Lyapunov-based Control Strategies for the Global Control of Symmetric VTOL UAVs

Rohin Wood

SCHOOL OF MECHANICAL ENGINEERING
The University of Adelaide
South Australia 5005
Australia

A thesis submitted in fulfillment of the requirements for the degree of Ph.D. in Mechanical Engineering on the 28th of June 2007.
Qualified on the 3rd of December 2007.
Acknowledgements

First and foremost, I would like to thank my academic supervisor Associate Professor Benjamin Cazzolato. I thank him for both stimulating my interest in control engineering throughout his undergraduate lectures, and for his support and guidance throughout my time as a postgraduate student.

I could have never made it to this point in my life and education without the love and support of my parents Christine and Garry Wood. I will always be grateful to them for the upbringing they gave me and for sparking my passions in the disciplines of science and mathematics from a young age.

There are three guys that have been largely responsible for making the last few years as a postgraduate student enjoyable ones. I of course refer to my office colleagues and good friends Dick Petersen, Will Robertson and Zebb Prime. In particular, I would like to thank Zebb for continuing to remind me what an engineer is supposed to be able to do with computers, Will for his ability to massage my documents with his typesetting wizardry and Dick for furthering the horizons of my musical tastes.

I am incredibly grateful to BAE Systems Australia for the financial support that they have provided me throughout my research. In particular, I would like to thank Mal Crozier for his insights into the real world of control system engineering, and his warm welcome on all of my visits to his Melbourne office. I would also like to thank Andrew Stonham for orchestrating the sponsorship.

I am somewhat indebted to Associate Professor Robert Mahony for his advice and guidance throughout the last year of my research. In particular, I would like to thank him for the time he spent with me and the discussions we had during my visit to ANU. I am constantly amazed at how someone to whom time is such a valuable commodity can be so generous with it.

Finally, I would like to thank the two most important people in my life; my beautiful partner Adriana who has supported me throughout all of my endeavors, and our young son Tavien who continues to inspire and amaze me every day. This thesis belongs to you both.
Abstract

The last decade has seen significant advances in the development of Vertical take-off and landing (VTOL) unmanned aerial vehicles (UAVs). The emergence of enabling technologies, in addition to the practical usefulness of such systems has driven their development to a point where numerous technology demonstrators and commercial products are now in existence. Of particular interest has been the development of small scale, VTOL UAVs commonly referred to as mini and micro-VTOL UAVs. The versatility and agility of such vehicles offers great potential for the use in clustered, urban environments.

Despite recent advancements, the autonomous navigation of VTOL UAVs remains a very challenging research area. The dynamics of VTOL UAVs are heavily nonlinear, underactuated and non-minimum phase. This, coupled with the aggressive maneuvers that such vehicles are expected to execute provides a stimulating problem in dynamic control. This is particularly true in the case of micro-VTOL UAVs. The fast, nonlinear nature of these systems render classical, linear control approaches inadequate.

The past twenty years has seen great interest in the development of nonlinear control strategies. This has led to the emergence of a number of standard design tools, most notably feedback linearisation and Lyapunov-based, backstepping approaches. Such design techniques offer a framework for the derivation of model based control laws capable of achieving global stabilisation and trajectory tracking control for heavily nonlinear systems. Recently, there has been significant interest in the application of such nonlinear control paradigms for the stabilisation and control of VTOL UAVs.

The aim of this thesis is to further the application and analysis of nonlinear control design techniques for the control of VTOL UAVs. In particular, focus is placed on Lyapunov-based, backstepping-type control approaches. The first half of this thesis investigates Lyapunov-based control strategies that cast the closed-loop VTOL dynamics into a globally stable, cascade structure. This work was directly inspired by, and builds on, a variety of previously published works. Firstly, an alternative design approach to that previously published is presented, resulting in an improved closed-loop dynamic structure. Although inspired by the VTOL system, this idea may be generalised for the control of a broad class of systems, and is presented as such. A singularity issue arising in the cascade control of VTOL vehicles is then investigated, and a novel approach to overcome this issue is formulated. The second half of this thesis is dedicated to the trajectory tracking control of VTOL UAVs at velocities where the influence of aerodynamics is significant. In general, the aerodynamic models of VTOL UAVs are heavily nonlinear and poorly known. The use of such models in a backstepping framework that uses explicit differentiation of these models for
dynamic inversion is questioned, due to the potential sensitivity of such nonlinear models. Consequently, an alternative approach utilising coupled filters to avoid such sensitivity issues is proposed. All control designs formulated in this thesis are accompanied by proofs guaranteeing their global stability, and numerical simulations demonstrating their time domain response characteristics.
Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, ev-
erything that is presented in this thesis is my own original work and has not been
presented previously for the award of any other degree or diploma in any univer-
sity or other tertiary institution. If accepted for the award of the degree of Ph.D.
in Mechanical Engineering, I consent that this thesis be made available for loan and
photocopying.

