PUBLISHED VERSION

Gourlay, Timothy Peter.
The supercritical bore produced by a high-speed ship in a channel, Journal of Fluid Mechanics, 2001; 434:399-409.

Copyright © 2001 Cambridge University Press

PERMISSIONS

http://journals.cambridge.org/action/stream?pageId=4088&level=2#4408

The right to post the definitive version of the contribution as published at Cambridge Journals Online (in PDF or HTML form) in the Institutional Repository of the institution in which they worked at the time the paper was first submitted, or (for appropriate journals) in PubMed Central or UK PubMed Central, no sooner than one year after first publication of the paper in the journal, subject to file availability and provided the posting includes a prominent statement of the full bibliographical details, a copyright notice in the name of the copyright holder (Cambridge University Press or the sponsoring Society, as appropriate), and a link to the online edition of the journal at Cambridge Journals Online. Inclusion of this definitive version after one year in Institutional Repositories outside of the institution in which the contributor worked at the time the paper was first submitted will be subject to the additional permission of Cambridge University Press (not to be unreasonably withheld).

2nd May 2011

http://hdl.handle.net/2440/508

J. Fluid Mech. (2001), *vol.* 434, *pp.* 399–409. Printed in the United Kingdom
© 2001 Cambridge University Press

The supercritical bore produced by a high-speed ship in a channel

By T. P. GOURLAY

Department of Applied Mathematics, The University of Adelaide, Australia 5005

(Received 8 December 1999 and in revised form 1 November 2000)

An experimental investigation is made into the various flow regimes accompanying a ship travelling in a channel at supercritical speeds. The phenomena of smooth solitons, broken solitons, bores, and steady supercritical flow are observed. We look at the conditions under which each phenomenon exists, and the depth-based Froude numbers at which the transitions occur. Special emphasis is placed on ship bores, and we put forward a simple theoretical model for predicting the form of the bores as well as the transition to steady supercritical flow.

1. Introduction

Thews & Landweber (1935) observed in model experiments that when a ship is travelling in a uniform channel at close to the 'critical speed', the flow is unsteady, with waves propagating forward of the ship almost periodically. This critical speed is the natural speed of long waves in shallow water, given by $\sqrt{gh}$, where g is the acceleration due to gravity and h is the undisturbed water depth. The form of these waves was later found (Huang *et al.* 1982) to be very similar to the solitary waves predicted by the Korteweg–deVries equation, which are one-dimensional crests capable of travelling unchanged in form along a uniform channel.

In recent years, much experimental and theoretical work has been done in order to understand and predict the form of these solitary waves, or 'solitons'. In terms of the depth-based Froude number $F_h = U/\sqrt{gh}$ (the ratio of the ship speed U to the critical speed $\sqrt{gh}$), one such finding is that pure solitons can no longer be generated when F_h is greater than about 1.2. This was predicted theoretically by Huang *et al.* (1982) who calculated that no solitons should exist for $F_h > 1.175$, in rough agreement with their experiments. Other experimental investigations (e.g. Ertekin, Webster & Wehausen 1985; Lee, Yates & Wu 1989) have also shown that the solitons radiating ahead of a ship begin to break when F_h is between 1.1 and 1.2.

The Korteweg–deVries/Boussinesq-type equations often used to model ship solitons (see e.g. Wu 1987) cannot be used directly to model the ship waves at Froude numbers greater than about 1.2, because of energy lost across the breaking waves. In this article we shall propose a simple one-dimensional method for modelling the waves ahead of the ship, which allows for the energy loss that occurs once the solitons have broken.

At even higher Froude numbers, Constantine (1961) predicted and observed that the ship eventually ceases to produce a bore travelling ahead of it, and the flow becomes steady. The transition between these different phenomena will also be studied here.

2. One-dimensional theory for a ship in a channel

Constantine (1961) proposed a one-dimensional theory for studying the flow past a ship in a channel, which states that the only significant velocity component is in the direction of the ship's motion, and this velocity is uniform across the channel. Similarly, the free surface height is also uniform across the channel. Although Constantine only considered a block-like ship, we shall consider here a ship of general shape.

