THE NEVER ENDING STORY OF MODELING CONTROL-DEVICES IN HYDRAULIC SYSTEMS ANALYSIS

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Citation: Deuerlein, J., Simpson, A.R. and Gross, E. (2008). “The Never Ending Story of Modeling Control Devices in Hydraulic Systems.” *10th Annual Symposium on Water Distribution Systems Analysis,* American Society of Civil Engineers, Kruger National Park, South Africa, 17-20 August.

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Abstract

Difficulties of simulation in existing hydraulic models arising from combinations of pressure and flow controlling devices in water distribution systems have been discussed in a number of previous papers. For instance, examples for non-convergence or wrong results of the hydraulic solver EPANET (version 2.00.10) were first published by Simpson in 1999. It may be shown that the problems were caused by a singularity of the equation system that appears if in an iteration two interacting control devices are active at the same time. In terms of graph theory the part of the network between the two active valves in this case is disconnected from the rest of the system leading to the singularity.

In the new EPANET version 2.00.12 that has been released recently this problem is tackled by adding a virtual coefficient to all matrix columns and rows corresponding to nodes of active flow control valves. Mathematically this method is equivalent to adding a very small diameter pipe to the actual network in parallel to the FCV resulting in a nonsingular system. The examples of networks published by Simpson (1999) where EPANET 2.00.10 failed to converge or converged to wrong results now can be solved successfully. Nevertheless the latest release of EPANET still has difficulties in modeling of combinations of control devices. Whereas the former version of EPANET (version 2.00.10) often failed to calculate the correct valve states (active, closed, open) the problems of the new version consist of numerical inexactness that is caused by the addition of the virtual matrix terms for FCVs. In addition examples can be found where version 2.00.12 of EPANET still fails to converge.

1. INTRODUCTION

Especially in developing countries water supply is often not continuous but intermittent. As a consequence separated subzones of the system are often disconnected from any water source like a reservoir, tank or pumping station. The supply is only for few hours a day or even worse only on certain days. As a consequence people try to get as much water as they can during supply hours. From a technical point of view that behavior leads to abnormally high velocities and headlosses leading to insufficient pressure conditions in some parts of the distribution system. Due to the low pressures and the large number of leaks contaminants may enter the pipe system leading to the possibility of very poor or even dangerous water quality. Often the water resources supply is sufficient. Thus one of the most important issues of a rehabilitation program of intermittently operated systems is the transition to continuous supply. After the definition of supply zones and sectors the transition process is executed zone by zone. The scenarios must be carefully planned enabling intermittent and continuous supply at the same time.

The planning and calculation of the transition scenarios often requires the application of control devices in the hydraulic model. In reality, control devices are used for different purposes. Pressure reducing valves are used to decrease the pressure at specified locations, for example, in order to reduce leakage losses. Flow control devices reduce the maximum flows to a given limit or are used to prevent backflows. In hydraulic modeling those devices are in addition to the representation of existing shutoff valves which are a valuable tool in planning and reconfiguration of supply areas. For instance they are used for the control of inflows and outflows of the supply zones or the operation of storage tanks. In the case where the modeler works with large simulation models consisting of several supply areas that are further subdivided into zones it is important that system states having infeasible valve settings are detected by the model and that the calculation results are reliable. It has been observed that EPANET sometimes fails to converge or even worse converges on the wrong results. The combination of flow and pressure controlling devices has been frequently discussed before. Simpson (1999) published the study of a simple example system with a FCV and a PRV in series and the comparison of the results of different network solvers.

In the first part of this paper the existence and uniqueness of the hydraulic steady-state of simple pipe networks including flow and pressure controlling devices is discussed from a theoretical point of view. At that stage only physical properties are considered. In the second part the results are used for the explanation of convergence problems and wrong results calculated by EPANET. The new version 2.00.12 that was released in March 2008 and the older version EPANET 2.00.10 show different behavior regarding the modeling of control devices. Examples have been calculated with both versions. It has been established that the problems discussed by Simpson (1999) can be solved with the new version. However there exist other examples where version 2.00.12 fails to converge or converges to wrong solutions. In turn some of those examples may be solved successfully with the older version. A selection of example systems is presented and the reasons for the new problems of the numerical algorithm are discussed. In addition, an alternative approach of tackling combinations of flow and pressure controlling devices in EPANET is outlined. The first method presented enables the detection of infeasible flow conditions before the iterative calculation takes place. The second makes use of the calculation of parameter sensitivities. The explanation of the calculation of the sensitivities using EPANET is followed by the identification of the control valves that are causing the singularity. The implementation of the method does not require the modification of the matrix by adding values corresponding to virtual pipes (version 2.00.12) with the associated problem of inexactness. In fact, the alternative approach implements a new function that replaces the former `badvalve(n)` function of the EPANET code. With this function the interdependent valves can be identified. For each pair of interacting valves the status of the valve that was already active during the previous iteration is changed to inactive. This is repeated for all valve pairs until a configuration is reached where the coefficient matrix is non-singular. With the correct estimation of active and inactive valves further calculations are straight-forward.

