



**THE UNIVERSITY OF ADELAIDE**

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School of Electrical and Electronic Engineering

**Analysis and Modelling of the Effects  
of Inertia and Parameter Errors on  
Wind Turbine Output Power**

**Chun Tang**

A thesis presented for the degree of Master of Engineering Science

2009

*Dedicated to my late grandmother, Honghui Xu*

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## **Abstract**

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Wind energy is an important renewable energy source. The average output power of a wind turbine is one of the main concerns in wind generation systems. The factors which affect the average output power include the location, the wind characteristics, the design of the blades and the control system etc. In this thesis, the effects of the inertia of a wind turbine under dynamic wind speed conditions, and the effects of the parameter errors under steady-state operation on the average output power are examined.

Maximum power point tracking is used to control the generator of a wind turbine in order to maximise the electrical output power of the wind turbine. However, under rapidly changing wind conditions, the output power of the wind turbines is reduced due to their inertia preventing them operating at the optimal turbine speed. Limited research into analysing this power reduction has been performed. Even under steady-state operating conditions, the maximum power coefficient and the optimal tip-speed ratio of the wind turbine generally need to be known for maximum power point tracking. Errors in the estimated parameters will result in an output power reduction for the wind turbine.

Therefore, an understanding of the sensitivity of wind turbine blade parameter errors to the output power reduction under steady-state conditions of wind turbines is also a significant issue.

The first part of the work in this thesis investigates the wind turbine output power reduction due to inertia under dynamic wind speed conditions. It is assumed that the wind turbine blade characteristics is known accurately and that a maximum power point strategy based on controlling the generator input torque as a function of generator speed is used (optimal torque control). The concept of the small-signal turbine time constant is introduced to denote the time constant of the response of a wind turbine for a small change in wind speed under the maximum power point operating conditions. It is shown that the turbine time constant is inversely proportional to the average wind speed, and the natural time constant is defined as the turbine time constant at the rated wind speed. An analytical equation for the small-signal output power reduction of a wind turbine with infinite inertia is then derived as the function of the ratio of the variance to the square of the average wind speed. For the small-signal finite inertia case, a scaling factor is added which is a function of the turbine time constant at the average wind speed and the “equivalent frequency” of the wind speed variations. Real wind speed data is utilised to test the analytical equation against simulation results for the power reduction with both infinite and finite inertia. As the wind speed profiles are not small-signal variations, the analytical results do not accurately predict the actual power reductions. The analytical results however provide useful physical insights into the differences in the power reductions with the different wind speed profiles and turbine inertia. Finally, some limited experimental measurements of the time-constant of a turbine are performed.

The second part of the work in this thesis investigates the effect of wind turbine blade parameter errors on the steady-state output power of a wind turbine. Two types of maximum power point tracking control strategies are investigated: constant tip-speed ratio control and optimal torque control. The analysis is carried out for a particular wind turbine blade characteristic. The steady-state output power reduction with errors in the maximum power coefficient and the optimal tip-speed ratio is shown graphically and compared for the two control strategies.

# Statement of Originality

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## List of Publications

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- [P1] C. Tang, M. Pathmanathan, W.L. Soong and N. Ertugrul, “Effects of Inertia on Dynamic Performance of Wind Turbines,” Australasian Universities Power Engineering Conference, AUPEC, Sydney 2008.
- [P2] M. Pathmanathan, C. Tang, W.L. Soong, N. Ertugrul, “Comparison of power converters for small-scale wind turbine operation”, Australasian Universities Power Engineering Conference, AUPEC, Sydney 2008.
- [P3] M. Pathmanathan, C. Tang, W.L. Soong, N. Ertugrul, “Detailed investigation of semi-bridge switched-mode rectifier for small-scale wind turbine applications”, IEEE International Conference on Sustainable Energy Technologies, 2008, Singapore, pp. 950-955.

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# Symbols

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$\omega$	turbine speed in rad/s
$\eta$	combined efficiency of the generator and the power electronics
$\rho$	air density or plastic density
$\lambda$	tip-speed ratio
$\gamma$	blade pitch angle
$\tau$	turbine time constant
$\sigma$	standard deviation of a wind speed variation
$\beta$	factor of the finite inertia effect on the power reduction
$\sigma^2$	variance of a wind speed variation
$\lambda_{(res)}$	resultant tip-speed ratio
$\lambda_{0(act)}$	actual optimal tip-speed ratio
$\lambda_{0(est)}$	estimated optimal tip-speed ratio

$\omega_{eq}$	equivalent frequency of a real wind speed variation
$\lambda_o$	optimal tip-speed ratio
$\tau_o$	natural time constant
$\Delta P$	power reduction
$\omega_{rated}$	rated turbine speed in rad/s
$\Delta T$	acceleration torque
$\Delta v$	peak variation of wind speed
$A$	swept area of blades
$C_p$	power coefficient of wind turbines
$C_{p(res)}$	resultant power coefficient
$C_{p0}$	maximum power coefficient
$C_{p0(act)}$	actual maximum power coefficient
$C_{p0(est)}$	estimated maximum power coefficient
$C_{pmax}$	maximum power coefficient
$C_t$	torque coefficient
$J_g$	rotor inertia of the generator
$J_t$	turbine inertia
$k$	ratio of the optimal turbine speed to the current wind speed
$k_0$	ratio of the optimal torque to the square of turbine speed
$m_R$	mass of the rectangular shape blade
$m_{rotor}$	mass of the rotor blades
$m_T$	mass of the triangular shape blade
$n$	turbine speed in rpm
$n_{opt}$	optimal turbine speed in rpm
$P_{(res)}$	resultant output power

$P_{0(act)}$	actual maximum output power
$P_{0(est)}$	estimated maximum output power
$P_J$	output power with finite inertia
$P_{J=\infty}$	output power with infinite inertia
$P_{J=0}$	output power with zero inertia
$P_m$	mechanical output power
$P_{out}$	output power of the wind turbine system
$P_{rated}$	power rating
$P_{reduction}$	power reduction
$R$	radius of the rotor of a wind turbine
$T_g$	generator torque
$T_{opt}$	optimal generator torque
$T_{opt}^*$	reference of the optimal generator torque
$T_{rated}$	rated torque of a wind turbine
$T_t$	turbine torque
$v$	wind speed
$v_{CMC}$	cube-root-mean-cube of wind speed
$v_m$	average wind speed
$v_{rated}$	rated wind speed



# Abbreviations

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CTC	constant tip-speed ratio control
MPPT	maximum power point tracking
OSP	over-speed protection
OTC	optimal torque control
PR	power reduction
TI	turbulence intensity
TSR	tip-speed ratio