Drought Predictions: Applications in Australia

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Abstract

Drought is a global and recurrent natural phenomenon, the inevitable consequence of meteorological variability. This natural hazard brings about devastating effects because water is one of the most fundamental commodities for human survival, and a lack of water can result in varying consequences, from mere inconvenience to life-threatening instances. Drought cannot be prevented but its effects can be mitigated through the design of appropriate water resource infrastructure and management strategies. The goal of this thesis is to model the spatial and temporal characteristics of drought occurrence in Australia, the driest continent. In doing so, predictions can be made, levels of risk can be evaluated, and conditional estimates of drought can be based on climatic state variables.

For insight into the nature of drought in Australia, multivariate models of drought characteristics are developed. Preliminary analysis demonstrates high correlations between several drought characteristics; these are the drought severity, intensity, and duration. This thesis applies the copula concept, which is a versatile means of modeling their dependence structure. Copulas are multivariate uniform distributions, which allow the joint behavior of variables to be modeled independently from their marginal distributions. This research extends the application of copulas by investigating the effect of climate variability on copula models and subsequent drought characteristics. Two different copula families are fitted to the drought characteristics to demonstrate the importance of tail dependence when modeling extreme climatic events. An important application of these models is the calculation of return periods of extreme drought events exceeding certain thresholds, taking account of variability in climatic indices.

A second objective is to forecast drought at various spatial resolutions. The most straightforward method are regression and ARMA models that incorporate global climatic indicators. The effect of climatic variation on Australia’s precipitation is examined by investigating the association between climatic indices and the multivariate distribution of drought at numerous sites across Australia. Two classification strategies for forecasting rainfall are compared using significance testing based on multiple comparison techniques. Further to this, rainfall forecasting relationships are explored using global sea-surface temperature anomalies.

The versatility of copula models is demonstrated through short-term rainfall predictions for neighboring rainfall districts, using separate copulas conditioned on antecedent climate conditions. This technique is shown to improve rainfall predictions in neighboring districts and improve estimates of drought probability.
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Declaration

I hereby declare this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Acknowledgments

“Trust in the LORD with all your heart and lean not on your own understanding; in all your ways acknowledge Him, and He will make your paths straight.”

Proverbs 3:5-6

This three-year journey has finally reached its ‘destination’ and I would like to give God all the glory and thanks, for His faithfulness and providence during this time.

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# Glossary of Technical Terms and Abbreviations

## Climate terminology

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<tbody>
<tr>
<td>ENSO</td>
<td>El-Niño Southern Oscillation</td>
</tr>
<tr>
<td>IOD</td>
<td>Indian Ocean Dipole</td>
</tr>
<tr>
<td>IPO</td>
<td>Inter-decadal Pacific Oscillation</td>
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<tr>
<td>MEI</td>
<td>Multivariate ENSO Index</td>
</tr>
<tr>
<td>PDO</td>
<td>Pacific Decadal Oscillation</td>
</tr>
<tr>
<td>SLP</td>
<td>Sea Level Pressure</td>
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<tr>
<td>SST</td>
<td>Sea-surface temperature</td>
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<td>SOI</td>
<td>Southern Oscillation Index</td>
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## Drought terminology

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>PDHI</td>
<td>Palmer Drought Hydrological Index</td>
</tr>
<tr>
<td>PDSI</td>
<td>Palmer Drought Severity Index</td>
</tr>
<tr>
<td>SPI</td>
<td>Standardized Precipitation Index</td>
</tr>
<tr>
<td>$S_t$</td>
<td>SPI at time $t$</td>
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## Statistical terminology

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<tr>
<td>AIC</td>
<td>Akaike’s Information Criterion</td>
</tr>
<tr>
<td>AEP</td>
<td>Annual Exceedance Probability</td>
</tr>
<tr>
<td>ARMA</td>
<td>Auto-regressive Moving Average</td>
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$S_t$ denotes the SPI at time $t$.
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<th>Term</th>
<th>Description</th>
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<tr>
<td>ARIMA</td>
<td>Auto-regressive Integrated Moving Average</td>
</tr>
<tr>
<td>$R^2$</td>
<td>Coefficient of determination</td>
</tr>
<tr>
<td>$\varphi(t)$</td>
<td>Copula generator</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Copula parameter</td>
</tr>
<tr>
<td>$cdf$</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>IFM</td>
<td>Inference Function of Margins</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>Lower tail dependence</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimation</td>
</tr>
<tr>
<td>$C(u_1, \ldots, u_n)$</td>
<td>$n$-dimensional copula</td>
</tr>
<tr>
<td>$pdf$</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>$\lambda_U$</td>
<td>Upper tail dependence</td>
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