The assumptions made in deriving this theory are that the channel's width and depth should be small compared to the length of the ship, so that the streamwise velocity becomes uniform across the channel; also that the ship's dimensions change only slowly along its length, so that transverse and vertical velocity components are of smaller magnitude than the streamwise component. It is a fully nonlinear theory, with no assumptions being made about the magnitude of the free surface elevation or streamwise velocity component.

The method is valid for a channel of arbitrary cross-sectional shape, provided that this shape is constant along the channel and that the ship and channel walls are approximately vertical at and near the waterline. We define the channel's waterline width as w and undisturbed average depth as h; this allows us to use the theory for either rectangular or non-rectangular channels. In order to simplify the analysis, we shall consider the ship to be vertically fixed in its rest position, so that it is unable to heave and pitch.

By considering the continuity and Bernoulli relations for this one-dimensional flow, Tuck (1974) showed that steady solutions are impossible for a range of Froude numbers extending above and below $F_h = 1$. The limits $F_h = F_{\text{lim}}$ of this range satisfy

$$3\left[F_{\text{lim}}^2\left(1-\frac{B}{w}\right)\right]^{1/3} - F_{\text{lim}}^2\left(1-\frac{B}{w}\right) = 2\left(1-\frac{S}{wh}\right) \tag{1}$$

where B is the ship's beam at any point along its length and S the corresponding hull section area.

Because of the local nature of this theory, each point along the ship's length, having different values of B and S, has its own range of Froude numbers for which no steady flow exists. Therefore the unsteady region for a given ship is the envelope of these Froude number ranges. In most cases, however, using the values of B and S from the hull section of largest cross-sectional area will give the correct 'unsteady' range.

The smaller solution of (1) for F_{lim} is denoted F_{lim}^-, which is the upper Froude number limit of steady subcritical flow; the larger solution F_{lim}^+ is the lower Froude number limit of steady supercritical flow. These will normally be less than and greater than unity respectively. When $F_{\text{lim}}^- < F_h < F_{\text{lim}}^+$, the steady flow of water around the hull cannot satisfy continuity without violating the Bernoulli condition (i.e. gaining energy). This leads to a piling up of water at the bow, as not all of the water can get past the ship. The piled-up water at the bow radiates ahead of the ship either as solitons or a bore.

2.1. *Unsteady bores*

Let us now consider the application of one-dimensional theory to the formation of an unsteady bore travelling ahead of the ship. Figure 1 shows a side-on view of the ship and channel in this situation. The still water ahead of the bore has depth h, while the shelf of water between the bore front and the ship has depth h_1. We wish to solve for this unknown depth h_1, as well as the speed of the bore front V and fluid speed behind the bore W. All of these quantities are considered constant.

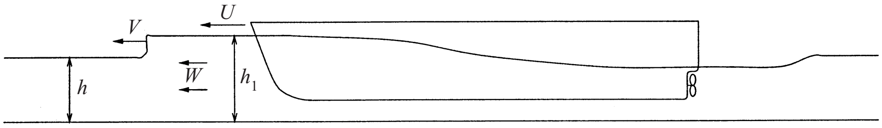

FIGURE 1. One-dimensional bore.

Conservation of mass in a moving control volume that includes the bore front requires (Stoker 1957, p. 321) that

$$Vh = (V - W)h_1 \tag{2}$$

while the rate of change of momentum of the control volume is used to give

$$Wh_1(V - W) = \tfrac{1}{2}g(h_1^2 - h^2). \tag{3}$$

No energy balance is applicable because energy is lost across the breaking bore front (Stoker 1957, p. 319). The conditions (2) and (3) can be thought of as determining V and h_1 in terms of the fluid speed W. As W increases, V and h_1 both increase monotonically. But what determines W?

We know that the bore is originally formed because not all of the water can move past the ship without violating energy conservation. Because energy is required to drive the bore, if W can be decreased by increasing the amount of water moving past the ship, it will be. Therefore the bore will be as small as possible, in order to pass the maximum amount of water under the ship.

To see how small W may be without violating Bernoulli's law, we first consider the case of steady subcritical flow. According to one-dimensional theory, there is a maximum Froude number F_{lim}^-, given by (1), for which steady subcritical flow may exist. F_{lim}^- depends only on B/w and $S/(wh)$. This defines the maximum ship speed into still water of a given depth for which steady flow is possible.

