The Development and Stability of some Non-Planar Boundary-Layer Flows

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Contents

Abstract

This thesis presents two problems in the field of fluid mechanics. Both problems concern the flow of a Newtonian viscous fluid in the laminar and early-transitional regimes. Geometrically, they also share the following features: a square corner; a wall boundary layer; and a semi-infinite physical domain.

Part 1 of this thesis, comprising Chapters 2–5, considers the laminar flow parallel to a streamwise corner. In Chapter 2 we present an in-depth study of the laminar flow internal to a square corner. The hydrodynamic stability of this flow is the subject of Chapter 3. For the special case of zero pressure gradient, our analysis suggests a critical Reynolds number of $Re_c \approx 44\ 000$ (based on streamwise distance from the leading edge), indicating that this flow is significantly less stable than the well-known Blasius boundary layer on a semi-infinite flat plate. In Chapter 4 we derive the laminar flow *external* to a square corner. Finally, in Chapter 5 we summarize our findings and offer some recommendations for future research on laminar and transitional corner flows.

Part 2, comprising Chapters 6–10, considers the sudden blockage of steady laminar flow within a circular pipe. Even though the blockage occurs almost instantaneously, the fluid takes an appreciable time to come to rest. Accordingly, Chapter 6 presents a detailed analysis of the laminar-decay process at an arbitrary location upstream of the blockage point. The hydrodynamic stability of this unsteady upstream flow is the subject of Chapters 7 and 8. Chapter 7 uses traditional linear eigenmode theory, originally developed for steady laminar flow, to estimate that the laminar flow is absolutely stable in the event that the pre-blockage Reynolds number does not exceed $Re_c \approx 450$. The linear pseudomode analysis of Chapter 8 yields the substantially lower estimate $Re_c \approx 115$, above which there exists the theoretical possibility of transient growth initiating a 'bypass' transition to turbulence. However, after accounting for the transient nature of the underlying flow itself, we hypothesize a significantly higher threshold $Re_c \approx 1000$ for full breakdown of the laminar structure.

Chapter 9 rounds off the present work by extending the laminar-flow analysis

of Chapter 6 to the immediate vicinity of the blockage point. We present a direct numerical simulation of the complete laminar-decay process within this end-region, highlighting the early-phase development of an unsteady corner boundary layer and the subsequent development of vortices in the interior of the pipe.

The thesis concludes in Chapter 10 by summarizing the findings from Part 2 and suggesting some fruitful directions for future research on unsteady pipe flows.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution, and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Publications

This work is associated with the following refereed publications:

- Jewell, N.D. and Denier, J.P. The instability of the flow in a suddenly blocked pipe. *Quarterly Journal of Mechanics and Applied Mathematics* **59** (4), 651–673 (2006).

- Denier, J.P. and Jewell, N.D. Boundary-layer separation and vortex generation in a suddenly blocked pipe. *Proceedings of the 22nd International Congress of Theoretical and Applied Mechanics (ICTAM), Adelaide* (2008).

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