Lyapunov-based Control Strategies for the Global Control of Symmetric VTOL UAVs

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Abstract

The last decade has seen significant advances in the development of Vertical take-off and landing (VTOL) unmanned aerial vehicles (UAVs). The emergence of enabling technologies, in addition to the practical usefulness of such systems has driven their development to a point where numerous technology demonstrators and commercial products are now in existence. Of particular interest has been the development of small scale, VTOL UAVs commonly referred to as mini and micro-VTOL UAVs. The versatility and agility of such vehicles offers great potential for the use in clustered, urban environments.

Despite recent advancements, the autonomous navigation of VTOL UAVs remains a very challenging research area. The dynamics of VTOL UAVs are heavily nonlinear, underactuated and non-minimum phase. This, coupled with the aggressive maneuvers that such vehicles are expected to execute provides a stimulating problem in dynamic control. This is particularly true in the case of micro-VTOL UAVs. The fast, nonlinear nature of these systems render classical, linear control approaches inadequate.

The past twenty years has seen great interest in the development of nonlinear control strategies. This has led to the emergence of a number of standard design tools, most notably *feedback linearisation* and Lyapunov-based, *backstepping* approaches. Such design techniques offer a framework for the derivation of model based control laws capable of achieving global stabilisation and trajectory tracking control for heavily nonlinear systems. Recently, there has been significant interest in the application of such nonlinear control paradigms for the stabilisation and control of VTOL UAVs.

The aim of this thesis is to further the application and analysis of nonlinear control design techniques for the control of VTOL UAVs. In particular, focus is placed on Lyapunov-based, backstepping-type control approaches. The first half of this thesis investigates Lyapunov-based control strategies that cast the closed-loop VTOL dynamics into a globally stable, cascade structure. This work was directly inspired by, and builds on, a variety of previously published works. Firstly, an alternative design approach to that previously published is presented, resulting in an improved closed-loop dynamic structure. Although inspired by the VTOL system, this idea may be generalised for the control of a broad class of systems, and is presented as such. A singularity issue arising in the cascade control of VTOL vehicles is then investigated, and a novel approach to overcome this issue is formulated. The second half of this thesis is dedicated to the trajectory tracking control of VTOL UAVs at velocities where the influence of aerodynamics is significant. In general, the aerodynamic models of VTOL UAVs are heavily nonlinear and poorly known. The use of such models in a backstepping framework that uses explicit differentiation of these models for dynamic inversion is questioned, due to the potential sensitivity of such nonlinear models. Consequently, an alternative approach utilising coupled filters to avoid such sensitivity issues is proposed. All control designs formulated in this thesis are accompanied by proofs guaranteeing their global stability, and numerical simulations demonstrating their time domain response characteristics.

Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, everything that is presented in this thesis is my own original work and has not been presented previously for the award of any other degree or diploma in any university or other tertiary institution. If accepted for the award of the degree of Ph.D. in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

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Rohin Wood

Contents

Errata

For copyright reasons, the following figures require removal from the thesis *"Lyapunov-Based Control Strategies for the Global Control of Symmetric VTOL UAVs"*:

FIGURE 1.1. Symmetric VTOL UAV examples. Left: Bertin Technology's LAAS-CNRS hovereye [60]. Right: Georgia Institute of Technology's GTSpy [33].

FIGURE 1.2. The Nulka active missile decoy system. Left: Mid-flight. Right: Deployed from HMAS Newcastle.

Nomenclature

Symbols

A Aerodynamic drag

A_{dir} Direction of aerodynamic drag

$\tilde{A}$ Approximate aerodynamic model

$a = [a_x, a_y, a_z]^T$
Acceleration

$a_d = [a_{dx}, a_{dy}, a_{dz}]^T$
Acceleration virtual control law (desired acceleration)

$a_d = [a_{dx}, a_{dy}]^T$
PVTOL desired acceleration

$\bar{a}_d = [\bar{a}_{dx}, \bar{a}_{dy}]^T$
PVTOL desired acceleration for controller utilising additional dynamics

$\bar{a}_{dx}, \bar{a}_{dy}, \bar{a}_{dz}$
Maximum input values of a_{dx}, a_{dy} and a_{dz} for polycubic interpolation function

