Nonlinear Dynamics Of
Magnetic Bearing Systems

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Abstract

The aim of the work outlined in this thesis was to gain a deep insight into the effects of nonlinearities of magnetic bearings and the influence of time delays occurring in the feedback control path on their dynamic behaviour and performance. Emphasis was on stability analysis, bifurcation control, determination of stable operating conditions, prediction of bearing performance, and on aspects of nonlinear dynamic behaviour including bifurcations, coexistence of multiple solutions and complicated motions.

Magnetic bearings use magnetic forces to support various machine components. Because of the non-contact nature of suspension, this new bearing technology offers a number of significant advantages over conventional bearings such as rolling element and fluid film bearings. These advantages include elimination of the lubrication system, friction free operation, no wear, high rotor speed, adjustable dynamic properties, vibration control, on-line identification and fault diagnosis capabilities, and active health monitoring of rotodynamic systems.

Magnetic bearings have found their applications in turbomachinery, centrifuges, vacuum machinery, machine tool spindles, medical devices, robotics, high-speed drives, spacecraft equipment, contactless actuators, and vibration isolation, etc.

The stable operation of a magnetic bearing system can only be achieved by feedback control. A magnetic bearing system is basically composed of sensors, controllers,
amplifiers and electromagnets. All of these components are characterized by nonlinear behaviour and therefore the entire system is inherently nonlinear. However, in simulations of the dynamic behaviour of magnetic bearing systems, the nonlinearities were usually neglected for simplicity and the components of magnetic bearing systems were simplified to linear models. Moreover, many of control techniques currently used in magnetic bearing systems were generally designed by ignoring the nonlinearity of the magnetic forces and the nonlinear modelling of the sensors and actuators. The main reason for simplification was the intractability of the complexity of the actual model. In fact, the inherent nonlinear properties of magnetic bearing systems can lead to a dynamic behaviour of the rotor-magnetic bearing system that is distinctly different from that predicted using a simple linearized model. Therefore the nonlinearities should be taken into account. For certain cases, it may perhaps be theoretically possible to compensate partially certain nonlinearities at a cost of highly complex control strategies. However, nonlinearities originating from hardware and physical system limitations cannot be eliminated by software and the control problem becomes very complicated due to the inherent nonlinearities associated with the electromechanical dynamics introduced into the magnetic bearing system.

The present thesis comprises 12 papers which were recently published in seven international journals. These papers contribute to study: 1) the effects of geometric coupling and nonlinear force relationships on the dynamic behaviour of magnetic bearings; 2) the effects of the nonlinear force relationship incorporating time delays or saturation of power amplifier (or saturation of magnetic material or limitation of the control current) on the dynamic behaviour and performance of magnetic
bearings; 3) the effect of a combination of up to three components of nonlinearity on the dynamic behaviour and performance of magnetic bearings; 4) the nonlinear response of rotor-magnetic bearing systems under primary, sub-harmonic, and super-harmonic resonance conditions. These papers also contribute to develop: 1) an analytical technique (which is referred to as the matching method) for constructing an approximate solution to a periodically excited nonlinear-linear system; and 2) develop a nonlinear control approach from the bifurcation control point of view to stabilize a subcritical Hopf bifurcation, thereby extending the operational region.

It is found that in the presence of nonlinear magnetic forces, rotor-magnetic bearing systems may exhibit a variety of nonlinear behaviours including saddle-node bifurcations, Hopf bifurcations, jump phenomena, coexistence of multiple solutions, quasi-periodic and chaotic motions. For a rotor-magnetic bearing system with saturation constraints, it is shown that the forced response of the system may accept symmetric and asymmetric period-one solutions, subharmonic and chaotic solutions.

It is also found that the time delays occurring in the feedback control path may have a significant impact on the stability and dynamics of rotor-magnetic bearing systems. For a rotor supported by a two-pole magnetic bearing, it is shown that a Hopf bifurcation can occur when time delays pass certain values. Co-dimension two bifurcations, which result either from a non-resonant or resonant Hopf-Hopf interaction or from an interaction of a Hopf and a steady state bifurcation, are also found to be possible after the trivial fixed point loses its stability through Hopf bifurcations. Increasing the values of time delays not only can increase the peak
amplitude of the forced response but also can shift the frequency-response curve to the right.

For a rotor-magnetic bearing system involving both geometric coordinate coupling and time delay, it is found that as the time delay increases beyond a critical value, the equilibrium position of the rotor motion becomes unstable via a Hopf bifurcation of multiplicity two and may bifurcate into two qualitatively different kinds of periodic motion. An interaction between the Hopf bifurcating periodic solutions and the external periodic excitation can induce primary and super-harmonic as well as sub-harmonic resonances in the neighbourhood of the Hopf bifurcation. The forced nonlinear response of the system may exhibit pitchfork bifurcations, Hopf bifurcations, symmetric and asymmetric phase-locked periodic motions, quasi-periodic motions, chaotic motions, and coexistence of two stable motions.
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