Londergan, J. Timothy; Murdock, D. P.; Thomas, Anthony William
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Experimental tests of charge symmetry violation in parton distributions

J. T. Londergan*
Department of Physics and Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405, USA

D. P. Murdock†
Department of Physics, Tennessee Technological University, Cookeville, Tennessee 38505, USA

A. W. Thomas‡
Jefferson Lab, 12000 Jefferson Ave., Newport News, Virginia 23606, USA

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Recently, a global phenomenological fit to high energy data has included charge symmetry breaking terms, leading to limits on the allowed magnitude of such effects. We discuss two possible experiments that could search for isospin violation in valence parton distributions. We show that, given the magnitude of charge symmetry violation consistent with existing global data, such experiments might expect to see effects at a level of several percent. Alternatively, such experiments could significantly decrease the upper limits on isospin violation in parton distributions.

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I. INTRODUCTION

Charge symmetry is a particular form of isospin invariance that involves a rotation of 180° about the "2" axis in isospin space. For parton distributions, charge symmetry involves interchanging up and down quarks while simultaneously interchanging protons and neutrons. In nuclear physics, charge symmetry is an extremely well respected symmetry, generally obeyed at the level of a fraction of a percent [1,2]. Since charge symmetry is so well satisfied at lower energies, it is natural to assume that it holds for parton distribution functions (PDFs). Furthermore, there is no direct experimental evidence that charge symmetry is violated in PDFs. Recently, the experimental upper limits on parton charge symmetry violation (CSV) have been improved, by comparing the $F_2$ structure functions for deep inelastic scattering induced by muons and neutrinos [3–6]. Until now, all phenomenological PDFs have assumed the validity of charge symmetry. But a recent global fit of PDFs by Martin et al. [7] included for the first time the possibility of charge symmetry violating PDFs for both valence and sea quarks. This provides us with particle distribution functions that agree with all of the experimental information used to obtain global fits to PDFs and which incorporate isospin violation.

The global fit of Martin et al. assumed a particular form for the charge symmetry violating PDFs, for both valence and sea quark CSV. In Sect. II, we will review the form for the CSV terms used by Martin, Roberts, Stirling, and Thorne (MRST), and we will discuss some of the features of their amplitudes. The allowed variation in the CSV terms is rather large. Consequently, there may be possibilities either to measure the magnitude of parton CSV distributions, or to provide more strict experimental limits on these quantities, with dedicated experiments that focus on specific observables. We will review two such possibilities in this paper, both of which are sensitive to valence quark CSV.

One promising approach would be to measure Drell-Yan cross sections induced by charged pions on an isoscalar target, for which the simplest would be the deuteron. In Sect. III, we review the possibilities for such measurements. We describe the observable which would be most sensitive to valence parton CSV distributions, and we use existing parton distributions, plus the CSV terms of MRST, to show potential variations in these observables which correspond to the current limits on CSV obtained in the MRST global fit.

A second possibility is through semi-inclusive deep inelastic scattering for electrons on isoscalar targets. In Sect. IV, we describe the relevant observable and show the magnitude of the effects which one must measure in order to establish charge symmetry violation in parton distribution functions, or to provide more strict upper limits on CSV terms.

We then review the information that could be extracted from these experiments, and the prospects for such measurements.

II. PHENOMENOLOGICAL CHARGE SYMMETRY VIOLATING PDFS

Because CSV effects are extremely small at nuclear physics energy scales [1,2], it is natural to assume that parton distribution functions obey charge symmetry. Furthermore, there is no direct evidence for violation of parton charge symmetry [8], even though the existing direct upper limits on parton charge symmetry violation

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*Electronic address: tlonderg@indiana.edu
†Electronic address: murdock@tntech.edu
‡Electronic address: awthomas@jlab.org

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are at roughly the 5–10% level [3]. Martin, Roberts, Stirling, and Thorne [7] have studied the uncertainties in parton distributions arising from a number of factors, including for the first time isospin violation.

Charge symmetry violating PDFs involve the difference between, say, the down quark PDF in the proton and the up quark in the neutron; thus we define

\[ \delta d_i(x) = d_i^q(x) - u_i^q(x), \quad \delta u_i(x) = u_i^q(x) - d_i^q(x), \]

with analogous relations for antiquarks. Now, from valence quark normalization, the first moment of the valence quark CSV PDFs must vanish, i.e.,

\[ \int_0^1 dx \delta d_i(x) = \int_0^1 dx [d_i^q(x) - u_i^q(x)] = 0, \]

with an analogous relation for the first moment of \( \delta u_i(x) \).

