



# **The discrete rigid body rotation of reinforced concrete beams using partial interaction and shear friction theory**



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## INTRODUCTION

In reinforced concrete members, two types of deformations exist: deformations due to curvature distribution and deformation due to discrete localised conditions such as cracks. The latter is the subject of this thesis, which presents a new approach for evaluating the discrete moment-rotation relationship of reinforced concrete members.

This thesis is a collection of submitted, accepted or published papers from internationally recognised Journals, where the titles of Chapters 1 through 13 reflect the titles of the Journal papers. Each chapter takes the following format: the key theory and results from each journal paper are presented in a short synopsis, after which the journal paper is presented in full. This provides the reader, if desired, with the ability to understand the research in full by only reading the synopses of each chapter.

In Chapter 1, the peripheral areas of shear friction theory, partial interaction theory and rigid body displacement are combined to quantify the moment-rotation response of any reinforced concrete member. It is discussed how partial interaction theory is used to model the behaviour of the reinforcement, shear friction theory the behaviour of the concrete, and that both these behaviours are combined through a rigid body displacement profile. This rigid body rotation approach is a structural mechanics model, and hence can be used to quantify the moment-rotation response of any reinforced concrete member (the subject of this thesis), and amongst other things the shear capacity of a concrete member and the influence of confinement on member behaviour (not covered in or the subject of this thesis). Importantly, closed form solution can be developed for all failure mechanisms, and flexural failure closed form solutions are presented in this thesis.

In Chapters 2-4, the partial interaction behaviour of steel reinforcing bars and externally bonded and near surface mounted FRP plates is described in detail. Specifically, Chapters 2 and 3 quantify the bond characteristics of steel reinforcing bars and embedded near surface mounted (NSM) fibre reinforced polymer (FRP) plates. Previously published pull test data is analysed to determine the local bond stress-slip relationship of deformed steel reinforcing bars, and experimental testing of embedded NSM FRP plates performed and analysed to determine the influence of embedment on the bond behaviour of embedded NSM FRP plates. In Chapter 4, partial interaction mathematical expressions are developed to model the elasto-plastic load-slip ( $P-\Delta$ ) behaviour of steel reinforcing bars. These mathematical expressions are later used to model the behaviour of externally bonded (EB) and NSM steel plates. These mathematical load-slip expressions are a critical component of the moment-rotation analysis technique.

In Chapter 5, the rigid body rotation model is described in detail and an iterative solution technique presented. The predicted moment-rotation behaviour of reinforced concrete beams is compared to that obtained experimentally with excellent accuracy. In Chapter 6, a closed form solution technique is presented for determining the moment-rotation response of an unplated or plated reinforced concrete member. This is a more user friendly analysis technique and can be used directly to determine the moment and rotation

at failure without developing the entire moment-rotation response from the start of loading to failure. In Chapter 7, the influence of bond on the rotation capacity of both unplated, externally bonded and near surface mounted plated reinforced concrete members is assessed, where it is shown that bond is a critical component of ductility.

In Chapters 8-11, an application of the moment-rotation model is presented. Moment redistribution expressions are derived for propped cantilevers and continuous members from elementary structural mechanics, and it is shown that the moment redistribution capacity of a member is proportional to the moment and rotation capacity of the section. Various examples of moment redistribution are presented, and finally a method to design for moment redistribution is presented where it is shown that the non-hinging region needs to remain elastic for the hinge to redistribute its maximum moment.

In Chapters 12 and 13, the shear friction behaviour of concrete is examined in detail and a method for extracting the shear friction parameters of initially uncracked hydrostatically confined concrete is presented. A generic expression for the shear friction parameters of initially uncracked concrete is developed, where the shear stress is a function of the normal stress across and displacement of the sliding plane, and the compressive strength of concrete. A previously developed expression for the shear friction parameters of initially cracked concrete is modified so that the shear stress is expressed as a function of the normal stress across and displacement of the sliding plane and the compressive strength of concrete. Expressing the shear friction parameters in this way allows bounds to be developed using Mattock's shear stress limits. These bounds for both initially cracked and uncracked concrete are expressed mathematically for the generic shear friction parameters. The development of these shear friction parameters and bounds will allow the rigid body rotation model to consider sliding failure of concrete, and quantify the effect of confinement through stirrups or FRP wrapping on the moment-rotation response.