..........................................................
Rohin Wood
Contents

Executive Summary i
Acknowledgements i
Abstract iii
Statement of originality v
Nomenclature xi
Symbols xi
Abbreviations xvi

Chapter 1. Introduction 1
1.1. Thesis overview 3
1.2. Publications arising from this thesis 5

Chapter 2. Background 7
2.1. Introduction 7
2.2. Background control theory 7
2.3. 3DOF VTOL rigid body dynamics: The PVTOL system 14
2.4. 6DOF VTOL rigid body dynamics 16
2.5. Previous works on the control of VTOL vehicles 34
2.6. Previous works on the nonlinear control of the PVTOL vehicle 36
2.7. Previous works on the Lyapunov-based control of the 6DOF VTOL vehicle. 44
2.8. Research Exposition 45

Chapter 3. Nonlinear cascade control design with application to the PVTOL vehicle 47
3.1. Introduction 47
3.2. Cascade control for a class of nonlinear systems 48
3.3. Cascade control for the PVTOL system 52
3.4. Modified cascade control of the PVTOL with varying weights 67
3.5. Conclusion 70

Chapter 4. Singularity issues in the nonlinear cascade control of the PVTOL vehicle 73
4.1. Introduction 73
4.2. Singularity issue in the cascade control of VTOL vehicles 73
4.3. Methods for overcoming singularity in cascade control designs 77
4.4. Conclusion 87

Chapter 5. Approximate backstepping control for symmetric, 6DOF VTOL vehicles 91
5.1. Introduction 91
5.2. System dynamics 91
5.3. Approximate backstepping control design 92
5.4. Closed-loop dynamics 95
5.5. Singularity issues 96
5.6. Modified approximate backstepping control design 97
5.7. Modified approximate backstepping control design with guaranteed non-negative thrust 101
5.8. Simulation Results 101
5.9. Conclusion 103

Chapter 6. Approximate backstepping/cascade control for symmetric VTOL vehicles: Trajectory tracking with significant aerodynamic effects 107
6.1. Introduction 107
6.2. System dynamics 108
6.3. Control design procedure 108
6.4. Function approximations for controller implementation 115
6.5. Numerical simulations 123

Chapter 7. Backstepping control for symmetric VTOL vehicles: Trajectory tracking with significant aerodynamic effects 137
7.1. Introduction 137
7.2. System dynamics 139
7.3. Conventional backstepping control 139
7.4. Backstepping-type control using coupled filters 149
7.5. Controller comparison 159
7.6. Singularity issues 169
7.7. Conclusion 175

Chapter 8. Conclusions and future work 179
8.1. Conclusions 179
8.2. Future work 181

Appendix A. Explicit control law expressions for Chapter 5 185

Appendix B. Explicit control law expressions for Chapter 6 189
B.1. Derivatives of approximation functions obtained via numerical method 190
B.2. Derivatives of approximation functions obtained via approximate aerodynamic model 194

Appendix C. Explicit control law expressions for Chapter 7 197

Appendix D. Partial derivatives of aerodynamic model 201
D.1. Partial derivatives of general aerodynamic model 201
Errata

For copyright reasons, the following figures require removal from the thesis “Lyapunov-Based Control Strategies for the Global Control of Symmetric VTOL UAVs”:

**Figure 1.1.** Symmetric VTOL UAV examples. Left: Bertin Technology’s LAAS-CNRS hovereye [60]. Right: Georgia Institute of Technology’s GTSpy [33].

**Figure 1.2.** The Nulka active missile decoy system. Left: Mid-flight. Right: Deployed from HMAS Newcastle.
Nomenclature

Symbols

\( A \)  
Aerodynamic drag

\( A_{\text{dir}} \)  
Direction of aerodynamic drag

\( \hat{A} \)  
Approximate aerodynamic model

\( a = [a_x, a_y, a_z]^T \)  
Acceleration

\( a_d = [a_{dx}, a_{dy}, a_{dz}]^T \)  
Acceleration virtual control law (desired acceleration)

\( a_d = [a_{dx}, a_{dy}]^T \)  
PVTOL desired acceleration

\( \bar{a}_d = [\bar{a}_{dx}, \bar{a}_{dy}]^T \)  
PVTOL desired acceleration for controller utilising additional dynamics

\( \bar{a}_{dx}, \bar{a}_{dy}, \bar{a}_{dz} \)  
Maximum input values of \( a_{dx}, a_{dy} \) and \( a_{dz} \) for polycubic interpolation function

\( \hat{a}_d = [\hat{a}_{dx}, \hat{a}_{dy}]^T \)  
PVTOL desired acceleration for controller utilising saturation functions