The MRST group chose a specific model for valence quark charge symmetry violating PDFs. They constructed a function that automatically satisfied the quark normalization condition of Eq. (2), namely:

\[ \delta u_i(x) = -\delta d_i(x) = \kappa f(x), \]

\[ f(x) = (1 - x)^4 x^{-0.5} (x - 0.0909). \]

The function \( f(x) \) in Eq. (3) was chosen so that at both small and large \( x \), \( f(x) \) has a form similar to the MRST2001 valence quark distributions [9], and the first moment of \( f(x) \) is zero. The functional form of the valence CSV distributions guaranteed that \( \delta u_i \) and \( \delta d_i \) would have opposite signs at large \( x \), in agreement with theoretical models for parton CSV [10,11]. Fixing the form of the CSV parton distribution leaves undetermined only the overall coefficient \( \kappa \), which was then varied in a global fit to a wide range of high energy data.

The value of \( \kappa \) which minimized the \( \chi^2 \) in the MRST global fit was \( \kappa = -0.2 \). The MRST \( \chi^2 \) vs \( \kappa \) is shown as the bottom curve in Fig. 1. Clearly \( \chi^2 \) has a shallow minimum with the 90% confidence level obtained for the range \(-0.8 \leq \kappa \leq -0.65 \). The figure to the left in Fig. 2 plots the valence quark CSV PDFs corresponding to the MRST best-fit value, \( \kappa = -0.2 \). The best-fit phenomenological valence CSV PDFs look extremely similar to the CSV PDFs calculated by Rodionov et al. [3,11], who implemented charge symmetry violation in simple quark models; this is shown as the figure to the right in Fig. 2. Note, however, that within the 90% confidence region for the MRST global fit, the valence quark CSV could be up to 4 times as large as that predicted by Rodionov et al. and it could even have the opposite sign.

The magnitude of the allowed CSV effects obtained by MRST is consistent with uncertainties in phenomenological PDFs. The total momentum carried by valence quarks is determined to within about 2%. Inclusion of a valence quark CSV term changes the momentum carried by valence quarks in the neutron from those in the proton. The total momentum carried by valence quarks is given by the second moment of the distribution, e.g., the momentum carried by up valence quarks in the neutron

\[ U^v = \int_0^1 x u^v_i(x) dx. \]

The 90% confidence limit in the valence quark CSV terms corresponds to a variation of roughly 2% of the momentum carried by valence quarks in the neutron, or just about the known experimental uncertainty in this quantity.

At the level allowed by MRST, isospin-violating PDFs are sufficiently large that, by themselves, they could account for the entire anomaly in the Weinberg angle suggested by the NuTeV experiment [12–15]. At present, this is the only single effect that appears capable of removing 100% of the NuTeV anomaly [16]; consequently, it is quite important that one be able to test the magnitude of parton isospin violation.
The MRST group also searched for the presence of charge symmetry violation in the sea quark sector. Again, they chose a specific form for sea quark CSV, dependent on a single parameter,

\[ \bar{u}(x) = \bar{d}(x)[1 + \delta], \quad \bar{d}(x) = \bar{u}(x)[1 - \delta]. \]  

With the form chosen, the total momentum carried by antiquarks in the neutron and proton are approximately equal.

Perhaps surprisingly, evidence for sea quark CSV in the global fit was substantially stronger than that for valence quark CSV. As shown in the top curve in Fig. 1, the best fit is obtained for \( \delta = 0.08 \), corresponding to an 8% violation of charge symmetry in the nucleon sea. The \( \chi^2 \) corresponding to this value is substantially better than with no charge symmetry violation, primarily because of the improvement in the fit to the NMC \( \mu - D \) deep inelastic scattering (DIS) data [5,6] when \( \bar{u} \) is increased. The fit to the E605 Drell-Yan data [17] is also substantially improved by the sea quark CSV term.

As explained in the following sections, we have used the MRST CSV parton distributions to calculate the differences one could expect in observables for pion-induced Drell-Yan processes, and semi-inclusive charged-pion production in electron-deuteron deep inelastic scattering.

III. PION-INDUCED DRELL-YAN PROCESSES AND PARTON CHARGE SYMMETRY

In Drell-Yan (DY) processes [18], two hadrons collide at high energies, and a quark in one hadron annihilates an antiquark of the same flavor in the other hadron, producing a virtual photon which subsequently radiates a pair of muons with opposite sign. This is shown schematically in Fig. 3, for \( NN \) and \( \pi N \) DY processes.