2. HYDRAULIC STEADY-STATE CALULATION OF SYSTEMS UNDER CONTROL

Problem Formulation

The hydraulic steady-state is defined by the continuity of flows (Eq. 1 (a)) at the nodes of the network, the compatibility of nodal pressures and headlosses along the pipes (Eq. 1 (b)) and a certain hydraulic relation between the flow and the headloss (Eq. 1 (c)) of each network feature

$$\mathbf{A}^T\mathbf{q} = \mathbf{Q} \quad (a), \quad \mathbf{h} + \mathbf{I}_{HG}\mathbf{z} + \mathbf{A}\mathbf{H} = -\mathbf{A}_R\mathbf{H}_R \quad (b), \quad \mathbf{D}\mathbf{q} = \mathbf{h} \quad (c), \tag{1}$$

where **A** is the incidence matrix of the network graph, **q** is the flow vector and **Q** is the vector of nodal demands. The vector **h** represents the headlosses due to friction, $\mathbf{I}_{HG}$ is the indicator matrix of links with given headloss values **z** (in EPANET denoted as Pressure Breaker Valves or PBVs) and the diagonal matrix **D** includes the derivatives of the hydraulic headloss equation with $D_{jj} = c_j|q_j|^{\alpha-1}$. The subscript R indicates matrices and vectors that belong to fixed grade nodes (Nielsen 1989). The question of existence and uniqueness of the hydraulic steady state has been discussed extensively in the past. For simple networks without control devices Birhoff (1963) published a variational principle proofing uniqueness under certain monotonicity assumptions for the headloss equation. The methods were renewed later by Collins et al. (1978). In that case the problem of calculating the steady-state of a pipe network was mathematically modeled by an equivalent minimization problem of the so called system content function (Collins et al., 1978). The content function Π^c is determined by

$$\Pi^{c}(\mathbf{u}) = \frac{1}{\alpha+1}(\mathbf{q}_{t} + \mathbf{Cu})^{T}\mathbf{D}(\mathbf{q}_{t} + \mathbf{Cu}) + \mathbf{u}^{T}\mathbf{C}^{T}\left[\mathbf{A}_{R}\mathbf{H}_{R} + \mathbf{I}_{HG}\mathbf{z}\right], \tag{2}$$

where $\mathbf{q}_t$ denotes a flow vector that solves the mass balance of the system (e.g. the flow distribution of a spanning tree), **u** is the vector of unknown loop flows and **C** is the loop matrix. Here, the minimization of the system content (Eq. 2) is formulated in the unknown loop flows $\mathbf{u} \in \mathbf{U}$. As long as no flow controlling devices are considered the feasible set **U** consists of the whole $\mathbf{R}^n$ (n: number of loops).

Flow constrained problems

The additional consideration of control devices leads to the formulation of inequality constraints of flows and pressures. Whereas the existence and uniqueness of the hydraulic steady-state of simple systems (without control devices) requires the monotonicity of the hydraulic functional relation between flow and headloss (Birkhoff 1963) for systems with flow control then the inequality conditions resulting from flow controlling devices must be proven additionally for consistency. It is necessary for the existence of a solution that the polyhedral set

$$\mathbf{U} = \left\{\mathbf{u} \in \mathbf{R}^{l} \,\middle|\, \mathbf{Gu} \le \mathbf{b}_1, \mathbf{Hu} = \mathbf{b}_2\right\} \text{with } \mathbf{G} = \mathbf{I}^{\mathbf{T}}_{\boldsymbol{IC}}\mathbf{C},\ \mathbf{G} = \mathbf{I}^{\mathbf{T}}_{\boldsymbol{EC}}\mathbf{C} \tag{3}$$

of feasible loop flows, which is described by the equality and inequality conditions resulting from the operation of flow control devices, is nonempty. **C** is again the loop matrix**,** $\mathbf{I}_{IC}$ and $\mathbf{I}_{EC}$ are the index matrices of links with inequality and equality constraints for the flows and $\mathbf{b}_1$ and $\mathbf{b}_2$ are the corresponding right hand side vectors of the constraints. From nonlinear optimization it is known that under a suitable constraint qualification (CQ) the Karush-Kuhn-Tucker (KKT) conditions hold at a local minimum $(\mathbf{u}^*, \boldsymbol{\mu}^*, \boldsymbol{\lambda}^*)$:

$$\nabla_{\mathbf{u}} L(\mathbf{u}^*, \boldsymbol{\mu}^*, \boldsymbol{\lambda}^*) = \mathbf{0}, \qquad \mu_i^*(\mathbf{Gu}^* - \mathbf{b}_1)_i = 0, \quad i = 1,\ldots,m, \quad \mu_i^* \ge 0, \quad i = 1,\ldots,m, \tag{4}$$

where $\mathrm{L}(\mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \Pi^c(\mathbf{u}) + \boldsymbol{\mu}^T(\mathbf{Gu} - \mathbf{b}_1) + \boldsymbol{\lambda}^T(\mathbf{Hu} - \mathbf{b}_2)$ is the Lagrangian and $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are the Kuhn-Tucker multipliers. The physical meaning of the multipliers is that they represent the minor headloss that has to be generated by the valve in order to observe the flow conditions. It can be shown that second order optimality conditions hold by guaranteeing strict convexity of the objective function. Together with the coercivity of Π^c and non-emptiness of the polyhedral set **U** (Eq. 3), the existence of a unique flow distribution can be proven.

