

**Seismic Wave Propagation and
Modelling in Poro-Elastic Media
with Mesoscopic Inhomogeneities**

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Abstract

Biot's theory when applied to homogeneous media (involving the macroscopic flow mechanism) cannot explain the high level of attenuation observed in natural porous media over the seismic frequency range. However, several successful mesoscopic inhomogeneity models have been developed to account for P wave attenuation. In this thesis I further develop the approaches to tackle S wave velocity and attenuation, to simulate transient wave propagation in poroelastic media, and to construct new models for determining the effective parameters of porous media containing mesoscopic inhomogeneities.

As an important application of the double-porosity dual-permeability (DPDP) model, I have reformulated the effective Biot model using the total-field variables. This gives rise to new and more general governing equations than the previous approach based on the host phase field variables (which become a special case of the more general treatment).

The analytical transient solution and dispersion characteristics for the double-porosity model and also for a poro-viscoacoustic model are derived over the entire frequency range for a homogeneous medium. The comparison between the results of the two models shows that dissipation by local mesoscopic flow of the double porosity model is very hard to fit by a single Zener element over a broad band. I chose the relaxation function to approximate the dispersion behaviour of the double porosity model just around the source centre frequency. It is shown that for most water-filled sandstones having a double porosity structure, wave propagation can be well described by the poro-viscoacoustic model with a single Zener element in the seismic frequency range. The transient solution for heterogeneous double porosity media is obtained by a numerical pseudospectral time splitting technique. This method is extended to 2.5-D poro-viscoelastic media to capture both P and S wave behaviour. I also demonstrate that if the frequency is below several Hz, then a single Kelvin-Voigt element gives an even better result than a single Zener element.

I propose a two-phase permeability spherical inclusion model and obtain the dispersion

curves of phase velocity and dissipation factor for the composite. I then determine the effective dynamic permeability of porous media with mesoscopic heterogeneities over the whole frequency range. This result is used to check the validity of other measures of effective dynamic permeability, deduced from the effective hydraulic permeability by replacing the permeability of the components with their dynamic values as determined from the Johnson, Koplik and Dashen (JKD) model.

I also investigate the scattering of plane transverse waves by a spherical porous inclusion embedded in an infinite poroelastic medium. The vector displacement wave equations of Biot's theory are solved as an infinite series of vector spherical harmonics for the case of a plane S-wave incidence. Then, the non-self-consistent theory is used to derive the dispersion characteristics of shear wave velocity and attenuation for a porous rock having mesoscopic spherical inclusions which are designed to represent either the patchy saturation model or the double porosity model with dilute concentrations of identical inclusions.

Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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