Since the PDFs for nucleons are rather accurately known, DY processes for pions on nucleons provide a sensitive way of extracting parton distributions in the pion. For example, the NA10 experiment at CERN [19] and experiment E615 at FermiLab [20] both studied Drell-Yan reactions induced by \( \pi^- \). Such reactions could be studied using the Main Injector at FermiLab to produce high energy protons that are scattered from a nuclear target to produce charged pions, and subsequently observing Drell-Yan reactions for these pions on an isoscalar target, which will assume is the deuteron. If this program were carried out, then from the DY reaction the pion PDFs could be very accurately determined at this value of \( Q^2 \). We will show that by comparing DY cross sections for \( \pi^+ \) and \( \pi^- \) on the deuteron, one can extract the CSV violating part of the nucleon valence PDFs—alternatively, through such reactions one could produce considerably stronger upper limits on the parton CSV terms. We use the nucleon CSV distributions recently extracted by MRST to indicate the range of CSV that could be probed in such experiments.

To test nucleon valence quark CSV, it is necessary to measure \( \pi^- - D \) DY processes at kinematics corresponding to large \( x \) for both pion and nucleon, e.g., \( x, x_e \gtrsim 0.3 \). In this region, where it is a reasonable first approximation to neglect sea quark effects, the Drell-Yan process will predominantly occur when a valence quark in the deuteron is annihilated by a valence antiquark in the pion. Then the lowest order (LO) DY cross sections for \( \pi^+ - D \) and

\[ \gamma \]

\[ q_i \]

\[ N \]

\[ q_j \]

\[ u \]

\[ d \]

\[ \mu^+ \]

\[ \mu^- \]

\[ \pi^+ \]

\[ N \]

\[ \mu^+ \]

\[ \mu^- \]

\[ \gamma \]

\[ q_i \]

\[ q_j \]

\[ u \]

\[ d \]

\[ \pi^+ \]

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\[ \pi^+ \]

\[ N \]

\[ \mu^+ \]

\[ \mu^- \]

\[ \gamma \]

\[ q_i \]

\[ q_j \]

\[ u \]

\[ d \]
\[ \pi^- - D \] have the approximate form

\[ \sigma_{\pi^- D}^{DY} = \frac{1}{9} [ d(x) + d(x) ] \bar{d}^+(x), \]

\[ \sigma_{\pi^- D}^{DY} = \frac{4}{9} [ u(x) + u(x) ] \bar{u}^+(x). \] (5)

Now, from charge conjugation invariance and the assumption of charge symmetry for pion PDFs, we can write the relations

\[ \pi_v(x) = u_v^+(x) = \bar{d}^+(x) = d_v^+(x) = \bar{u}^+(x), \]

\[ \pi_s(x) = q_s^+(x) = \bar{q}_s^+(x) = q_v^+(x) = \bar{q}_v^+(x), \]

\[ [q = u, d], \]

\[ \bar{\pi}_v(x) = s_v^+(x) = \bar{s}^+(x). \] (6)

Inserting the results of Eq. (6) into Eq. (5), we see that in the valence-dominated region for both pion and deuteron, the \( \pi^- - D \) DY cross section should be 4 times the \( \pi^- - D \) term. Thus, in the quantity

\[ 4\sigma_{\pi^- D}^{DY} - \sigma_{\pi^- D}^{DY}, \]

the valence-valence contributions will cancel. The remaining terms will contain sea-valence interference terms and charge symmetry violating terms for both nucleon and pion. Introducing the nucleon CSV terms into the PDFs, the DY cross section for \( \pi^- - D \) has the form

\[ 9\sigma_{\pi^- D}^{DY}(x, x) = \pi_v(x) [ d_v(x) + u_v(x) - \delta u_v(x) \]

\[ + 5(\bar{u}(x) + \bar{d}(x)) - \delta \bar{u}(x) - 4\delta \bar{d}(x)] \]

\[ + \pi_v(x) [ d_v(x) + u_v(x) ] + 10(\bar{u}(x) \]

\[ + \bar{d}(x) - \delta u_v(x) - 4\delta \bar{d}(x) - 2\delta \bar{d}(x) \]

\[ - 8\delta \bar{d}(x)] + 2\bar{\pi}_v(x) [ s_v(x) + \bar{s}(x)]. \] (8)

in Eq. (8) we have also introduced the charge symmetry violating PDFs from Eq. (1).

