

Excitations of the Nucleon in Lattice QCD

Thesis for the degree of Doctor of Philosophy

by

Md. Selim Mahbub

Supervisors: D. B. Leinweber and A. G. Williams



Centre for the Subatomic Structure of Matter
School of Chemistry and Physics
University of Adelaide
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To my **father**

Abstract

A comprehensive study of the positive and negative parity, isospin = $\frac{1}{2}$, spin = $\frac{1}{2}$ energy eigenstates of the nucleon is presented from lattice QCD.

The existence of the Roper resonance, $N^{\frac{1}{2}+}$ (1440 MeV) P_{11} , of the nucleon is experimentally well determined as signified by a four star rating in the Review of Particle Physics. The Roper state is puzzling as this state has a significantly lower mass than the adjacent $N^{\frac{1}{2}-}$ ground state. In a constituent quark model the Roper state sits at almost 100 MeV higher than the $N^{\frac{1}{2}-}$ (1535 MeV) state.

In this thesis our principal investigation is centred in finding the Roper resonance using the “variational method”. The analysis is presented in quenched and dynamical QCD with two degenerate flavours of sea quarks. We extract the elusive Roper resonance using the variational approach for the first time, by using smeared-smeared correlators in constructing the correlation matrices. The Roper state has a tendency to approach the physical state at light quark masses and a significant curvature is observed as the chiral region is approached. We also observe a physical level ordering between the Roper and the negative parity ground states of the nucleon, $N^{\frac{1}{2}-}$, at light quark masses.

The consistency of the Roper state is demonstrated with the further analysis of larger correlation matrices. We present results that illustrate how excited-state energies are split with larger dimensions of the correlation matrices ultimately revealing true eigenstates. The need to use larger dimensions of correlation matrices to reliably isolate the energy eigenstates of QCD is emphasized. We conclude with a calculation of the Roper state in dynamical QCD, and find that the results are consistent with the quenched simulations.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Md. Selim Mahbub and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

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Md. Selim Mahbub

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