
**APPLICATIONS OF
CONDITIONAL VALUE-AT-RISK
TO
WATER RESOURCES MANAGEMENT**

PhD Thesis by Roger Brian Webby

Principal Supervisor: Associate Professor A. V. Metcalfe

Co-Supervisors: Associate Professor J. Boland
Professor P. G. Howlett

March 2009

SCHOOL OF MATHEMATICAL SCIENCES



Contents

1	Introduction	1
1.1	Research Problem	1
1.2	Stochastic Hydrology	5
2	Conditional Value-at-Risk	9
2.1	Value-at-Risk	10
2.2	Conditional Value-at-Risk	13
2.3	Calculation of VaR and CVaR	23
2.4	CVaR and expected utility	28
2.5	CVaR and EMV	29
3	Literature Review	31
3.1	Applications of VaR and CVaR other than in water resources . .	31
3.2	Optimisation in water resource applications	35
3.3	CVaR as a criterion in water resources management	36
4	Synthesis	39
5	Future Directions	61
6	Conclusion	67
7	The Papers	71

Abstract

In this thesis I develop mathematical models of freshwater resources and assess the application of a risk measure, Conditional Value-at-Risk, as a criterion for making decisions on the allocation of these resources. The nature of hydrological systems is such that they are well represented by stochastic models. The models considered are: time simulation; stochastic and deterministic linear programming; and stochastic dynamic programming. The hydrological applications are: draw down of dams; allocation and blending of water resources; operation of a small-scale solar-powered desalination plant; and insurance against fishery and crop shortfall. In water resource applications, optimisation models usually have the goal of maximising expected return, or utility, but here I demonstrate that the minimisation of the risk metric is a relevant additional criterion to expected return for water resource management.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

SIGNED:

DATE: 07/04/09.....

The author acknowledges that copyright of published works contained within this thesis (as listed below) resides with the copyright holders of those works and thanks the publishers for their kind permission to reproduce the works in this thesis.

- Webby, RB, Adamson, PT, Boland, J, Howlett, PG, Metcalfe, AV and Piantadosi, J. 2006. *The Mekong - applications of Value-at-Risk (VaR) and Conditional-Value-at-Risk (CVaR) simulation to the benefits, costs and consequences of water resources development in a large river basin*. Ecological Modelling, 201: pp. 89-96.
- Webby, RB, Boland, J, Howlett, PG, Metcalfe, AV and Sritharan, T. 2006. *Conditional value-at-risk for water management in Lake Burley Griffin*. ANZIAM J. 47, pp. C116–C136.
- Webby, RB, Adamson, PT, Boland, J, Howlett, PG and Metcalfe, AV. 2007. *Conditional Value-at-Risk analysis of flooding in the Lower Mekong Basin*. IAHS Red Book 317: pp. 297-302.
- Webby, RB, Boland, J, Howlett, PG and Metcalfe, AV 2008. *Stochastic linear programming and Conditional Value-at-Risk for water resources management*. ANZIAM J. 48, pp C885–C898.
- Webby, RB, Boland, J and Metcalfe, AV 2007. *Stochastic programming to evaluate renewable power generation for small-scale desalination*. ANZIAM J. 49, pp. C184–C199.
- Webby, RB, Green, DA and Metcalfe, AV. 2008. *Modelling water blending – sensitivity of optimal policies*. Environmental Modeling and Assessment (to appear).
- Fisher, AJ, Green, DA, Metcalfe, AV. and Webby, RB. 2008. *Optimal Control of Multi-reservoir Systems with Time-dependent Markov Decision Processes*. Proceedings of Water Down Under 2008, Engineers Australia.

Acknowledgements

I express my deepest gratitude to my principal supervisor, Andrew Metcalfe, for suggestions and criticisms of my research, for support through adverse times and good, and for enhancing my graduate experience.

I also thank my co-supervisors, John Boland and Phil Howlett, and research colleagues David Green, Peter Adamson and Julia Piantadosi for their assistance and guidance.