As proposed by Londergan et al. [21], once the DY cross sections for \( \pi^- \) and \( \pi^+ \) on deuteron are measured, one can extract values for the pion valence and sea quark parton distributions. Using those distributions, one can then focus on the region of large \( x \) for both pion and nucleon. In this region, one can construct ratios of the DY cross sections, for example

\[ R_{\pi^- D}^{DY}(x, x) = \frac{4\sigma_{\pi^- D}^{DY} - \sigma_{\pi^- D}^{DY}}{\sigma_{\pi^- D}^{DY} - \sigma_{\pi^- D}^{DY}}. \] (9)

As we will see, in order to extract CSV terms in this ratio, the large quantities in the two terms in the numerator of Eq. (9) will very nearly cancel. This requires that one be able to obtain very accurate relative normalization of DY cross sections for charged pions. This can be achieved by normalizing the charged-pion cross sections to the \( J/\psi \) peak.

Inserting the DY cross sections from Eq. (8) (and the analogous \( \pi^- - D \) term), one obtains

\[ R_{\pi^- D}^{DY}(x, x) = \frac{5\pi_v(x)}{\pi_v(x)} + C(x, x)[R_{CS}(x) + R_{SV}(x)] + R_1(x, x), \]

\[ R_{CS}(x) = \frac{4(\delta d_v(x) - \delta u_v(x))}{3q_v(x)}, \]

\[ R_{SV}(x) = \frac{5(\bar{u}(x) + \bar{d}(x)) - \delta \bar{u}(x) - 4\delta \bar{d}(x)}{q_v(x)}. \]

\[ C(x, x) = \left( 1 + \frac{2\pi_v(x)}{\pi_v(x)} \right), \]

\[ q_v(x) = u_v(x) + d_v(x). \] (10)

Equation (10) is calculated to lowest order in small terms. With the exception of the CSV terms, all of the parton distribution functions should be known (the pion PDFs at that \( Q^2 \) could be extracted from the same Drell-Yan experiment). Our proposal is that one construct the ratio \( \lambda_{QM}^{DY} \) and measure its \( x \) dependence for fixed \( x, x \geq 0.3 \). In this case the first term in Eq. (10) is a constant, and that constant will decrease as \( x \) increases. From the known nucleon and pion valence PDFs, one can predict the value of this ratio assuming charge symmetry. Deviations of the ratio from this prediction would be evidence for CSV in nucleon parton distributions. The term \( R_{CS}(x) \) in Eq. (10) contains contributions from valence quark CSV. Although the CSV term is small, with increasing \( x \) it will become a progressively larger fraction of the ratio, since all other terms are proportional to sea quark distributions that fall rapidly with increasing \( x \).

We used phenomenological parton distributions for nucleons and pions, that could be evolved to the \( Q^2 \) region of interest for Drell-Yan processes. To obtain their CSV parton distributions, MRST started with the MRST2001 set of PDFs and varied these using a global fit, to obtain the best distributions including charge symmetry violation. The resulting PDFs differ slightly from the best-fit MRST2001 distributions [22]. The value of \( \Lambda_{QCD} \) and the gluon distributions are essentially identical to those in the MRST2001 set [9]. The input parameters for the nucleon PDFs, allowing for valence quark CSV, are

\[ xu_v(x) = 0.129x^{0.223}(1 - x)^{3.31}[1 + 4.89x^{0.5} + 69.86x], \]

\[ xd_v(x) = 0.0163x^{0.241}(1 - x)^{3.75}[1 + 123.6x^{0.5} + 76.04x]. \]

\[ xS(x) = 0.215x^{-0.269}(1 - x)^{3.77}[1 + 3.34x^{0.5} + 11.80x]. \] (11)

MRST define the quark sea from the function \( S(x) \) using the relations,
\[ \tilde{u}(x), \tilde{d}(x), \tilde{s}(x) = 0.2S - 0.5\Delta, \quad 0.2S + 0.5\Delta, \quad 0.1S, \]
\[ x\Delta(x) = x(\tilde{d}(x) - \tilde{u}(x)) = 1.195x^{1.24}(1 - x)^{0.10}[1 + 14.05x - 45.52x^2]. \]

(12)

The starting scale for these PDFs is \( Q_0^2 = 1 \text{ GeV}^2 \), and MRST provide interpolation matrices that allow one to evolve these PDFs to higher \( Q^2 \). For the valence quark CSV we used the MRST form of Eq. (3), and we varied the overall parameter \( \kappa \) between the range \(-0.8 \) and \(+0.65 \) corresponding to the 90\% confidence limit obtained by MRST. We have used the same MRST CSV distributions given in Eqs. (3) and (4) at all \( Q^2 \), in agreement with MRST, who did not include any \( Q^2 \) dependence in the CSV terms in their global fits.

In the MRST global fits, the valence and sea quark CSV were varied separately, and slightly different minima were obtained for the quark PDFs in these two cases. Thus there is some inconsistency in our using sea quark CSV terms with the quark PDFs appropriate for the valence CSV global fit. However, the contribution of sea quark CSV to the ratio \( F_{\pi D}^{DY} \) of Eq. (9) is extremely small since we restrict our attention to the region where both \( x \) and \( x_\pi \) are large. In this region the inconsistency in our procedure has only a very small effect.