\( \mathcal{B} = \{e_x, e_y, e_z\} \)  
Body fixed reference frame

\( b_0 \)  
Saturation function upper limit

\( C^n \)  
Class of functions with finite \( n^{th} \) order derivatives

\( C_1, C_2 \)  
PVTOL translation control matrices

\( C_N \)  
Normal drag coefficient

\( \hat{C}_N \)  
First order approximation of normal drag coefficient

\( \hat{C}_N \)  
RBFN representation of normal drag coefficient

\( \tilde{C}_N \)  
Approximate normal drag coefficient

\( c \)  
Distance from CG to CP
\( c_0 \)  
Radius of region surrounding singularity, defined for additional dynamics embedded within PVTOL control law

\( c_{11}, c_{12}, c_{21}, c_{22} \)  
PVTOL translation control gains

\( D \)  
Reference length

\( D_1, D_2 \)  
Control gains for additional dynamics embedded within PVTOL control law

\( d_1, d_2 \)  
PVTOL orientation control gains

\( E \)  
Storage function

\( \mathcal{E} = \{E_x, E_y, E_y\} \)  
Inertial reference frame

\( F \)  
Nonlinear function defined for additional dynamics embedded within PVTOL control law

\( F_c \)  
Control force

\( g \)  
Scalar gravitational acceleration

\( \tilde{g} \)  
Vector gravitational acceleration

\( g_i \)  
Radial basis functions

\( h_1, h_2 \)  
Functions comprising \( F \)

\( H \)  
Matrix defined for conventional backstepping control law

\( \bar{H} \)  
Matrix defined for backstepping-type control law

\( I_{xx}, I_{yy}, I_{zz} \)  
Principal moments of inertia in X, Y and Z directions

\( I \)  
Inertia tensor

\( I_1 \)  
Constant matrix

\[
I_1 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\( I_2 \)  
Constant matrix

\[
I_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\( \dot{j} = [j_x, j_y, j_z]^T \)  
Jerk

\( \dot{j}_d = [j_{dx}, j_{dy}, j_{dz}]^T \)  
Jerk virtual control law (desired jerk)

\( K \)  
Control gain for additional dynamics embedded within PVTOL control law

\( k \)  
Yaw control loop gain

\( k_1 \)  
PVTOL desired thrust
\(k_2\)  
PVTOL desired roll angle

\(\tilde{k}_1\)  
PVTOL desired thrust from control law utilising saturation functions

\(\tilde{k}_2\)  
PVTOL desired roll angle from control law utilising saturation functions

\(\tilde{k}_3, \tilde{k}_4\)  
Backstepping-type control law gains

\(\hat{k}_1\)  
PVTOL desired thrust for control law utilising additional dynamics

\(\hat{k}_2\)  
PVTOL desired roll angle for control law utilising additional dynamics

\(\hat{k}_3\)  
Control gain

\(\tilde{k}_3, \tilde{k}_3\)  
Filter gains

\(k_i\)  
Control gains  
\(i = 1, ..., 4\)

\(L_f\)  
Lie derivative with respect to function \(f\)

\(L_1, L_2\)  
Filter Lyapunov functions

\(l\)  
Eccentricity of roll control inputs with respect to CG

\(M\)  
Matrix defined for backstepping-type control law

\(m\)  
Mass

\(P_D\)  
Dynamic pressure

\(q\)  
Modified feedback law for primary thrust

\(\tilde{q}\)  
Weighted, modified feedback law for primary thrust

\(\tilde{\tilde{q}}\)  
Non negative, modified feedback law for primary thrust

\(\mathbb{R}^+\)  
Space of all real, non-negative numbers

\(\mathbb{R}^n\)  
Space of all real, \(n\)-dimensional numbers

\(R\)  
Rotation matrix

\(Re\)  
Reynolds number

\(r\)  
Variables defined for ease of cascade control law notation  
\(r = a_d + gE_z + \frac{1}{m}A_1\)

\(\tilde{r}\)  
PVTOL unit vector defining direction of thrust

\(S\)  
Reference Area

\(s\)  
Dummy variable

\(s_x, s_y, s_z\)  
Elliptical reference trajectory spatial parameters

\(T\)  
Variable defined to ensure non negative thrust  
\(\bar{T}_{zd} = \|T\|\)

\(T\)  
Velocity Jacobean relating angular velocity to orientation angle derivatives