When a bore is produced, the flow is no longer steady and the ship is no longer advancing into still water (because the water has speed W ahead of the ship). However, in a frame of reference moving forward with speed W, the ship has relative speed $U - W$, and the flow is steady in the vicinity of the ship since h_1 is constant. The local Froude number is therefore

$$\frac{U - W}{\sqrt{gh_1}} \tag{4}$$

in this frame of reference. Decreasing W and h_1 increases this Froude number; as such, the minimum possible values of W and h_1 will occur when the local Froude number is at its upper limit. Therefore the remaining condition on W that is needed is

$$\frac{U - W}{\sqrt{gh_1}} = F_{\text{lim}}^-, \tag{5}$$

with F_{lim}^- being given by equation (1) for a given ship and channel.

Conditions (2), (3) and (5) define three simultaneous equations for the three unknowns h_1, W and V. We can eliminate W and V from these equations to give

$$2\frac{h_1}{h}\left(F_h - \sqrt{\frac{h_1}{h}}F_{\text{lim}}^-\right)^2 = \left(\frac{h_1}{h} - 1\right)^2\left(\frac{h_1}{h} + 1\right) \tag{6}$$

for the non-dimensional free surface height h_1/h. This can be solved numerically and then the corresponding values of W and V found.

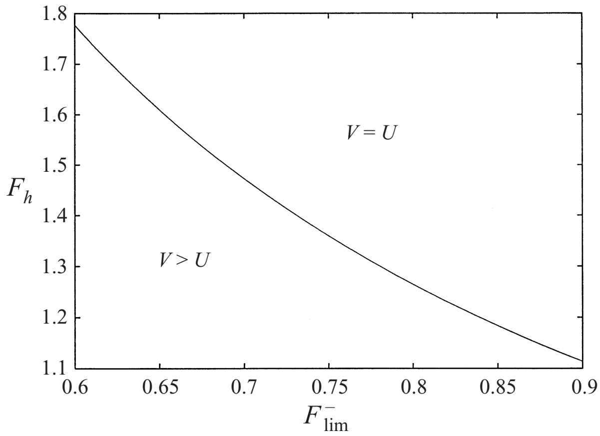

FIGURE 2. Froude number ranges for which the bore speed V is greater than or equal to the ship speed U.

The above analysis has been carried out assuming that $V > U$, i.e. that the bore is able to travel faster than the ship. Calculations suggest that for practical ship and channel configurations this should normally be the case at low supercritical Froude numbers. As the Froude number increases, however, there will be a point at which $V = U$. If this is less than F^+_{lim}, there will be a range of Froude numbers for which the bore predicted by this theory is unable to travel faster than the ship, yet steady supercritical flow is not possible. So what will happen according to one-dimensional theory in this region?

Provided that we are still in the critical range, i.e. $F_h < F^+_{\text{lim}}$ as given by (1), steady flow is still unable to exist without changed conditions ahead of the ship. Therefore a bore must still be produced in order to decrease the local Froude number (4) below F^-_{lim}. Note that the local Froude number cannot be increased to F^+_{lim}, as this would involve a negative bore propagating ahead of the model, which is impossible (Stoker 1957, p. 321).

In this case it is clear that the smallest possible bore will move at the same speed as the ship, i.e. $V = U$. The condition (5) is no longer true in this case, since the local Froude number is less than F^-_{lim}. With V known, we only need the bore conditions (2) and (3) to solve simultaneously for h_1 and W.

The range of F_h for which V is greater than or equal to U is shown in figure 2. We see that for each value of F^-_{lim}, there is a value of F_h above which any bore must travel at the same speed as the ship. This changeover value of F_h is seen to decrease as F^-_{lim} comes closer to unity.

We can now identify four possible wave-making regimes that could occur for a ship travelling in a channel at faster than the critical speed. In order of increasing Froude number, these are:

(i) solitons travelling ahead of and faster than the ship;
(ii) a bore travelling ahead of and faster than the ship;
(iii) a bore travelling ahead of the ship at the same speed;
(iv) steady supercritical flow.