For the pion PDFs we took the parton distributions of Sutton et al. [23], extracted from pion Drell-Yan and prompt photon experiments [19,20,24]. The Sutton analysis obtained different pion PDFs, depending upon the amount of the pion momentum assumed to be carried by the sea. We used those PDFs that correspond to the sea carrying 10\% of the pion momentum. These PDFs are defined for a starting scale \( Q_0^2 = 4 \text{ GeV}^2 \) and can be evolved upwards using interpolation matrices provided by the Durham group [25]. For an actual \( \pi - D \) Drell-Yan experiment, the pion PDFs would be extracted directly from the Drell-Yan data at that \( Q^2 \).

We have plotted the quantity \( F_{\pi D}^{DY} \), defined in Eq. (10) as a function of \( x \), for fixed \( x_\pi \), at \( Q^2 = 25 \text{ GeV}^2 \), which is a representative value of \( Q^2 \) that could be obtained in fixed-target Drell-Yan experiments with charged pions at Fermilab. In fixed-target experiments the detector configurations preferentially detect results corresponding to large \( |x_F| = |x - x_\pi| \). The top graph in Fig. 4 plots \( F_{\pi D}^{DY} \) vs \( x \) for \( x_\pi = 0.4 \), while the bottom graph shows the same quantity for \( x_\pi = 0.8 \).

The solid curve in Fig. 4 corresponds to no CSV contribution; the dashed and dotted curves include the limits of the phenomenological CSV contributions at the 90\% confidence level as extracted by the MRST group. The nucleon PDFs are extracted from global fits, and the pion PDFs would have previously been measured in this DY experiment. The difference between the solid curve and the two dashed curves gives the magnitude of CSV effects (almost entirely nucleon valence CSV) allowed by the MRST phenomenological fit. Note that at reasonable kinematic values, the differences between the CSV contributions are substantial. At \( x = 0.4 \) the two extremes of the CSV contributions are 30–50\% smaller or larger than the value corresponding to no CSV contribution, while for \( x \sim 0.7 \) the difference between the terms is more like a factor of 2.

The magnitude of the solid curve gives the precision necessary in the DY measurements in order to extract meaningful information. For \( x_\pi = 0.4 \), the two DY cross sections are predicted to cancel to within about 10\%. Therefore the charged-pion DY cross sections need to be measured to a few percent. For \( x_\pi = 0.8 \) and \( x \sim 0.7 \), the two DY cross sections cancel almost completely with the pion PDFs used in these calculations. However, the CSV contributions at the 90\% level would allow values between roughly \(-0.2 \) and \(+0.2 \), so measurements of DY cross sections at the few percent level would allow one to discriminate between these limits.

There is an additional contribution to the Drell-Yan ratio of Eq. (10) arising from the possibility of charge symmetry violation in the pion PDFs. The largest contribution would arise from differences between the valence\( \tilde{d} \) in the \( \pi^+ \) and the \( \tilde{u} \) in the \( \pi^- \). The additional contribution to the DY ratio...
has the form
\[
\delta R_{\pi \pi D}^{N}(x, x) = \frac{4(\hat{d}_{s}^\pi(x) - \hat{u}_{s}^\pi(x))}{3P(x)}.
\] (13)

The contribution from pion CSV is a function only of \(x_\pi\). For fixed \(x_\pi\), it will contribute a constant amount to the DY ratio. This was estimated by Londergan et al. [21], by taking quark mass effects into account in a Nambu-Jona Lasinio model that was used to calculate pion valence quark distributions. The charge symmetry violating pion PDFs were calculated at a small value of \(Q^2\) and then evaluated at higher \(Q^2\) values using Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution [26–28].

The predicted effects from pion CSV were quite small. For \(x_\pi \approx 0.4\), the effects were almost zero, while for \(x_\pi \approx 0.8\) the contribution from Eq. (13) was less than 0.01 [21]. This term is added to all of the curves in Fig. 4, and increases slightly the uncertainty in nucleon CSV that can be extracted from these ratios.

