis Using Second Harmonic of Rotor Frequency Sidebands of the Fundamental

Figure B.9 shows how the magnitude of the frequency components in (Eq. 6.7) from the motor vibration signal varies under different loading conditions and different static eccentricity severities.
B.8. Experimental Results for Static Eccentricity Fault Analysis Using RMS Vibration

Figure B.9 – Magnitude of the second harmonic of the rotor frequency sidebands of the fundamental (Eq. 6.7) in the vibration signal as a function of load (top) and as a function of static eccentricity level (bottom).

B.8. Experimental Results for Static Eccentricity Fault Analysis Using RMS Vibration

Figure B.10 shows how the RMS vibration level varies under different loading conditions and different static eccentricity severities.
Figure B.10 – Magnitude of the RMS vibration as a function of load (left) and as a function of static eccentricity level (right).
Reference List