At small blockage coefficients $S/(wh)$, F^+_{lim} is close to 1, so we would expect that soliton generation will cease before the solitons start to break. In this case no bores will be produced at all. For larger blockage coefficients, F^{+}_{lim} is larger and bores will be produced in the approximate region $1.2 < F_h < F^{+}_{\text{lim}}$. Depending on the ship and channel geometry, these bores may travel faster than the ship for all, part of, or none of this range.

For the cases considered in this article, $V > U$ for all $F_h < F^{+}_{\text{lim}}$, so that the bore should always be travelling faster than the ship. We saw in figure 2 that as F^{-}_{lim} comes closer to unity, $V = U$ at smaller Froude numbers. However, F^{+}_{lim} also normally decreases in that case, so that bores moving at the same speed as the ship may still not be produced.

According to (1), it is possible to increase F^{+}_{lim} while keeping F^{-}_{lim} constant, by increasing B/w significantly and $S/(wh)$ slightly (Gourlay 1999). Therefore a ship and channel with a large B/w ratio should have a greater range of Froude numbers over which bores moving at the same speed as the ship will occur.

It is important to note that this one-dimensional theory yields bore solutions for all $F_h > F^{-}_{\text{lim}}$. However, its range of validity lies only in the Froude number range for which solitons are no longer generated and steady supercritical flow has not yet started.

3. Experimental setup

Experiments to observe ship bores were carried out in the 60 m towing tank at the Australian Maritime College in Tasmania. A 1.6 m AME CRC model #11 was used (Bojovic 1997). This is a transom-stern round-bilge monohull which has a parent hull the same as that of the High Speed Displacement Hull Form series (Robson 1988). It has block coefficient $C_B = 0.5$, length/beam ratio $L/B = 4.00$, beam/draught ratio $B/T = 4.00$ and midship section coefficient $C_M = 0.799$.

The model was fixed to the carriage in its design waterline position, and was not allowed to sink or trim. It was moved along the centre of the channel, which is of width 3.5 m and has vertical walls. Two wave probes, 0.72 m from one of the channel walls, were positioned at different distances along the tank near the end of the run. These measured the free surface elevation as a function of time; the time difference for the bore front passing the two probes was used to determine the bore's average speed.

Two different water depths were used, corresponding to h/T values of 1.14 and 2.05. Runs were made for gradually incremented speeds corresponding to $F_h = 1.05$ and upwards. For each run, video footage was taken to see the changing nature of the waves ahead of the ship. As noted by earlier investigators, these waves spanned perfectly across the tank in the soliton and bore-producing Froude number ranges. The more complicated trailing waves are not amenable to one-dimensional analysis and were not studied in detail.

4. Experimental results

4.1. $h/T = 1.14$

For $F_h < 1.12$, pure solitons were periodically radiated forward of the ship. We shall not discuss the exact nature of these solitons here, as this has been thoroughly done both experimentally and theoretically by earlier authors (e.g. Huang *et al.* 1982; Ertekin *et al.* 1985; Lee *et al.* 1989).

The solitons first began to break at $F_h = 1.12$, at which stage they had maximum

