

A Fourier-Mukai Transform for Invariant Differential Cohomology

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A	<i>Harmonic Cheeger-Simons characters with applications.</i>	
	R. Green and V. Mathai.	
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Abstract

Considering real tori as a differential-geometric analogue of abelian varieties, we consider the corresponding analogue of the Fourier-Mukai transform. The transform at the level of ordinary cohomology is refined to a Fourier-Mukai transform for the invariant differential cohomology of a torus or more generally a trivial torus bundle.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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1. Richard Green and Varghese Mathai, *Harmonic Cheeger-Simons characters with applications*, Journal of Geometry and Physics 59 (2009), no. 5, 663–672.

SIGNED: DATE:

Statement of Contributions

The appendix contains the following joint work with V. Mathai:

1. Richard Green and Varghese Mathai, *Harmonic Cheeger-Simons characters with applications*, Journal of Geometry and Physics 59 (2009), no. 5, 663–672.

I, Richard Green, declare that the contributions to the work (1.) were as follows. The original concept for the paper was due to V. Mathai. We jointly developed the proofs of the final results, working through initial ideas, modifying where necessary and expanding the level of detail in the proofs so as to make them suitable for publication. The production of the final text was also carried out collaboratively, with writing and editing contributions from both authors as the project developed.

SIGNED: DATE:

I, Mathai Varghese, declare that the above is an accurate description of the contributions made to the work (1.), and give permission for the inclusion of said work in this thesis.

SIGNED: DATE:

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