Flow and pressure constrained problems

Modeling of distributed feedback devices where the set value is not within the same device (e.g. PRV: the head of the downstream node of the PRV is controlled by the headloss of the PRV-link) is not possible to be posed as a single optimization problem. In this case the identification of the correct value for the headloss generated by the PRV is a matter of inverse modeling. The vector **z** in Eq. 1 has to be determined by additional conditions for the pressure at the set pressure nodes. For each of the *q* pressure regulating devices an additional optimization problem is formulated minimizing the difference between the calculated pressure and the set pressure $H_{set,i}$:

$$\min_{z_i} \frac{1}{2}\left|H_{u,i}(\mathbf{u},\overline{\mathbf{z}}) - z_i - H_{set,i}\right|^2, \quad z_i \geq 0, \quad i = 1,...,q. \tag{5}$$

The variable of the minimization problem in Eq. 5 is the value of the minor headloss z_i that is operated by the i-th pressure regulating device. The pressure at the upstream node $H_{u,i}$ of valve *i* depends on the flow distribution of the system that is determined by **u** and the headlosses $\overline{\mathbf{z}}$ generated by the other pressure regulating devices ($\overline{\mathbf{z}} = \mathbf{z}_{\backslash i}$). The corresponding KKT-conditions are (with Lagrange-multipliers ν_t):

$$H_{u,i}(\mathbf{u},\overline{\mathbf{z}}) - z_i - H_{set,i} + \nu_i = 0, \nu_i(-z_i) = 0, \nu_i \geq 0, -z_i \leq 0, \quad i = 1,...,q. \tag{6}$$

Combining the KKT-conditions given by Eq. 4 and Eq. 6 the hydraulic steady state of systems with general control devices can be modeled as a Nash equilibrium of q+1 parametric nonlinear optimization problems that derive from mathematical game theory (Deuerlein et. al., 2005). An alternative formulation of the Nash-equilibrium is possible by a Variational Inequality (VI) problem representing a generalization of the nonlinear optimization model. Harker and Pang (1990) published a comprehensive survey of variational inequalities. For a solution, a so-called generalized KKT-Point of the VI can be calculated. The proof of the existence and uniqueness of such a point is more complicated than that of the convex programming problem and is beyond the scope of this paper.

3. EXISTENCE AND UNIQUENESS OF THE HYDRAULIC STEADY-STATE

Working with large systems it is often not easy to detect inconsistent flow conditions. Especially in the case of interacting flow and pressure controlling devices configurations can be found where either the hydraulic steady-state is non-unique or does not exist at all. In the following the problem will be clarified using a simple system consisting of two reservoirs that are connected by a substantial length of pipeline. The flows and pressures are controlled by two control devices.

Non-uniqueness of the hydraulic steady-state: Example with FCV-PRV in series

The first example includes the network of Simpson (1999) with one FCV and one PRV in series. Imagine now that starting with an active FCV (Q_s = 400 L/s) the system in Fig. 1 is run over a period of time. Then, the water level of tank T_2 is increasing because its volume is finite and there is no outflow from the tank. Along with the water level in the tank the pressure head at the set pressure node of the PRV is also increasing until the set pressure is reached. At this time, both the flow through the FCV and the pressure at the downstream node of the PRV are both exactly at their set values. As a consequence, both valves could theoretically be in an active state. Trying to draw the hydraulic grade line (HGL) of this state it is obvious that this state of the system is unstable with respect to the heads between the FCV and the PRV (Fig. 1). There exist an infinite number of combinations of headlosses generated by the PRV and the FCV that could lead to the desired conditions of both valves. For example, if the flow is controlled by the FCV alone the headloss generated by the PRV is still zero. However the set pressure for the PRV is already reached (HGL “a” in Fig. 1). If the control passes over to the PRV a linear combination of both headloss generators is possible (HGL “b” in Fig. 1). For this case both control valves are in an active state. This situation is physically and mathematically unstable. The HGL “c” in Fig. 1 indicates the case where the control is borne only by the PRV and the headloss generated by the FCV is zero.

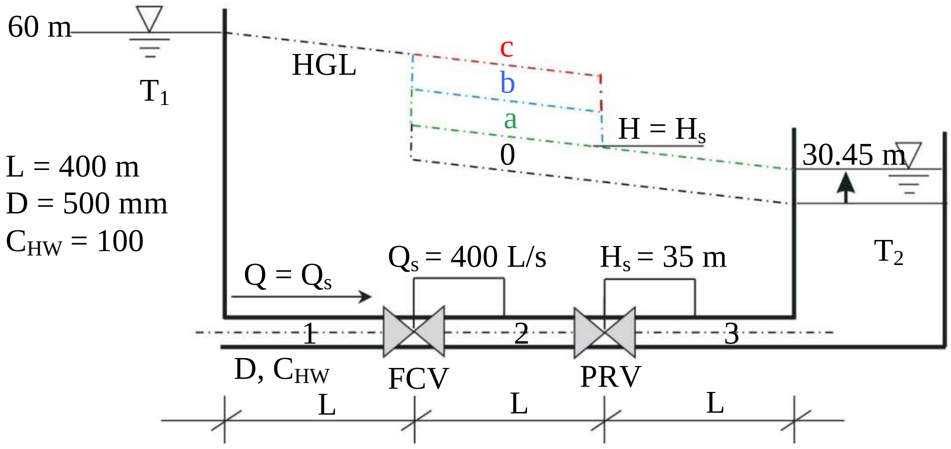

Figure 1: Non-uniqueness of the hydraulic grade line (HGL)