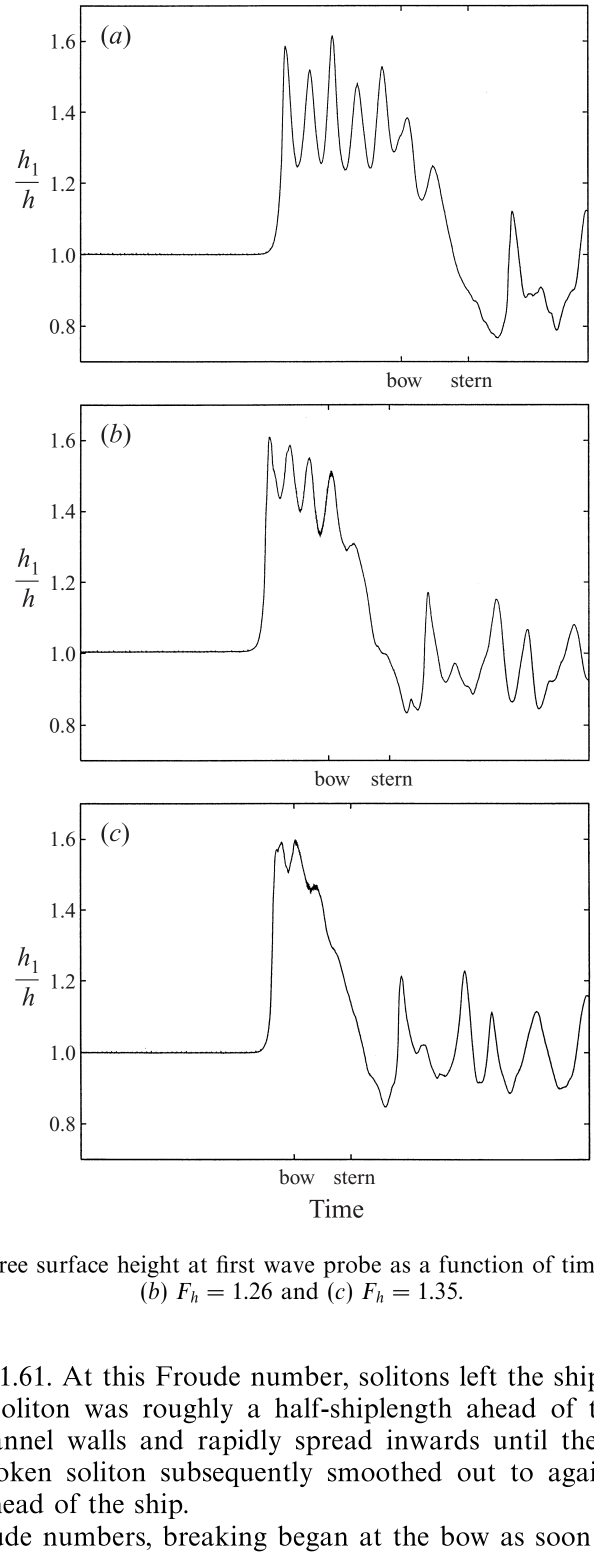

FIGURE 3. Scaled free surface height at first wave probe as a function of time, for (*a*) $F_h = 1.15$, (*b*) $F_h = 1.26$ and (*c*) $F_h = 1.35$.

elevation $h_1/h = 1.61$. At this Froude number, solitons left the ship's bow unbroken; then, when the soliton was roughly a half-shiplength ahead of the bow, breaking began at the channel walls and rapidly spread inwards until the whole wave was broken. Each broken soliton subsequently smoothed out to again become a pure soliton further ahead of the ship.

At higher Froude numbers, breaking began at the bow as soon as the model was started and the waves subsequently remained broken. For $1.12 < F_h < 1.35$ there was

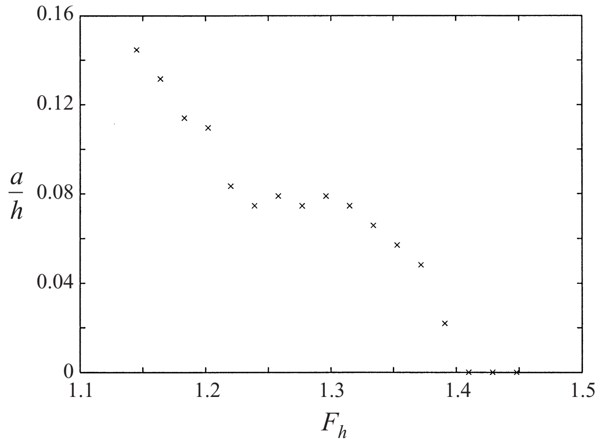

FIGURE 4. Scaled mean wave oscillation amplitude a/h.

a gradual transition from broken solitons at lower Froude numbers to an almost-pure bore at higher Froude numbers. Figure 3(*a*–*c*) illustrates this transition as the Froude number is increased. Here we have plotted the scaled free surface height h_1/h at the first wave probe as a function of time, for F_h =1.15, 1.26 and 1.35. The time at which the bow and stern pass the probe is also indicated. The figures could also be taken to roughly represent the free surface profile as a function of distance along the tank, at a given moment in time.