$\hat{a}_d = [\hat{a}_{dx}, \hat{a}_{dy}]^T$
PVTOL desired acceleration for controller utilising saturation functions

$\mathcal{B} = \{e_x, e_y, e_z\}$
Body fixed reference frame

b_0 Saturation function upper limit

C^n Class of functions with finite n^{th} order derivatives

C_1, C_2 PVTOL translation control matrices

C_N Normal drag coefficient

$\bar{C}_N$ First order approximation of normal drag coefficient

$\hat{C}_N$ RBFN representation of normal drag coefficient

$\tilde{C}_N$ Approximate normal drag coefficient

c Distance from CG to CP

c_0 Radius of region surrounding singularity, defined for additional dynamics embedded within PVTOL control law

$c_{11}, c_{12}, c_{21}, c_{22}$
PVTOL translation control gains

D Reference length

D_1, D_2 Control gains for additional dynamics embedded within PVTOL control law

d_1, d_2 PVTOL orientation control gains

E Storage function

$\mathcal{E} = \{E_x, E_y, E_y\}$
Inertial reference frame

F Nonlinear function defined for additional dynamics embedded within PVTOL control law

F_c Control force

g Scalar gravitational acceleration

$\tilde{g}$ Vector gravitational acceleration

g_i Radial basis functions

h_1, h_2 Functions comprising F

H Matrix defined for conventional backstepping control law

$\bar{\mathrm{H}}$ Matrix defined for backstepping-type control law

I_{xx}, I_{yy}, I_{zz}
Principal moments of inertia in X, Y and Z directions

I Inertia tensor

I_1 Constant matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

I_2 Constant matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$j = [j_x, j_y, j_z]^T$
Jerk

$j_d = [j_{dx}, j_{dy}, j_{dz}]^T$
Jerk virtual control law (desired jerk)

K Control gain for additional dynamics embedded within PVTOL control law

k Yaw control loop gain

k_1 PVTOL desired thrust

k_2	PVTOL desired roll angle
$\bar{k}_1$	PVTOL desired thrust from control law utilising saturation functions
$\bar{k}_2$	PVTOL desired roll angle from control law utilising saturation functions
$\bar{k}_3, \bar{k_4}$	Backstepping-type control law gains
$\hat{k}_1$	PVTOL desired thrust for control law utilising additional dynamics
$\hat{k}_2$	PVTOL desired roll angle for control law utilising additional dynamics
$\hat{k}_3$	Control gain
$\tilde{k}_3, \tilde{k}_3$	Filter gains
$k_i\ i = 1, ..., 4$	Control gains
L_f	Lie derivative with respect to function f
L_1, L_2	Filter Lyapunov functions
l	Eccentricity of roll control inputs with respect to CG
M	Matrix defined for backstepping-type control law
m	Mass
P_D	Dynamic pressure
q	Modified feedback law for primary thrust
$\bar{q}$	Weighted, modified feedback law for primary thrust
$\tilde{q}$	Non negative, modified feedback law for primary thrust
$\mathbb{R}^+$	Space of all real, non-negative numbers
$\mathbb{R}^n$	Space of all real, n-dimensional numbers
R	Rotation matrix
Re	Reynolds number
r	Variables defined for ease of cascade control law notation $\left(r = a_d + gE_z + \frac{1}{m}A_\perp\right)$
$\tilde{r}$	PVTOL unit vector defining direction of thrust
S	Reference Area
s	Dummy variable
s_x, s_y, s_z	Elliptical reference trajectory spatial parameters
T	Variable defined to ensure non negative thrust $\bar{T}_{zd} = \|T\|$
T	Velocity Jacobean relating angular velocity to orientation angle derivatives
T_f	Elliptical reference trajectory completion time

T_x, T_y Roll control thrust inputs

T_z Primary thrust input

$\bar{T}_z$ Augmented primary thrust input

$\bar{\bar{T}}_z, \bar{\bar{\bar{T}}}_z$ Additional state and new input arising from dynamic extension of $\bar{T}_z$ input

