Fundamental bigroupoids and 2-covering spaces

David Michael Roberts

Thesis submitted for the degree of
Doctor of Philosophy
in the School of Mathematical Sciences
University of Adelaide

December 2009
Contents

Abstract v
Statement of originality vii
Acknowledgements ix
Introduction 1

Chapter 1. Internal categories and anafunctors 11
  1. Internal categories and groupoids 11
  2. Sites and covers 15
  3. Weak equivalences 19
  4. Anafunctors 22
  5. Localising bicategories at a class of 1-cells 28
  6. Anafunctors are a localisation 31

Chapter 2. A fundamental bigroupoid for topological groupoids 41
  1. Preliminaries on topological groupoids and n-partitions 42
  2. Paths and surfaces in topological groupoids 51
  3. Thin homotopies and structure morphisms 58
  4. 2-tracks 64
  5. Compositions and concatenations 71
  6. The fundamental bigroupoid 76
  7. Calculations and comparisons 96

Chapter 3. Interlude: Pointed topological anafunctors 125

Chapter 4. 2-covering spaces I: Basic theory 135
  1. Review of covering spaces 135
  2. Locally trivial and weakly discrete groupoids 140
  3. 2-covering spaces 147
  4. Lifting of paths and surfaces 153
  5. 2-covering spaces and bundle gerbes 168

Chapter 5. 2-covering spaces II: Examples 173
  1. Some topology on mapping spaces 173
  2. The topological fundamental bigroupoid of a space 182
  3. The canonical 2-connected cover 196
  4. A 2-connected 2-covering space 199
  5. Vertical fundamental groupoid 213
Appendix A. Bicategories, bigroupoids and 2-groups

1. Bicategories 217
2. 2-groups 222

Bibliography 225
Abstract

This thesis introduces two main concepts: a fundamental bigroupoid of a topological groupoid and 2-covering spaces, a categorification of covering spaces. The first is applied to the second to show, among other things, that the fundamental 2-group of the 2-covering space is a sub-2-group of the fundamental 2-group of the base. Along the way we derive general results about localisations of the 2-categories of categories and groupoids internal to a site at classes of weak equivalences, construct a topological fundamental bigroupoid of locally well-behaved spaces, and finish by providing a rich source of examples of 2-covering spaces, including a functorial 2-connected 2-covering space.
Statement of originality

This thesis contains no material which has been accepted for the award of any other degree or diploma at any other university or other tertiary institution to me and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

I also give permission for the digital versions of my thesis to be made available on the web, via the University’s digital research repository, the Library catalogue, the Australasian Digital Theses Program (ADTP) and also through web search engines.

David Michael Roberts
Adelaide, South Australia
December 2009
Acknowledgements

I must first thank my principal supervisor, Michael Murray, for his patience and for trusting me to pretty much direct my own research. I also thank my supervisor Mathai Varghese for imparting to this one-time physicist a sense of thoroughness and mathematical rigor. His emphasis on the need for existence of nontrivial examples was a major motivation for this thesis. Mathai also helped organise a visit to the Erwin Schrödinger Institute in Vienna which gave me the opportunity to meet my now coauthor Urs Schreiber. The visit to ESI was jointly funded by the Institute and a Cathy Candler Award from AFUW-SA. I must also thank Jim Stasheff for agreeing to join my supervisory panel late in my degree – it was a step into the unknown to ask him and a most pleasant surprise to hear a ‘yes’. Nick Buchdahl, while his time as a supervisor was only a few months at the start of my candidature, always helped me when I turned up with bewildered questions, for which I say merci.

The postgraduate students in the School of Mathematical Sciences have formed a wonderful group of friends, and special thanks are due to David Butler and Jono Tuke for being there from the beginning. These two, together with Richard Green, Rongmin Lu, Ryan Mickler, Tyson Ritter and Raymond Vozzo, provided not only listeners to whom I could pour out my sometimes obscure theories, but welcome distractions and opportunities to learn things I would otherwise never have considered. The hosts of the $n$-category café have also provided a wonderful environment for the reading and discussion of many fascinating topics.

I greatly appreciate help with proofreading and suggestions for improvements from Richard Green, Rongmin Lu, Michael Murray, Tyson Ritter, Jim Stasheff and Raymond Vozzo. During the first four and a half years of my candidature I was supported by an Australian Postgraduate Award, and I am also grateful for study leave from my employer, the National Centre for Vocational Education Research, during the final stages of the writing up of this thesis.

Lastly, I would like to thank my mother, Shirley Woodhouse, for her constant patience and support throughout my entire time at university, and my fiancée, Dani-Marie Breen, for joining me on my journey, and filling it with many wonderful moments.