Multiple FCVs in series

The same unstable behavior may be observed when analyzing networks with multiple FCVs in series. As an example, the system of Fig. 1 is considered with the PRV replaced by a second FCV also having a set value of 500 L/s (Fig. 2). It is easy to determine the slope of the HGL because the flow is known from the active state of the valve. In contrast the location of the HGL of the inner network part between the two FCVs is not well defined. There are infinitely many solutions between full control by the first valve and full control by the second valve. For an active flow or pressure regulating device there is no functional relation between headloss and flow through the valve. In fact, the head at the upstream and downstream nodes of the FCVs is determined by the hydraulics of the remainder of the system. Based on graph theoretical mapping the active FCVs can be replaced by a constant in- and outflow meeting the set value at the inlet and outlet node of the valve (Fig. 2).

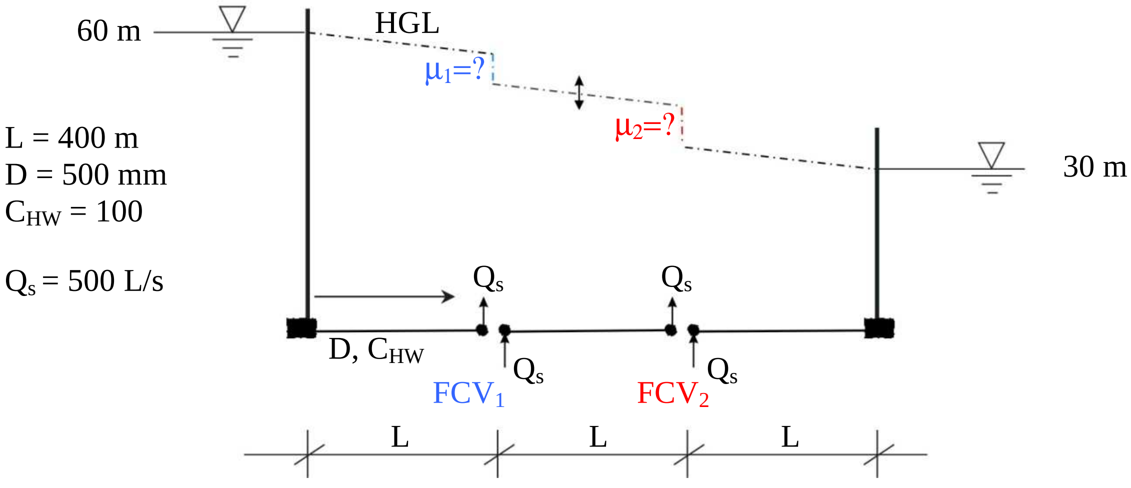

Figure 2: Non-uniqueness of the Lagrange multipliers μ_1 and μ_2

As a result, the inner part of the example network is disconnected from all of the fixed grade nodes. Thus, the vertical position of the HGL of the link is undefined. From this example a general condition for the placement and the size of the set values of flow and pressure controlling devices is deduced:

Condition 1: To ensure the stability and uniqueness of the hydraulic steady-state with respect to pressure heads of flow and pressure controlled networks it has to be proven that the system graph after reducing all of the active flow and pressure controlling devices is still connected.

In the former example (Fig. 2) uniqueness of the Lagrange multipliers (Kyparisis, 1985) is not observed because the constraints involved by the FCVs are violating the linear independency constraint qualification (LICQ). After removing one arbitrary FCV from the system the LICQ holds and uniqueness of the Lagrange multipliers can be proven.

Non-existence of the hydraulic steady-state: Example system with two FCVs

The examples above have shown that in some cases the hydraulic steady-state under the assumption of ideal control conditions is not unique and therefore not stable. Now the existence of the hydraulic steady state is investigated. For that purpose the system of Fig. 2 is slightly modified. In the center of the system a demand node (Q = 500 L/s) is added and the water level of the second tank is increased to 60 m similar to the water level of the first tank on the left hand side (see Fig. 3). Because of the symmetry in the system without control both tanks supply 250 L/s each. Now, upper bounds for the input of the tanks are introduced. For example if FCV 1 allows only 200 L/s the remaining 300 L/s are delivered from tank 2. But what happens if the flows through both valves are at their limit? In that case the demand **Q** cannot be satisfied and there exists no feasible solution to the problem.