### IV. SEMI-INCLUSIVE ELECTRON-DEUTERON REACTIONS AND PARTON CHARGE SYMMETRY VIOLATION

Another possibility to measure charge symmetry violating terms in parton distributions arises in measurements of semi-inclusive charged-pion production from lepton DIS on isoscalar nuclear targets. Considering semi-inclusive electroproduction of a hadron \(h\) from a nucleon \(N\), \(e^+ N \rightarrow h + X\), the yield of hadron \(h\) in such processes is given by
\[
N^{Nh}(x, z) = \sum_{i} e_{i}^{N} q_{i}^{N}(x) D_{i}^{h}(z).
\] (14)

In Eq. (14), \(N^{Nh}(x, z)\) is the yield of hadron \(h\) from the semi-inclusive DIS (SIDIS) electroproduction on nucleon \(N\), \(q_{i}^{N}(x)\) is the parton distribution for flavor \(i\) in the nucleon. Similarly \(D_{i}^{h}(z)\) is the fragmentation function for a quark of flavor \(i\) to fragment into hadron \(h\), where \(z\) is the fraction of the hadron energy carried by the quark. Semi-inclusive production of a charged hadron from a proton thus has the form
\[
N^{p\pi^\pm}(x, z) = \frac{4}{9} [u^{p}(x)D_{u}^{\pi^\pm}(z) + \bar{u}^{p}(x)D_{u}^{\pi^\pm}(z)] + \frac{1}{9} [d^{p}(x)D_{d}^{\pi^\pm}(z) + \bar{d}^{p}(x)D_{d}^{\pi^\pm}(z)] + s^{p}(x)D_{s}^{\pi^\pm}(z) + \bar{s}^{p}(x)D_{s}^{\pi^\pm}(z).
\] (15)

Charge conjugation invariance requires that
\[
D_{u}^{\pi^\pm}(z) = D_{u}^{\pi^\pm}(z), \quad D_{d}^{\pi^\pm}(z) = D_{d}^{\pi^\pm}(z).
\] (16)

The above equations hold for electroproduction of any charged hadrons. In the remaining discussion we look specifically at electroproduction of charged pions. If we assume charge symmetry for fragmentation functions we have the additional relations
\[
D_{u}^{\pi^+}(z) = D_{u}^{\pi^+}(z), \quad D_{d}^{\pi^+}(z) = D_{d}^{\pi^+}(z).
\] (17)

Thus, under the assumption of charge symmetry, the fragmentation of light quarks to charged pions can be written in terms of only two independent fragmentation functions, which are defined as “favored” or “unfavored” depending upon whether the quark that produces a hadron exists in the valence configurations of that hadron. Thus for up quarks \(D_{u}^{\pi^+}\) is favored while \(D_{u}^{\pi^+}\) is unfavored.

From Eq. (14), the yields depend on favored and unfavored fragmentation functions, as well as the fragmentation of strange quarks to pions. Levelt, Mulders, and Schreiber [29] derived an expression for measuring the ratio of fragmentation functions. They showed that
\[
\frac{\langle N^{p\pi^+}(x, z) - N^{n\pi^+}(x, z) + N^{p\pi^-}(x, z) - N^{n\pi^-}(x, z) \rangle}{\langle N^{p\pi^+}(x, z) - N^{n\pi^+}(x, z) - N^{p\pi^-}(x, z) + N^{n\pi^-}(x, z) \rangle} = \frac{9S_{G}}{5} \frac{\Delta(z) + 1}{\Delta(z) - 1}.
\] (18)

In Eq. (18), the brackets denote integration of both numerator and denominator over \(x\), the quantity \(\Delta(z)\) is the unfavored/favored ratio
\[
\Delta(z) = \frac{D_{u}^{\pi^+}(z)}{D_{u}^{\pi^+}(z)},
\] (19)

and \(S_{G}\) is the Gottfried sum rule [30]. Thus the semi-inclusive electroproduction of charged pions in \(e - D\) reactions can be used to extract the unfavored/favored ratio of fragmentation functions of quarks to pions. Londergan et al. [31] showed that these semi-inclusive reactions could also be used to investigate the presence of charge symmetry violation in nucleon parton distributions. This follows from the fact that, at large \(x\) and assuming parton charge symmetry, the favored production of charged pions from valence quarks obeys the relation
\[
N^{D\pi^+}(x, z) = 4N^{D\pi^+}(x, z).
\] (20)

Consequently, they proposed measuring the ratio
\[
R(x, z) = \frac{4N^{D\pi^+}(x, z) - N^{D\pi^-}(x, z)}{N^{D\pi^+}(x, z) - N^{D\pi^-}(x, z)}.
\] (21)

It is convenient to define the quantity
\[
R^{D}(x, z) = \frac{1 - \Delta(z)}{1 + \Delta(z)} R(x, z).
\] (22)

