Resistance, Wave-Making and Wave-Decay of Thin Ships, with Emphasis on the Effects of Viscosity

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Abstract

Three interrelated topics in ship hydrodynamics - resistance, wave-making and wave decay - are investigated in an attempt to improve the accuracy of some simple methods used in the preliminary design of thin ships.

Several published sets of data from classical and recent boundary layer experiments on flat plates are used to estimate boundary layer quantities such as thicknesses and eddy viscosities. These quantities are subsequently used to modify the hull shape and the free-surface boundary condition as a means of including viscous effects on wave-making and ship-wave decay.

A recent technique is used to analyse 161 experimental flat-plate turbulent boundary layer velocity profiles, and a new skin-friction line is derived.

Some practical methods are proposed for the numerical quadrature of integrals arising in thin-ship hydrodynamics. We demonstrate that for some integrals, rapid oscillation, rather than being a hindrance to accurate quadrature, can actually be beneficial if appropriate techniques are employed.

We find that boundary layer displacement thickness effects on wave resistance are very small and can be safely ignored for full-size vessels. On the other hand, the idea of a detachment layer, an indication of where the boundary layer begins to thicken rapidly, is shown to have a significant effect on wave resistance.

A modification to the Kelvin free-surface boundary condition is used as a means of including viscous effects on wave-making. Detailed comparisons of total resistance predictions and experiments are made for three model-size Wigley hulls. It is shown that inclusion of viscous effects smooths out the well-known humps and hollows in the wave resistance curves calculated using Michell's (inviscid) integral.

Predictions of the total resistance of a model Wigley hull using Michell's integral and a simple skin-friction line are shown to be as good as those of a modern CFD computer code. Furthermore, the simple method does so in a very small time on an inexpensive computer.

The effect of employing a form factor on the skin-friction is shown to improve correlations between resistance predictions and experiments. It has recently been proposed that a form factor should also be applied to the wave resistance. We show that good predictions are indeed possible, but that the use of a modified form of Michell's integral and an "appropriate" value of the eddy viscosity leads to even better agreement.

Two existing wave-decay models are examined and a new formulation is suggested that combines the theoretical $-1/2$ decay rate of transverse waves with the $-1/3$ decay rate of diverging waves.

The effects of viscosity on ship-wave decay are considered. It is found that large values of the viscosity, of the order required to have a significant effect on wave resistance, lead to an over-damping of far-field waves at low Froude numbers.

We show that it may be possible to get a rough estimate of the (ambient) eddy viscosity from an analysis of the decay of ship-waves with transverse distance from the sailing line, without resorting to computationally expensive Fourier transform methods. Three wave decay models are used to estimate the eddy viscosity from the behaviour of the wave decay. The model that uses the theoretical decay rates of transverse and of diverging waves is found to be slightly better at recapturing the original eddy viscosity than the other two models.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

SIGNED: .... DATE: ....

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It is with all my admiration and appreciation that I dedicate this thesis to Ernie Tuck.

Notation

ACRONYMS AND ABBREVIATIONS

BL	Boundary Layer
CFD	Computational Fluid Dynamics
FSBC	Free Surface Boundary Condition
ITTC	International Towing Tank Conference
LL08	Convenient name for a friction line derived in the present thesis
N-S	Navier-Stokes
RANS	Reynolds-number Averaged Navier-Stokes
SES	Surface Effect Ship
TBL	Turbulent Boundary Layer

ROMAN SYMBOLS

A	Complex wave amplitude
A_{wp}	Waterplane area
B	Hull beam
B_0	Log law constant in BL calculations
c_f	Planar local skin friction coefficient
C_F	Planar drag coefficient
F_r	Froude number based on ship length
g	Gravitational acceleration
G	Green function
H_m	maximum crest-to-trough (or trough-to-crest) distance
H_{12}	BL shape factor $= \delta^*/\Theta$
k_f	Form factor on skin-friction
k_w	Form factor on wave resistance
k_0	Fundamental wave number, $k_0 = g/U^2$
k_1	x-wise wave number: $k_1 = k_0 \sec\theta$
k_2	$k_2 = k_0 \sec^2\theta$

$\log$	Natural logarithm
$\log_{10}$	Base 10 logarithm
L	Hull length
N_2	Multiplier in estimates of eddy viscosity
p	Pressure
R_n	Reynolds number
R_δ	Reynolds number based on BL thickness
$R_{\delta *}$	Reynolds number based on BL displacement thickness
R_θ	Reynolds number based on BL momentum thickness
R_F	Skin friction
R_T	Total resistance
R_V	Viscous resistance
R_W	Wave resistance
S	Hull wetted surface area
T	Hull draft
Tu	Free-stream turbulence
u, v, w	Velocity components
u_τ	BL friction velocity $u_\tau = \sqrt{\tau/\rho}$
u_d	Defect velocity $U - u/u_\tau$
u^+	Abbreviation for u/u_τ in BL calculations
U	Ship speed
U_∞	Free-stream velocity
x, y, z	Co-ordinates of a point in the wave field
y^+	Abbreviation for yu_τ/ν in BL calculations
z_{range}	Difference between highest and lowest elevations along a wave cut.

GREEK SYMBOLS

α_2	Speed-independent viscosity parameter
α_m,β_m	Constants in modified Winter-Gaudet TBL wake model
α_w,β_w	Constants in Winter-Gaudet TBL wake model
β_2	Viscous parameter in modified FSBC
β_K	Kelvin angle
δ	BL thickness
δ^+	BL thickness (in wall units) aka the Karman number
δ^*	BL displacement thickness
δ_v^+	Distance from wall at which flow becomes fully turbulent
Δ	Rotta-Clauser thickness: $\Delta = \delta^* \sqrt{2/c_f}$
η	BL outer lengthscale: $\eta = y/\delta$

η_m	η-ordinate where φ attains its maximum
η_s	Start of BL wake region
∇	Hull displacement volume
ζ	Wave elevation
θ	Wave propagation angle
Θ	BL momentum thickness
κ	von Karman constant in BL calculations
ν	Molecular kinematic viscosity
ν_t	(Turbulent) eddy kinematic viscosity
Π	Coles' wake strength parameter in BL calculations
τ	Shear stress
ρ	Fluid density
ϕ	Disturbance velocity potential
ϖ	Wave co-ordinate: $\varpi = x\cos\theta + y\sin\theta$
φ	BL wake function
ς	BL Parameter: $\varsigma = U_\infty/u_\tau = \sqrt{2/c_f} = \Delta/\delta^*$