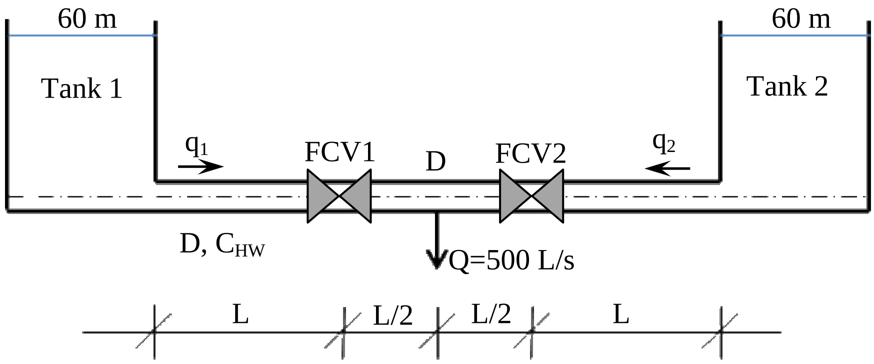

Figure 3: Example for non-existence of the hydraulic steady state

In demand driven analysis the continuity equation at each node of the system (Eq. 1 (a)) requires that the sum of inflows and outflows at a node exactly meets the given demand of the node. If for example the second FCV in Fig. 3 has also a flow limit of 200 L/s like the first FCV the demand of Q = 500 L/s cannot be reached without violating the inequality conditions $q_1 \leq 200$ L/s and $q_2 \leq 200$ L/s. Mathematically formulated in this case the feasible set **U** (Eq. 3) is empty.

Mixed problem: Combination of FCV and PSV

A similar problem occurs if the second flow control valve is replaced by a pressure sustaining valve (PSV). Whereas the FCV continues to regulate the inflow of tank 1 the PSV tries to keep the pressure head at the inlet node above a given set value. The inflow of tank 2 is determined by the head difference between the water level in tank 2 and the set value of the PSV and the characteristics of the connecting pipeline. If the demand Q exceeds the sum of possible inflows a feasible solution does not exist anymore. In that case the consideration of the set **U** is not sufficient for the proof of existence of a feasible solution. In addition the pressure conditions at the first node of the PSV have to be considered. The system can also be used for demonstrating the limitations of demand driven analysis. Whereas the system shown in Fig. 1 with a FCV and a PRV in series results in hydraulically unstable conditions the difficulties resulting from the combination of FCV and PSV can be resolved by a more realistic pressure driven analysis.

4. COMPARISON OF EPANET VERSIONS 2.00.10 AND 2.00.12 REGARDING THEIR CAPABILITY OF SOLVING FLOW AND PRESSURE CONSTRAINT NETWORKS

The results of the last section will now be used for the discussion of the convergence properties of the EPANET-algorithm. Similar systems to the examples above are used for the comparison of the convergence properties of the older version EPANET 2.00.10 (for the following the abbreviation v10 will be used) and the most recent version 2.00.12. (abbreviated by v12) for the solution of flow and pressure constraint pipe systems. From different publications it is known that v10 fails to converge for certain combinations of flow and pressure controlling devices as shown in Fig. 1. The problems could be explained by the singularity of the linear equation system that is solved in each iteration of the nonlinear calculation. The status of the valves is assumed at the beginning of the iterative process, checked between the iterations and if necessary adjusted by specific heuristics. In some cases the heuristic results in conditions where a couple of interacting valves are active at the same time leading to a singular equation system. For instance, the singularity appears if the two valves in Fig. 1 are active at the same time or in general if the system state contradicts the Condition 1 as stated above.

In version v12 the described problem is tackled by adding the fixed value 1.0/CBIG with CBIG = 10^8 to the matrix coefficients belonging to the two end nodes of the valve. Physically, the approach is equivalent to adding a very small diameter pipe in parallel to the valve. In the case where the active valves would contradict Condition 1 the virtual pipe guarantees the connectivity of the system. It could be proven that the modified approach solves most of the problems resulting from interacting flow and pressure controlling devices as stated in Simpson (1999). However, in some examples where the demand driven analysis has no feasible solution the new method results in misleading output. In addition there exist combinations of pressure regulating devices where v12 fails to converge or to calculate the correct results. In the following different valve combinations of the example pipe system that is shown in Fig. 4 are considered and the results of v10 and v12 are stated in Table 1.

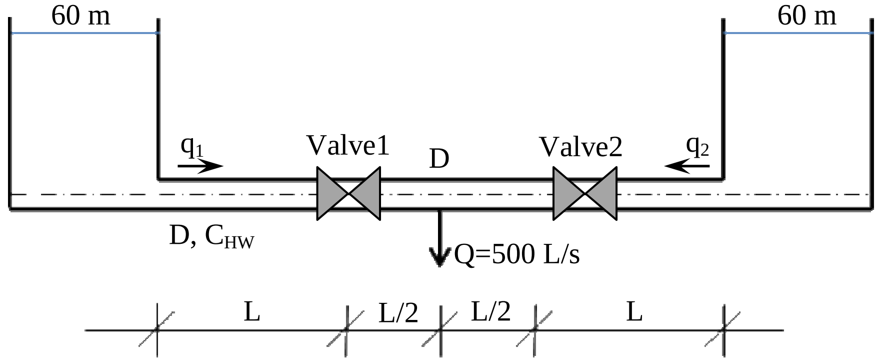

Figure 4: Example network with two valves

Four different cases are distinguished:

Case 1: Hydraulic solution feasible, EPANET converges to right solution.

