

# Fundamental bigroupoids and 2-covering spaces

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## Abstract

This thesis introduces two main concepts: a *fundamental bigroupoid of a topological groupoid* and *2-covering spaces*, a categorification of covering spaces. The first is applied to the second to show, among other things, that the fundamental 2-group of the 2-covering space is a sub-2-group of the fundamental 2-group of the base. Along the way we derive general results about localisations of the 2-categories of categories and groupoids internal to a site at classes of weak equivalences, construct a topological fundamental bigroupoid of locally well-behaved spaces, and finish by providing a rich source of examples of 2-covering spaces, including a functorial 2-connected 2-covering space.



## Statement of originality

This thesis contains no material which has been accepted for the award of any other degree or diploma at any other university or other tertiary institution to me and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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