I am grateful to the School of Mathematical Sciences for financial and administrative support. And for advice and fun from my postgrad colleagues, particularly Aiden, Ariella, Geraldine, James, Jason, Kate and Rongmin. I will always recall room G12.

I would acknowledge the suggestion of Barry Clark that started this journey.

STATEMENT OF AUTHORSHIP

THE MEKONG - APPLICATIONS OF VALUE-AT-RISK (VaR) AND CONDITIONAL VALUE-AT-RISK (CVaR) SIMULATION TO THE BENEFITS, COSTS AND CONSEQUENCES OF WATER RESOURCES DEVELOPMENT IN A LARGE RIVER BASIN

Ecological Modelling 2006; 201: 89–96.

Webby, R.B. (Candidate)

Developed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate

Signed ...

Adamson, P.T.

Provided river flow data and manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed .

Date.....

Boland, J.

Co-supervised development of work and provided critical evaluation of manuscript

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Howlett, P.G.

Co-supervised development of work and provided critical evaluation of manuscript

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date... 2/4/8

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the ~~paper in the thesis~~ paper in the thesis

Signed

Date... 3rd April 2008

Piantadosi, J.

Provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date... 2/04/08

STATEMENT OF AUTHORSHIP

CONDITIONAL VALUE-AT-RISK FOR WATER MANAGEMENT IN LAKE
BURLEY GRIFFIN

ANZIAM J. 2006; 47: C116-C136.

Webby, R.B. (Candidate)

Developed concept for paper, developed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate

Signed

Date. 21/03/09

Boland, J.

Provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 2/04/08

Howlett, P.G.

Provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 2/4/8

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date..... 3rd April 2008

Sritharan, T.

Provided quantitative and qualitative evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed .

...Date..... 21st March 2009

STATEMENT OF AUTHORSHIP

CONDITIONAL VALUE-AT-RISK ANALYSIS OF FLOODING IN THE LOWER MEKONG BASIN

IAHS Red Book 2007; 317: 297-302.

Webby, R.B. (Candidate)

Developed concept for paper, developed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate

Signed

Date. 21/03/09

Adamson, P.T.

Provided data and graph, contributed quantitative and qualitative evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed ...

Date. 21st March 2009

Boland, J.

Provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 2/04/08.

Howlett, P.G.

Provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date... 2/4/8.....

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed .

Date... 3rd April 2008

STATEMENT OF AUTHORSHIP

STOCHASTIC PROGRAMMING TO EVALUATE RENEWABLE POWER
GENERATION FOR SMALL-SCALE DESALINATION

ANZIAM J. 2007; 49: C184–C199.

Webby, R.B. (Candidate)

Developed concept for paper, developed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate

Signed .

Date. 21/03/09.

Boland, J.

Contributed data, co-supervised development of work and provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 2/04/08.

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed .

Date. 3rd April 2008

STATEMENT OF AUTHORSHIP

**STOCHASTIC LINEAR PROGRAMMING AND CONDITIONAL VALUE-AT-RISK
FOR WATER RESOURCES MANAGEMENT**

ANZIAM J. 2008; 48: C885–C898.

Webby, R.B. (Candidate)

Developed concept for paper, developed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate

Signed .

Date. 21/03/09

Boland, J.

Provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed .

Date. 2/04/08

Howlett, P.G.

Co-supervisor

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 2/4/8

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis //

Signed

Date. 3rd April 2008

STATEMENT OF AUTHORSHIP

MODELLING WATER BLENDING - SENSITIVITY OF OPTIMAL POLICIES

Environmental Modeling and Assessment, 2008

Webby, R.B. (Candidate)

Developed concept for paper, developed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate

Signed .

Date. 21/03/09

Green, D.A.

Assisted in developing mathematical model and provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 20 MAR 2009

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis,

Signed .

Date. 20th March 2009

STATEMENT OF AUTHORSHIP

OPTIMAL CONTROL OF MULTI-RESERVOIR SYSTEMS WITH TIME-DEPENDENT MARKOV DECISION PROCESSES

Proceedings of Water Down Under 2008, 2008

Webby, R.B. (Candidate)

Researched and developed application as decision problem, researched and fitted rainfall - runoff model, assisted in developing mathematical model and contributed to the writing of the manuscript

I hereby certify that the ~~statement~~ statement of contribution is accurate

Signed .