We notice in these figures that the maximum free surface height remains approximately constant at the breaking threshold ($h_1/h = 1.61$) as the Froude number is increased. It is the wave troughs that elevate as F_h increases, until at $F_h = 1.35$ the wave ahead of the ship is very close to a pure bore.

Although the wave front in figure 3(*c*) is only a half-shiplength ahead of the model, it is still travelling faster than the model; at these higher speeds there is less time for the bore to form, and the speed difference between it and the model is smaller. This means that the bore is only slightly ahead of the model by the time the first wave probe is reached. For this reason, results for wave heights were taken at the second wave probe, where the waves were more completely developed.

The progression from solitons to a pure bore as F_h increases is illustrated in figure 4. Here we have plotted the experimental mean wave amplitude a, which is equal to half of the difference between the mean peak and mean trough elevations ahead of the ship. This is scaled with respect to h and plotted as a function of Froude number. We can see that a/h tends to zero as F_h increases, meaning that the waves ahead of the ship change gradually to a flat shelf of water.

At this depth, the bore was seen to travel faster than the model whenever it was produced. For $1.43 < F_h < 1.48$, however, the bore was travelling only slightly faster than the model. The measured bore speed will be compared to the theoretical predictions in the following section.

At $F_h = 1.48$ the bore that had spanned the channel at lower Froude numbers became a bow wave that swept back at a slight angle; this indicated the imminent commencement of steady supercritical flow. As the Froude number increased further, this bow wave swept back at a gradually increasing angle, with the flow being steady at each Froude number.

4.2. $h/T = 2.05$

In the deeper water with $h/T = 2.05$, smooth solitons were still observed at Froude numbers up to $F_h = 1.11$. At this Froude number (almost exactly the same as in the shallower water) the solitons started to break slightly ahead of the model, before re-forming further ahead. The breaking solitons had a maximum free surface height of $h_1/h = 1.60$, which is also very similar to the shallower water result.

At higher Froude numbers ($1.2 < F_h < 1.3$) a single breaking wave spanning the tank was produced at the ship's bow. This was moving only slightly faster than the ship. Because of the high model speed, the wave was unable to move far ahead of the ship before the end of the tank was reached. The transition to steady supercritical flow occurred at $F_h = 1.31$, and the bow wave then swept back at a rapidly increasing angle at higher Froude numbers.

5. Comparison with one-dimensional theory

5.1. $h/T = 1.14$

According to the one-dimensional theory described in § 2, the production of bores at a given depth (governed by (2), (3) and (5)) depends only on the Froude number F_h and the limiting Froude number F_{lim}^- of steady subcritical flow. F_{lim}^- as given by (1) depends only on the beam/channel-width ratio B/w and blockage coefficient $S/(wh)$. The values of these are taken at the hull cross-section of the largest area.

This example will serve to illustrate the simplicity of the theory. In this case we have $B/w = 0.4/3.5 = 0.1143$ and $S/(wh) = 0.80(BT/wh) = 0.0802$. According to (1), the limiting Froude numbers are $F_{\text{lim}}^- = 0.702$ and $F_{\text{lim}}^+ = 1.437$. Therefore, we should expect a transition from bores being produced to steady supercritical flow at the limiting Froude number $F_{\text{lim}}^+ = 1.44$. The transition occurred experimentally at $F_h = 1.48$, which is in reasonable agreement with the theory.

At lower Froude numbers, the one-dimensional theory predicts a bore of constant height and speed for each Froude number. Using (2), (3) and (5), to predict the scaled quantities h_1/h, $V/\sqrt{gh}$ and $W/\sqrt{gh}$ only requires input of F_{lim}^- and F_h. With $F_{\text{lim}}^- = 0.702$ in this case, we have plotted the scaled bore height h_1/h and scaled bore speed $V/\sqrt{gh}$ as functions of F_h in figures 5 and 6 respectively. Experimental results are also shown on the same axes. The bore height h_1 represents the mean free surface height ahead of the ship, while the bore speed V is calculated using the time difference between the arrival of the bore front at each probe.