Note that the overall \(z\)-dependent factor in Eq. (22) is just the factor that is extracted using the Levelt et al. ratio defined in Eq. (18). To lowest order in small quantities it can be shown that
\[ R^D(x, z) = R^D_C(z) + R^S_C(x) + R^S_V(x) + R^P(x, z), \]
\[ R^D_C(z) = \frac{5\Delta(z)}{1 + \Delta(z)}, \]
\[ R^S_C(x) = \frac{D^+_{\pi^-}(z) + D^-_{\pi^+}(z)(s(x) + \bar{s}(x))}{D^-_{\pi^+}(z)(1 + \Delta(z))} q_s(x). \]

The quantities \( R^S_C(x), R^S_V(x), \) and \( q_s(x) \) are those defined in Eq. (10). Thus the quantity \( R^D(x, z) \) can be divided into three parts: a term that depends only on \( z \); a term that contains the parton CSV contribution, and that depends only on \( x \); and terms that contain contributions from sea quarks. The term \( R^D_C(z) \) depends on the unfavored/favored parton CSV contribution, which can be accurately measured in these semi-inclusive reactions using the method proposed by Levelt et al. (there is a contribution from charge symmetry violation in the fragmentation functions; this term was discussed in Ref. [31]. It is expected to be small, and it depends only on \( z \). In order to isolate the CSV contribution, one should examine the ratio \( R^D(x, z) \) of Eqs. (21) and (22) as a function of \( x \) at fixed \( z \). The CSV term is expected to peak at \( x \sim 0.35 \) (see Fig. 2), and the sea quark contributions decrease very rapidly at large \( x \), it is possible that at fixed \( z \) and sufficiently large \( x \), the CSV terms will be substantial, and might even dominate.

For our calculations we used the fragmentation functions extracted by Kretzer, Leader, and Christova [32]. They used the information on \( e^+e^- \) production on charged pions at the \( Z^0 \) peak [33], together with the HERMES data on SIDIS charged-pion production [34]. It turns out that the \( e^+e^- \) data at the \( Z^0 \) peak essentially fixes the combination
\[ D^\pi^- = 2[D^+_{\pi^+} + D^-_{\pi^-} + D^+_{\pi^-}]. \]

This data, combined with the HERMES SIDIS data, allowed Kretzer et al. to extract the individual fragmentation functions, despite the fact that the \( Z^0 \) data need to be evolved over a great distance to match up with the HERMES data. They obtained the relations
\[ D^+_{\pi^+}(z) = 0.217z^{-1.805}(1 - z)^{2.037}, \]
\[ D^-_{\pi^-}(z) = 0.164z^{-1.927}(1 - z)^{2.886}. \]

The fragmentation functions in Eq. (25) are appropriate for an average \( \langle Q^2 \rangle = 2.5 \text{ GeV}^2 \). In this process, the fragmentation functions \( D^+_{\pi^+} \) and \( D^-_{\pi^-} \) are quite accurately determined, while the strange fragmentation function is determined to within about a factor of 2.

In Fig. 5, we plot the ratios of fragmentation functions that enter into Eq. (23). The solid curve is \( R^D_C(z) \), while the dashed curve is the \( z \)-dependent factor in the term \( R^S_C(x, z) \). In Fig. 6, we plot the contributions to the quantity \( R^D(x, z) \) vs \( x \) for fixed \( z = 0.4 \). At this value, \( R^D_C(z) \sim 1.4 \). The solid curve corresponds to the case with no parton CSV contribution (this includes the sea and strange quark contributions). The dashed and dotted curves include valence parton CSV contributions corresponding to the MRST phenomenological PDFs with \( x = -0.8 \) and \( +0.65 \), respectively. These values demarcate the 90% confidence limits using the MRST valence quark CSV function of Eq. (3). The dot-dashed curve is the strange contribution \( R^D(x, z) \) calculated at \( z = 0.4 \). Except for extremely small values of \( x \), the strange quark contribution is negligible. Hence the factor 2 uncertainty in the strange fragmentation function does not affect this ratio.

For \( x \gtrsim 0.4 \), the contributions from charge symmetry violating PDFs are substantial, and they rapidly become

\[ \begin{align*}
R^D_C(z) & = \frac{5\Delta(z)}{1 + \Delta(z)}, \\
R^S_C(x) & = \frac{D^+_{\pi^-}(z) + D^-_{\pi^+}(z)(s(x) + \bar{s}(x))}{D^-_{\pi^+}(z)(1 + \Delta(z))} q_s(x). 
\end{align*} \]
the dominant contribution at larger $x$. Thus, at the levels determined by the MRST global fit, it would appear that precise measurements of charged-pion production in semi-inclusive DIS electroproduction reactions on deuterium have the possibility of observing these isospin-violating effects. At the very least they would be able to lower the current allowed limits on partonic CSV effects.