Date. 21/03/09

Fisher, A.J.

Constructed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 20/3/09

Green, D.A.

Supervised development of work and provided critical evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 20 MAR 2009

Metcalf, A.V.

Supervised development of work and provided manuscript evaluation

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis

Signed

Date. 20th Mar 2009

Preface

The University of Adelaide has recently reformed its rules for submission of theses by higher degree research students. These now encourage postgraduate students to submit a thesis based on publications during their candidature. I have chosen to submit my thesis under these rules; I reproduce the clause specifying the content of the main part of the work below.

(c) the main body of work should contain in addition to the relevant publications a contextual statement which normally includes the aims underpinning the publication(s); a literature review or commentary which establishes the field of knowledge and provides a link between publications; and a conclusion showing the overall significance of the work and contribution to knowledge, problems encountered and future directions of the work. The discussion should not include a detailed reworking of the discussions from individual papers within the thesis.

The following list gives citations of the seven publications in which I have reported my research. For the sake of brevity and to assist in the recall of their content, I refer to each paper by a short title based on the application considered in the paper. The full citations, in the order in which they were written, are:

1. Webby, RB, Adamson, PT, Boland, J, Howlett, PG, Metcalfe, AV and Piantadosi, J. 2006. *The Mekong - applications of Value-at-Risk (VaR) and Conditional-Value-at-Risk (CVaR) simulation to the benefits, costs and consequences of water resources development in a large river basin*. Ecological Modelling, 201: pp. 89-96.
2. Webby, RB, Boland, J, Howlett, PG, Metcalfe, AV and Sritharan, T. 2006. *Conditional value-at-risk for water management in Lake Burley Griffin*. ANZIAM J. 47, pp. C116–C136. Proceedings of the 7th Biennial Engineering Mathematics and Applications Conference, Melbourne, Australia, September 2005, Editors: A. Stacey, W. Blyth, J. Shepherd & A. J. Roberts.
3. Webby, RB, Adamson, PT, Boland, J, Howlett, PG and Metcalfe, AV. 2007. *Conditional Value-at-Risk analysis of flooding in the Lower Mekong Basin*. IAHS Red Book 317: pp. 297-302. Proceedings of the Third International Symposium on Integrated Water Resources Management, Bochum, Germany, September 2006. Editors M. Pahlow & A. Schumann.
4. Webby, RB, Boland, J, Howlett, PG and Metcalfe, AV 2008. *Stochastic linear programming and Conditional Value-at-Risk for water resources management*. ANZIAM J. 48, pp C885–C898. Proceedings of the 13th Biennial Computational Techniques and Applications Conference, CTAC-2006 Editors: Wayne Read, Jay W. Larson and A. J. Roberts.
5. Webby, RB, Boland, J and Metcalfe, AV 2007. *Stochastic programming to evaluate renewable power generation for small-scale desalination*. ANZIAM J. 49, pp. C184–C199. Proceedings of the 8th Biennial Engineering Mathematics and Applications Conference, Hobart, Australia. Editors: Geoffry N. Mercer and A. J. Roberts.

6. Webby, RB, Green, DA and Metcalfe, AV. 2009. *Modelling water blending – sensitivity of optimal policies*. Environmental Modeling and Assessment, 14: pp. 749 - 757.
7. Fisher, AJ, Green, DA, Metcalfe, AV. and Webby, RB. 2008. *Optimal Control of Multi-reservoir Systems with Time-dependent Markov Decision Processes*. Proceedings of Water Down Under 2008. Editors: M Lambert, TM Daniell and M Leonard.

The corresponding short titles are:

1. *Mekong - Tonle Sap*
2. *Lake Burley Griffin*
3. *Mekong - Delta*
4. *Crop selection*
5. *Sizing for desalination*
6. *Use of stormwater*
7. *Wivenhoe*