\(T_f\)  
Elliptical reference trajectory completion time
\( T_x, T_y \) Roll control thrust inputs
\( T_z \) Primary thrust input
\( \bar{T}_z \) Augmented primary thrust input
\( \hat{T}_z, \tilde{T}_z \) Additional state and new input arising from dynamic extension of \( \bar{T}_z \) input
\( \dot{T}_{zd} \) Estimate of \( \bar{T}_{zd} \)
\( t \) Time
\( u_1 \) PVTOL thrust input
\( \bar{u}_1 \) PVTOL augmented thrust input
\( u_2 \) PVTOL roll control input
\( V_i \), \( i = 1, ..., 4 \) Control Lyapunov functions
\( v = [v_x, v_y, v_z]^T \) CG velocity
\( W \) Weighting matrix
\( w_i \) RBFN weights
\( w_{\text{opt}} \) Optimal RBFN weights
\( w_x \) Component of weighting matrix influencing horizontal dynamics
\( w_y \) Component of weighting matrix influencing vertical dynamics
\( Y \) RBFN measured data
\( X = [\lambda^T, \sigma^T, \eta^T, \omega^T]^T \) 6DOF VTOL state vector
\( x \) RBFN input parameters
\( x = [x_x, x_y, x_z]^T \) CG displacement
\( x_{Ci} \) RBFN center locations
\( x_i \) RBFN input parameters for measured data \( Y \)
\( x_1, y_1 \) PVTOL horizontal and vertical components of CG displacement
\( x_2, y_2 \) PVTOL horizontal and vertical components of CG velocity
\( Z \) Space of all real integers
\( \alpha \) Vehicle attitude
\( \beta \) Vehicle heading
\[ \delta = [\delta_x, \delta_y]^T \]

Perturbation to desired PVTOL acceleration

\[ \delta_1 = [\delta_{1x}, \delta_{1y}, \delta_{1z}]^T \]

Tracking error variable

\[ \bar{\delta}_3, \bar{\delta}_4 \]

Error variables used in backstepping-type control law design

\[ \hat{\delta}_3 \]

Orientation angle set error

\[ \tilde{\delta}_3, \tilde{\delta}_4 \]

Error variables used in backstepping-type control law filter designs

\[ \delta_i, i = 1, ..., 4 \]

Error variables used in backstepping-based controller designs

\[ \delta_a \]

Error variable resulting from error in \( \bar{T}_{zd} \) and \( \eta_d \) estimates

\[ E \]

6DOF VTOL input coupling matrix

\[ \epsilon \]

PVTOL input coupling parameter

\[ \zeta_1 \]

PVTOL roll error

\[ \zeta_2 \]

PVTOL roll error derivative

\[ \eta = [\phi, \theta, \psi]^T \]

Orientation angle set (Pitch, Roll Yaw)

\[ \eta_d = [\phi_d, \theta_d, \psi_d^*]^T \]

Desired value of orientation angle set

\[ \hat{\eta_d} = [\hat{\phi_d}, \hat{\theta_d}, \hat{\psi_d}^*]^T \]

Estimate of \( \eta_d \)

\[ \theta \]

PVTOL roll angle

\[ \kappa \]

Radial basis function width

\[ \lambda = [\lambda_x, \lambda_y, \lambda_z]^T \]

CP displacement

\[ \lambda_d = [\lambda_{dx}, \lambda_{dy}, \lambda_{dz}]^T \]

Demand trajectory (desired displacement)

\[ \lambda_x, \lambda_y \]

PVTOL horizontal and vertical components of CP displacement

\[ \mu \]

Air viscosity

\[ \rho \]

Air density

\[ \sigma = [\sigma_x, \sigma_y, \sigma_z]^T \]

CP velocity

\[ \sigma_d = [\sigma_{dx}, \sigma_{dy}, \sigma_{dz}]^T \]

Velocity virtual control law (desired velocity)

\[ \sigma_x, \sigma_y \]

PVTOL horizontal and vertical components of CP velocity
$\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_z$  
Maximum input values of $\sigma_x, \sigma_y$ and $\sigma_z$ for polycubic interpolation function

$\tau = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$  
Torque input

$\varphi$  
Saturation function

$\psi_i, \ i = 1, \ldots, 4$  
Interconnection terms between cascaded subsystems

$\Omega = [\omega_x, \omega_y, \omega_z]^T$  
Angular velocity

$\dot{\Omega}_d = [\dot{\omega}_xd, \dot{\omega}_yd, \dot{\omega}_zd]^T$  
Angular velocity virtual control law (desired angular velocity)

$\omega$  
PVTOL angular velocity

**Abbreviations**

- **3DOF**: Three Degrees Of Freedom
- **6DOF**: Six Degrees Of Freedom
- **CG**: Center of Gravity
- **CLF**: Control Lyapunov Function
- **CP**: Center of Percussion/Control Point
- **LHS**: Left Hand Side
- **MAV**: Mobile Air Vehicle
- **PVTOL**: Planar Vertical Take-Off and Landing
- **RBFN**: Radial Basis Function Network
- **UAV**: Unmanned Aerial Vehicle
- **VTOL**: Vertical Take-Off and Landing