Note that the expected effects shown in Fig. 6 are small. They would require extremely precise measurements, in order to be able to distinguish between various predictions for valence quark CSV terms. In addition, Eq. (23) shows that the $x$-dependent terms plotted in Fig. 6 sit atop an $x$-independent term. For $z = 0.4$ this term has a value of approximately 1.4; hence this term is much larger than the CSV term one wants to extract. This difficulty could be overcome, provided that one can measure very accurate values for the fragmentation functions. In this case one can construct the ratio

$$R^A(x, z) = \frac{N^{D^*} \pi^- (x, z) - N^{D^*} \pi^+ (x, z)}{N^{D^*} \pi^+ (x, z) - N^{D^*} \pi^- (x, z)}. \quad (26)$$

It is straightforward to show that

$$R^A(x, z) = C^A(z) [R_{CS}(x) + R_{SV}(x) + R_s^D(x, z)],$$

$$C^A(z) = \frac{8(1 + \Delta(z))}{(1 + 4\Delta(z))(4 + \Delta(z))}. \quad (27)$$

Equation (26) has the advantage that it eliminates the large $z$-dependent term in Eq. (23). However, uncertainties in the favored/unfavored fragmentation ratio now play an important role in determining the uncertainty associated with extracting the charge symmetry violating PDFs. The term $C^A(z)$ in Eq. (27) is plotted as the dot-dashed curve in Fig. 5, at $Q^2 = 2.5$ GeV$^2$. It is normalized so that $C^A \sim 1$ for moderate values of $z$.

Using either of the ratios defined in Eqs. (23) or (26), in order to test for CSV terms in electron induced SIDIS reactions it is essential that one be able to vary $x$ while keeping $z$ constant. Furthermore, the ability to write the ratio as the sum of terms in $x$ and $z$ requires that the cross section factorize as in Eq. (14). Thus the data must be taken at sufficiently high energies that factorization is valid to within a few percent. SIDIS charged-pion experiments have been carried out at HERMES. Another possibility is measurements of $e + D \rightarrow \pi^\pm + X$ at Jefferson Laboratory. However, 6 GeV is most probably too low an energy for factorization to be valid, and even 12 GeV may not suffice. It will be necessary to perform checks of the validity of the factorization hypothesis. If an electron-ion collider were built, these experiments could be carried out at sufficiently high energies.

The CSV effects shown in Fig. 6 are slightly underestimated. The relative magnitude of CSV and sea quark terms is the same for the Drell-Yan ratio and electroproduction, as can be seen by comparison of Eqs. (10) and (23). The DY cross sections are evaluated at much larger $Q^2$ values; at these values the sea quark distribution is shifted to significantly smaller $x$. However, we have not evolved the CSV distributions, in agreement with the procedure used by MRST in their global fit. If we evolved the CSV distributions in $Q^2$, this would increase the CSV contributions relative to the sea in Fig. 6. Finally, our results have been derived in lowest order QCD; it is important to check how these results change in next to leading order (NLO).

V. CONCLUSIONS

The MRST phenomenological global fit to parton distribution functions provides limits on the magnitude of isospin-violating PDFs. The CSV effects allowed by the MRST fit are substantially larger than theoretical predictions of charge symmetry violation in parton distributions [10,11]. In this paper, we have analyzed two dedicated experiments that might detect parton CSV effects. We used the range of values allowed by MRST to assess the magnitude of effects that might be expected in Drell-Yan reactions induced by charged pions on the deuteron, and in semi-inclusive production of charged pions in electron-deuteron deep inelastic scattering.

In the Drell-Yan reaction one compares the cross sections for $\pi^+ - D$ and $\pi^- - D$ reactions at reasonably large values of $x$ for both pion and nucleon. Using parton distributions for both nucleon and pion, we predict quite large values for this ratio, depending on the sign and magnitude of the CSV distributions.

In the electron SIDIS measurements, once again one compares rates for production of $\pi^+$ and $\pi^-$ in $\nu$ reactions at reasonably large $Q^2$. In order to extract CSV terms in SIDIS reactions, it is necessary to make very precise measurements of both the cross sections and fragmentation functions. It is essential that the factorization hypothesis for the cross section be valid to a few percent. Nevertheless, given the importance of these quantities and their relevance in experiments like the NuTeV neutrino cross sections, it is of great interest now to investigate this issue experimentally.

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EXPERIMENTAL TESTS OF CHARGE SYMMETRY


R. S. Thorne, private communication.


The URL for the Durham parton distributions is http://www-spires.dur.ac.uk/hepdata/mrs.html.