Case 2: Hydraulic solution feasible, EPANET converges to wrong solution.
Case 3: Hydraulic solution feasible, EPANET does not converge.
Case 4: Hydraulic solution infeasible, EPANET converges to wrong solution

Table 1: Calculation results of example network with different combinations of valves (v12)

Ex.	Valve 1			Valve 2			Pressure Node D	Case No.	Results differ
No.	Type1	Set.1	q_1 (L/s)	Type2	Set.2	q_2 (L/s)	(m)	v12 (v10)	v10-v12
0	None	-	250	None	-	250	57.16	1 (1)	No
1a	FCV	240	250	FCV	240	250	$-1E10^7$	4 (4)	Yes
1b	CHV	Rev. Flow	50	FCV	400	450	$-5E10^7$	4 (4)	Yes
2a	PSV	59	177	PSV	56	323	55.44	1 (1)	No
2b	PSV	56	250	PSV	59	250	57.16	2 (1)	Yes
2c	PSV	55	250	PSV	59	250	57.16	2 (1)	Yes
2d	PSV	54	---/250	PSV	59	---/250	---/57.16	3/2 (1)	Yes
2e	PSV	53	323	PSV	59	177	55.44	1 (1)	No
3a	PSV	59	177	FCV	300	323	$-2E10^7$	4 (4)	Yes
3b	PSV	54	466	Closed link		34	$-4E10^7$	4 (4)	No
3c	Closed link		0	PSV	54	500	49.74	4 (4)	No
3d	PSV	54	500	Deleted link		0	49.74	4 (4)	No

Results calculated by EPANET version 2.00.12

Examples 1a and 1b belong to a group of systems, where no physically feasible solution exists. In these simple examples it is easy to find out that for example 1a both FCV set flows are exceeded and either 50% of the excess flow is allocated to each FCV. With the above mentioned CBIG large negative pressures are calculated. The CHV of example 1b is intended to allow flow only from the right to the left reservoir. Instead of this, a reverse flow of 50 L/s through the pipes adjacent to the CHV (but not through the CHV itself) is computed. In contrast for examples 2a to 2e hydraulically feasible solutions exist. The only difference between 2a and 2b is the location of the PSV with the lower setting. While v12 converges to the right solution after a few trials in 2a, in 2b the same flows as in example 0 without control are calculated. In the solution the constraint of the PSV with the setting of 59 m is not observed. After decreasing the setting of valve 1 by 1.0 m in three subsequent steps the results calculated for scenarios 2c to 2e are wrong in 2c, correct in 2e and no solution is found in 2d when using the default values (2, 10, 0) for CHECKFREQ, MAXCHECK and DAMPLIMIT as described in the help function of v12. With the other choice of (10, 100, 0.01) that is suggested in the help function for networks that have difficulties in converging the algorithm converges in all cases but the results are also wrong as found in 2b and 2c.
Examples 3a to 3d deal with an overloaded PSV. One can argue whether these overloadings may be allowed or not. In case 3a the total excess flow is assigned to the FCV, also leading to a large negative pressure at the demand node. Examples 3b to 3d demonstrate the behavior of EPANET according to closed links: After closing either the right link (case 3b) or the left link (case 3c) in the first case a flow of about 34 L/s flows through the pipes adjacent to the closed pipes again leading to large negative pressures, while in the second case (3c) the total flow of 500 L/s passes through the PSV with violation of its constraint but a pressure of 49.74 m at the demand node. The same result as in 3c is reached if the closed link is deleted from the network (case 3d).

Results calculated by EPANET version 2.00.10

The cases 1a, 1b and 3a in table 1 include example configurations where the constraints representing the operational behavior of control devices cannot be fulfilled: In Example 3a only the constraint of the FCV is violated, in example 1a the constraints of both FCVs are violated by the same amount. In v10 for example 3a, a warning is given that the system may be unstable. The flows are equivalent to v12. In contrast the pressure at the demand node is 55.44 m. In v10 no additional headloss due to the overload of the FCV is considered. The same situation is found for example 1a: v10 increases the flow of the right valve up to 260 L/s and a warning, that the system may be unstable, is given. Because no additional headloss is calculated the pressure at the demand node is 56.94 m. Similar to case 1a the excess flow for example 1b passes through the check valve in the reverse direction – resulting in a flow of 100 L/s through the pipes adjacent to the check valve and a pressure of $-1E10^8$ at the demand node. Unlike to v12, in cases 2b – 2d v10 converges to the right solution after a few iterations. For the other examples the results calculated by v10 and v12 are equivalent.