We can see that h_1 and V both increased experimentally and theoretically with increasing F_h. The general form of the experimental results matched the theoretical results, although the theory underpredicted h_1 by 3–6% and V by 1–4%. Due to the limited tank length, the waves were still evolving slightly when they reached the end of the tank, which resulted in small oscillations in the experimental values for h_1 and V. The theory predicted that the bore would travel faster than the ship for all $F_h < 1.47$. Since $F_{\text{lim}}^+ = 1.44$ in this case, the bores should travel faster than the ship whenever they are produced, which was also borne out by experiment.

5.2. $h/T = 2.05$

At this water depth the relevant parameters governing the flow are $B/w = 0.1143$ and $S/(wh) = 0.0446$. According to (1), the limiting Froude numbers are then $F_{\text{lim}}^- = 0.792$ and $F_{\text{lim}}^+ = 1.341$. Because of the smaller blockage coefficient, these are both closer to unity than for the shallow water case. The theoretical prediction for the transition to

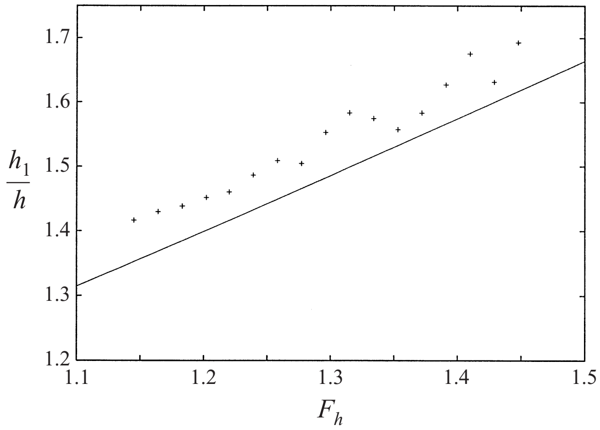

FIGURE 5. Scaled bore height as a function of F_h for $h/T = 1.14$; full line shows theoretical results and points show experimental results.

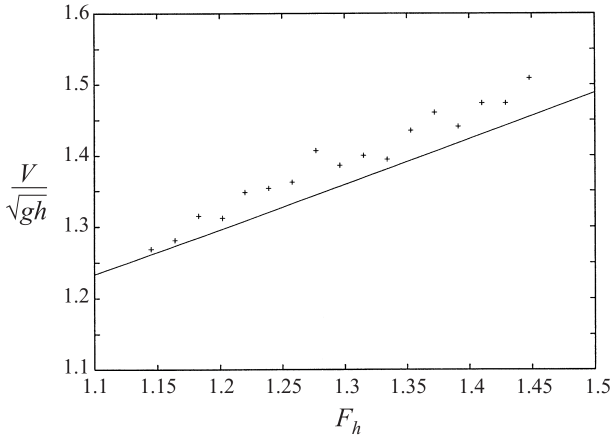

FIGURE 6. Scaled bore speed as a function of F_h for $h/T = 1.14$; full line shows theoretical results and points show experimental results.

steady supercritical flow ($F^{+}_{\text{lim}} = 1.34$) again agreed quite closely with the experimental value at $F_h = 1.31$.

Because the transition to steady supercritical flow occurred at a relatively low Froude number in this water depth, pure bores were not really produced. The waves more closely resembled broken solitons; because these were only slightly ahead of the ship's bow by the end of the tank, it was impossible to measure an average bore height to compare with the theory. The wave speed (not shown) was still quite close to the theoretical values over the small Froude number range of bores being produced; however, because the waves were still evolving, the experimental results were more erratic than in the shallower water.

6. Conclusions

For F_h greater than about 1.12, the solitons produced by a ship in a channel begin to break, and remain broken as F_h increases. While the maximum height of these waves stays approximately constant at the breaking threshold, the troughs between these broken waves rise in elevation as F_h increases, until the waves resemble a flat shelf of water travelling ahead of the ship.

In the entire speed range where solitons and bores are generated, the free surface ahead of the ship is uniform across the channel. This fact, along with the shallow water assumption, permits analysis of the flow ahead of the ship using a simple unsteady one-dimensional theory. We have developed this theory for a ship of general shape that is fixed in its design waterline position, and compared the results to experimental values for the bore's height and speed. The general form of the results agreed with the theory, although the height and speed of the bore both exceeded predictions by 1–6%.