$\hat{\bar{T}}_{zd}$ Estimate of $\bar{T}_{zd}$

t Time

u_1 PVTOL thrust input

$\bar{u}_1$ PVTOL augmented thrust input

u_2 PVTOL roll control input

$V_i\ i = 1, ..., 4$
Control Lyapunov functions

$v = [v_x, v_y, v_z]^T$
CG velocity

W Weighting matrix

w_i RBFN weights

w_{opt} Optimal RBFN weights

w_x Component of weighting matrix influencing horizontal dynamics

w_y Component of weighting matrix influencing vertical dynamics

Y RBFN measured data

$\mathrm{X} = \begin{bmatrix} \lambda^T & \sigma^T & \eta^T & \omega^T \end{bmatrix}^T$
6DOF VTOL state vector

x RBFN input parameters

$x = [x_x, x_y, x_z]^T$
CG displacement

x_{Ci} RBFN center locations

x_i RBFN input parameters for measured data Y

x_1, y_1 PVTOL horizontal and vertical components of CG displacement

x_2, y_2 PVTOL horizontal and vertical components of CG velocity

$\mathbb{Z}$ Space of all real integers

α Vehicle attitude

β Vehicle heading

$\delta = [\delta_x, \delta_y]^T$
Perturbation to desired PVTOL acceleration

$\delta_1 = \begin{bmatrix} \delta_{1x} & \delta_{1y} & \delta_{1z} \end{bmatrix}^T$
Tracking error variable

$\bar{\delta}_3, \bar{\delta}_4$ Error variables used in backstepping-type control law design

$\hat{\delta}_3$ Orientation angle set error

$\tilde{\delta}_3, \tilde{\delta}_4$ Error variables used in backstepping-type control law filter designs

$\delta_i \; i = 1, ..., 4$
Error variables used in backstepping-based controller designs

δ_a Error variable resulting from error in $\bar{T}_{zd}$ and η_d estimates

E 6DOF VTOL input coupling matrix

ϵ PVTOL input coupling parameter

ζ_1 PVTOL roll error

ζ_2 PVTOL roll error derivative

$\eta = [\phi, \theta, \psi]^T$
Orientation angle set (Pitch, Roll Yaw)

$\eta_d = [\phi_d, \theta_d, \psi_d^\star]^T$
Desired value of orientation angle set

$\hat{\eta}_d = \left[\hat{\phi}_d, \hat{\theta}_d, \psi_d^\star\right]^T$
Estimate of η_d

θ PVTOL roll angle

$\varkappa$ Radial basis function width

$\lambda = [\lambda_x, \lambda_y, \lambda_z]^T$
CP displacement

$\lambda_d = [\lambda_{dx}, \lambda_{dy}, \lambda_{dz}]^T$
Demand trajectory (desired displacement)

λ_x, λ_y PVTOL horizontal and vertical components of CP displacement

μ Air viscosity

ρ Air density

$\sigma = [\sigma_x, \sigma_y, \sigma_z]^T$
CP velocity

$\sigma_d = [\sigma_{dx}, \sigma_{dy}, \sigma_{dz}]^T$
Velocity virtual control law (desired velocity)

σ_x, σ_y PVTOL horizontal and vertical components of CP velocity

$\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_z$
Maximum input values of σ_x, σ_y and σ_z for polycubic interpolation function

$\tau = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$
Torque input

φ Saturation function

$\psi_i,\ i = 1, .., 4$
Interconnection terms between cascaded subsystems

$\Omega = [\omega_x, \omega_y, \omega_z]^T$
Angular velocity

$\hat{\Omega}_d = [\hat{\omega}_{xd}, \hat{\omega}_{yd}, \hat{\omega}_z^\star]^T$
Angular velocity virtual control law (desired angular velocity)

ω PVTOL angular velocity

Abbreviations

3DOF Three Degrees Of Freedom

6DOF Six Degrees Of Freedom

CG Center of Gravity

CLF Control Lyapunov Function

CP Center of Percussion/ Control Point

LHS Left Hand Side

MAV Mobile Air Vehicle

PVTOL
Planar Vertical Take-Off and Landing

RBFN Radial Basis Function Network

UAV Unmanned Aerial Vehicle

VTOL Vertical Take-Off and Landing