Handling of constraints

During this research it has been observed that both versions of EPANET handle constraints in a way that leads to the risk of misinterpreting the results. According to p 190 of the EPANET manual (Rossman, 2000) closed links obey a linear headloss relation with a large resistance factor, i.e., $h = 10^8 Q$, so that $p = 10^{-8}$ and $y = Q$. In some cases this leads to a significant flow through a pipe assigned as closed resulting in wrong results. The only warning is that there exist negative pressures. Dealing with large networks of deficient water supply systems where negative pressures are expected the modeler may ignore the fact that in such cases the continuity of flows in combination with the valve settings does not hold. Additionally, as shown in example 3d a PSV can be overloaded resulting in pressures at the upstream junction slightly below its setting. For overloaded combinations of PSV and FCV in v12 the flow that exceeds the valve settings is mismatched to the FCV with the result of large negative pressures although in other calculations an overloading of a PSV leads to pressures slightly under the setting. Examples 3b to 3d show, that the EPANET algorithm does not distinguish between “harder” and “weaker” constraints. While in examples 3c and 3d an overloading of the PSV is allowed, in example 3b the flow exceeding the maximum possible flow through the left pipe without violating the constraint of the setting of the PSV is related to the closed link with the result of large negative pressures. Similar to examples 2a and 2b, examples 3c and 3d are symmetric and must lead to the same computational results. An interesting detail is that in example 3b the flow through the closed link is stated as 0, while the flow in the adjacent links is 34 L/s.

5. DISCUSSION OF THE EXAMPLES

Pressure Control

In order to explain the reasons for the non-convergence or wrong results of the EPANET algorithm the network of Fig. 4 with a couple of tanks supplying demand node D via two pipelines serves as an example. Let the pressures at the inlet side of the pipelines be controlled by a PSV on each side (case 2a - 2e). As demonstrated the calculations with EPANET version v12 show different results depending on the respective set value of the PSVs. In case 2c the set value of valve 1 is chosen to 55 m and the set value of valve 2 is 59 m. For that configuration a physically feasible solution exists where the valve 1 is in an OPEN state and valve 2 is ACTIVE. EPANET 2.00.12 converges after 5 iterations. The calculated flows through the valves are both 250 L/s and the valves states are OPEN (Table 1). The results contradict the condition of valve 2 that should guarantee a minimum pressure of 59 m at the inlet node. If the iterative process in the source code is followed, it appears that the setting of valve 2 is set to XPRESSURE in function `badvalve(n)`. The function is called within the first iteration because the system matrix is singular. The singularity can be explained by consideration of Condition 1 above. In v12 the status of pressure and flow controlling devices is set to ACTIVE at the beginning of the iterative calculations. The resulting system is in contradiction to the Condition 1 because if both active valves are replaced the inner part of the system is disconnected from all head nodes. During the following iterations the valve 2 cannot be reactivated since the heuristics allow only a switch from XPRESSURE to CLOSED if the flow changes its direction.

Now let us decrease the set value of valve 1 by 1.0 m (case 2d, table 1). In that case v12 fails to converge within 1000 possible iterations. The small difference to the conditions above is that after fixing the status of valve 2 to XPRESSURE a change of the flow direction in iteration 3 occurs. As a consequence the status of valve 2 is set to CLOSED. In the subsequent iterations the status is changed to ACTIVE at the same time with valve 1 resulting again in the singularity of the equation system. From that point the algorithm is circling between different valve states and the flows and pressures are out of reasonable bounds. Now the set values of the PSVs are exchanged. Consequently the set value of valve 1 is 59 m and the one of valve 2 is 55 m. In that case v12 has no problems to calculate the correct solution within seven iterations. The second scenario with a set value of 54 m at valve 2 is correctly calculated within seven iterations as well. With a view to the source code it is easy to explain that behavior. Whereas in the first case the valve with the higher set pressure (valve 2) was set to XPRESSURE in the case with the exchanged values the valve with the lower set value is set to XPRESSURE (valve 2 again). In effect the status XPRESSURE is treated like OPEN and the algorithm converges to the correct results. The same results can be reached if in the first case the order of valves and nodes in the EPANET-Input file is changed. The given example shows that the convergence and correctness of the solution sometimes is still left to chance in version 2.00.12.

Flow Control

The problem of invalidating the continuity condition for systems with flow control and infeasible valve settings can be explained by the method of dealing with closed valves and active FCVs. Under normal conditions the effect of adding the matrix coefficients corresponding to a very small diameter pipe in parallel to links with active flow control, as explained above, is negligible. However, if the flow conditions result in physically infeasible constraints continuity is forced during the iterative calculations. As a consequence the flow constraints are invalidated and large headlosses are calculated. For the modeler the only way to find the infeasible valve setting is to investigate the region with very small pressures and the adjacent valves.

6. SUGGESTIONS FOR THE IMPROVEMENT OF CALCULATION RESULTS

Proving non-existence of the hydraulic steady-state

In the previous section it has been shown that there exist different sources of incorrectness of the results of EPANET calculations. In this section a preprocessing method is proposed that calculates a flow vector that solves, firstly, the continuity equations at the nodes and, secondly, the flow constraints representing the valve operation. It is well documented in literature that the convergence properties of the hydraulic solver often rely on the initial guesses of the valve settings. In the present version v12 the valves are all set to ACTIVE at the beginning of the calculation runs. In v10 all valves were initially set to OPEN. As it has been shown in section 4 examples exist where one version fails to converge and the other calculates the correct results and vice versa. The presented approach of the preprocessing procedure has the additional benefit that the initial valve states are estimated correctly (i.e. in accordance to the initial flow vector). The non-existence of a feasible solution (e.g. case 1a in table 1) is also detected.