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## NOTATION

$A$	cross sectional area
$a_{el}$	development length required to yield the steel reinforcement
$a_{fract}$	additional embedment length required to increase the load in the reinforcement from the yield to fracture
$A_{rebar}$	cross sectional area of a reinforcing bar
$B$	bond force
$c$	cohesive component of Mohr-Coulomb failure plane
$c$	cover
$c_{\tau}$	shear friction parameter
$d$	depth of FRP or steel plate
$d_{asc}$	depth of the non-softening region of concrete
$d_b$	diameter of reinforcing bar
$d_{prism}$	diameter of a concrete prism
$d_{rebar}$	distance from the crack apex to the location of the reinforcing bars
$ds/dx$	slip strain
$d_{soft}$	depth of the softening region of concrete
$E$	elastic modulus
$E_c$	Elastic modulus of concrete
$EI$	flexural rigidity
$E_s$	Young's modulus of steel
$E_{sh}$	strain hardening modulus of steel
$f_c$	compressive strength of concrete
$f_{co}$	unconfined compressive strength of concrete
$f_y$	yield stress
$G_f$	interfacial fracture energy of bond stress-slip relationship
$h_{cr}$	crack height in rigid body rotation model
$h_{cr}$	crack separation across a sliding plane
$h_{plate}$	crack width at the location of the plate
$h_{rebar}$	crack width at the location of the reinforcing bar
$h_{soft}$	vertical displacement due to the formation of a softening wedge
$k_b$	bond stress factor
$K_{MR}$	moment redistribution factor
$k_u$	neutral axis parameter
$L$	span length
$L_{deb}$	length over which debonding can occur without a drop in axial load
$L_{hinge}$	plastic hinge length
$L_{per}$	perimeter of failure plane of axial reinforcing
$L_{prism}$	length of a concrete prism
$L_{soft}$	length of the softening region of concrete
$m$	frictional component of Mohr-Coulomb failure plane
$M$	moment
$M_{cap}$	moment capacity
$M_h$	moment at the support in a continuous member
$M_{hog}$	moment at support in a continuous member

$M_s$	moment at mid-span in a continuous member
$M_{st}$	static moment
$M_{yield}$	moment at yield of steel
$m_\tau$	shear friction parameter
$P$	point load
$P_{asc}$	force in the non-softening region of concrete
$P_{cc}$	total compressive force in the concrete
$P_{IC}$	load at which intermediate crack debonding commences
$P_{plate}$	axial force in a plate
$P_r$	force in the reinforcement
$P_{reinf-bar}$	axial force in a reinforcing bar
$P_{soft}$	force in the softening region of concrete
$R_{pl}$	dimensionless parameter that indicates if yield occurs before debonding in EB and NSM plates
$s$	slip
$sp$	stirrup spacing
$s_{slide}$	displacement of the softening wedge at which uncontrollable sliding occurs
$s_{soft}$	displacement of the softening region of concrete
$\nu_c$	Poisson's ratio of concrete
$w$	applied uniformly distributed load
$w_b$	width of a reinforced concrete beam
$\%C$	volumetric percentage weight loss due to corrosion
$\alpha$	angle the softening wedge of concrete forms
$\Delta$	global displacement
$\Delta_{deb(end)}$	displacement at the end of debonding allowing for localised debonding
$\Delta_{fract}$	displacement (slip) at fracture of axial reinforcement
$\Delta_L$	displacement (slip) of axial reinforcement at the loaded end from a pull test
$\Delta_{plate}$	slip of a plate (steel or FRP) across a crack front
$\Delta_{rebar}$	slip of a reinforcing bar across a crack front
$\Delta_{sliding}$	slip at sliding failure
$\Delta w$	proportional increase in applied load due to moment redistribution
$\Delta_{yield}$	slip at which steel reinforcement yields
$\Delta_{yp-ult}$	slip at fracture of the reinforcing bar
$\chi_{fract}$	curvature at bar fracture
$\delta$	local displacement (slip)
$\delta_1$	slip at which the peak bond stress is first achieved in local bond stress-slip relationship
$\delta_{axial}$	axial displacement in a concrete prism under hydrostatic confinement
$(\delta_{lat})_{sm}$	lateral displacement of wedge in a prism
$\delta_{max}$	peak slip from the bond stress-slip relationship
$\varepsilon$	strain

$\epsilon_{\text{axial}}$	axial strain in a concrete prism
$\epsilon_{\text{c}}$	strain in concrete
$\epsilon_{\text{cone}}$	strain in the “cone region” of a concrete prism
$\epsilon_{\text{fract}}$	strain at bar fracture
$\epsilon_{\text{IC}}$	strain at commencement of debonding
$\epsilon_{\text{lat}}$	lateral strain in a concrete prism
$\epsilon_{\text{pk}}$	concrete strain at the achievement of the peak compressive strength
$\epsilon_{\text{r}}$	strain in the reinforcement
$\epsilon_{\text{wedge}}$	strain in the softening wedge of concrete
$\varphi_{\text{f}}$	aspect ratio of failure plane of axial reinforcement
$\lambda_{\text{el}}$	elastic partial interaction parameter
$\lambda_{\text{sh}}$	plastic partial interaction parameter
$\theta$	rotation
$\theta_{\text{cap}}$	rotation capacity
$\theta_{\text{emp}}$	empirical rotation from plastic hinge length
$\theta_{\text{fract-limit}}$	rotation at fracture of reinforcing bar
$\theta_{\text{h}}$	rotation at support
$\theta_{\text{s}}$	rotation at midspan
$\theta_{\text{total}}$	total rotation in a continuous member due to moment redistribution
$\theta_{\text{yield}}$	rotation at yield of steel
$\kappa$	initial stiffness of the bond stress-slip relationship
$\sigma$	stress
$\sigma_{\text{lat}}$	lateral confinement
$\sigma_{\text{N}}$	stress normal to a sliding plane
$\sigma_{\text{soft}}$	softening stress
$\sigma_{\text{residual}}$	residual stress in concrete
$\tau$	shear stress
$\tau_{\text{max}}$	peak bond stress from the bond stress-slip relationship
$\tau_{\text{N}}$	shear stress along a sliding plane