It seems likely that the discrepancy at lower Froude numbers is due to the broken solitons having a higher natural speed than a pure bore would. At higher Froude numbers, the bore may have a slightly higher speed when it first moves ahead of the ship, before settling down to a constant speed at larger times. A longer tank is required to verify this experimentally.

In the experiments performed here, only bores travelling faster than the ship were observed; however, in cases of larger ship-beam/channel-width ratio, the theory predicts a range of Froude numbers over which bores should travel at the same speed as the ship. We showed how the height of such bores can be calculated using one-dimensional theory.

The experiments showed a clear transition from bore production to steady supercritical flow, and the Froude numbers at which this occurred agreed quite well with the theory.

Although the case of a fixed ship was chosen here in order to simplify the analysis and gauge more precisely the accuracy of the method, extension to the case of a ship that is free to sink and trim is possible. The Froude number limits of steady flow have already been found in this case (Gourlay 1999). In order to calculate the form of the bores, we must consider the inverse problem of solving the flow around the ship in its (as yet unknown) squatted position. Alternatively, the sinkage and trim could be found approximately by finding the force and moment on the ship in its rest position.

If the sinkage and trim are known, the method will still require only the maximum section area and corresponding beam as input; however, these must be adjusted to include any increase due to local sinkage below the ship's rest position. The main squat effect will be a large bow-up trim while bores are being produced. We might expect that, despite the large trim, the point along the ship's length at which the maximum section area occurs will not be significantly displaced from its rest position. If this is the case, the current theory could be used unmodified. Further theoretical and experimental studies are required to clarify this.

I would like to thank the Australian Maritime College in Launceston for allowing me to conduct the tests, particularly J. Duffy, G. McFarlane and M. Renilson; also, Professor E. O. Tuck of the University of Adelaide for valuable discussions on the subject.

REFERENCES

Bojovic, P. 1997 Resistance of AME CRC Systematic Series of high speed displacement hull forms. *Proc. 4th Symp. on High Speed Marine Vehicles, Naples, Italy*. ATENA/CETENA/ Dipartimento di Ingegneria Navale, Universita di Napoli Frederico II.

Constantine, T. 1961 On the movement of ships in restricted waterways. *J. Fluid Mech.* **9**, 247–256.

Ertekin, R. C., Webster, W. C. & Wehausen, J. V. 1985 Ship-generated solitons. *15th Symp. Naval Hydrodynamics*, pp. 347–361. Washington DC. National Academy Press.

Gourlay, T. P. 1999 The effect of squat on steady nonlinear hydraulic flow past a ship in a channel. *Schiffstechnik* **46**(4), 217–222.

Huang, D. B., Sibul, O. J., Webster, W. C., Wehausen, J. V., Wu, D. M. & Wu, T. Y. 1982 Ships moving in the transcritical range. *Proc. Conference on Behaviour of Ships in Restricted Waters, Varna*, Vol. II, 26/1–10.

Lee, S. J., Yates, G. T. & Wu, T. Y. 1989 Experiments and analyses of upstream advancing solitary waves generated by moving disturbances. *J. Fluid Mech.* **199**, 569–593.

Robson, B. L. 1988 Systematic series of High Speed Displacement Hull Forms for naval combatants. *Trans. R. Inst. Nav. Archit.* **130**, 241–252.

Stoker, J. J. 1957 *Water Waves.* Interscience.

Thews, J. G. & Landweber, L. 1935 The influence of shallow water on the resistance of a cruiser model. *US Experimental Model Basin, Navy Yard, Washington DC, Rep.* 408.

Tuck, E. O. 1974 One-dimensional flows as slender-body problems, with applications to ships moving in channels. *Proc. Workshop on Slender Body Theory, Ann Arbor, Michigan. University of Michigan Rep.* NAME, vol. 164. pp. 27–35.

Wu, T. Y. 1987 Generation of upstream advancing solitons by moving disturbances. *J. Fluid Mech.* **184**, 75–99.