It follows from the KKT-conditions (Eq. 4) that for each active flow constraint (in other words if it is solved by an equality constraint) there exists a positive Lagrangian multiplier representing the local headloss that is caused by the device. Since the proposed preprocessing step deals only with the continuity equations and inequalities but not with the head balance the pressure drops are not known at that stage. As a consequence it is not sufficient to find a flow distribution on the boundary of the feasible set **U**. Since the heads are not known it cannot be decided if the constraint is active or not. That problem can be solved by starting with a feasible flow vector that is chosen from the inner of the feasible set **U**. Such a vector can be found by a slight modification of Phase I of the Simplex Algorithm known from Linear Programming. The initial flow vector $\mathbf{q} = \mathbf{q}_t + \mathbf{u}$ is calculated by the solution of the following minimization problem:

$$\min_{(\mathbf{u},\xi)\in\Phi} \xi \quad \text{with} \quad \Phi = \left\{(\mathbf{u},\xi)\in \mathbf{R}^n \times \mathrm{R} : \left[\mathbf{Gu}-\mathbf{b}_1\right]_i \leq \xi,\ i=1,\ldots,m \wedge \mathbf{Hu}-\mathbf{b}_2=\mathbf{0}\right\} \qquad (7)$$

If the polyhedral set **U** (Eq. 3) is nonempty and therefore a feasible solution to the problem exists the value of the objective function ξ in Eq. 7 is negative. Then, the solution vector **u** lies in the inner of **U** and solves all of the inequality constraints with “<”. Consequently all multipliers are zero and the flow control devices are OPEN at the beginning of the iterative process and subsequently activated if needed.

Dealing with singularities during the iterations

It has been shown above that under certain conditions (e.g. for the pair of PSVs in case 3, Table 1) the matrix of the equation system of the new version v12 has the same problems resulting from the singularity of the coefficient matrix as was observed in v10. If the solver of EPANET detects such a singularity the function `badvalve(n)` is called. The original function proves if the initial or end node of an active control device corresponds to the line `n` of the matrix with a non-positive diagonal element. The index `n` indicates a node that is disconnected from any node with given potential if all of the active and closed valves would be replaced from the graph. As a result the head of the disconnected nodes is undefined. If such a valve is found its setting is changed to XFLOW (FCV) or XPRESSURE (PSV, PRV). The problem was that the node `n` is not always adjacent to a valve causing the singularity with the consequences for the convergence as explained above.

Here, a modified version of the function `badvalve(n)` is outlined that makes use of parameter sensitivities for the identification of the correct valve that causes the singularity. The first step of the new approach includes the identification of such a valve. For that purpose all valves are opened and the sensitivity of the nodal heads against a change in demand at node `n` is calculated. The valve in the search is a formerly active or closed valve whose nodes have the highest sensitivity. For the next step that valve, (say valve k for example), is kept open whereas all of the other valves are set to their actual valve state at that iteration. Then, the sensitivity of nodal heads against a change in headloss (dz) of the opened valve is calculated. In some cases the calculation of the sensitivity fails because there are still valves causing a singularity in the system of equations. If this appears, the valves are opened step by step by calling again the function that identifies the valve. This is repeated until a feasible solution is calculated and the result is the sensitivity dH/dz where dz is the headloss change for valve k. The second valve that is communicating with valve k is a valve that is still active and whose nodal heads have significant sensitivity against dz. The last step is to prove which of the two valves should be active in the next iteration. This can be done by application of the common valve heuristic in EPANET.

Warnings given by EPANET

From our point of view it would be very helpful for the users of EPANET to receive more information in the status report. As mentioned before, in developing countries negative pressures in some parts of the network are not unusual. For that reason, a user might not pay a lot of attention to EPANETs hints of negative pressures. It would be useful to be warned about the lowest pressure in the whole network. A sophisticated user would search for the reason if a note like “Minimum network pressure is -4E07 at junction 968” was given. It is suggested that the messages for valves should be changed into: *open*, *active*, *constraints violated* and *reverse flow* (e.g. FCV, PBV). It would also be very useful to flag when significant flow is calculated through closed links or check valves in the opposite direction than intended. Warnings like “PSV 7 open but cannot deliver pressure” in v12 are now less helpful than in v10 because they are given when the valve is open or constraints are violated (in v10 only for violated constraints).

7. SUMMARY AND CONCLUSIONS

In the first part of the paper a description is given of the problem of physical non-existence and non-uniqueness of the hydraulic steady-state of simple pressurized pipe systems with flow and/or pressure controlling devices. The results have been used for the explanation of difficulties of EPANET version 2.00.12 with convergence or calculation of incorrect results. It appears that in some cases the older version 2.00.10 delivers more reliable results. In the last part of the paper modifications of the EPANET source code have been proposed. The first one includes a preprocessing approach and requires more extensive code additions because it relies on extra topological information (spanning tree, loops). In contrast, the second approach includes only few modifications of the source code and allows the identification of interacting control valves that cause a singularity of the equation system. It should be noted that both modifications can be only regarded as improvements of the existing heuristics. A mathematically more exact way of calculating the hydraulic steady state of pipe networks under control is the use of the Nash-Equilibrium as stated in section 2.

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