Robust Scheduling Control of Aeroelasticity

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Summary

Aeroelasticity is a broad term describing the often complex interactions between structural mechanics and aerodynamics. Aeroelastic phenomena such as divergence and utter are potentially destructive, and thus must be avoided. Passive methods to avoid undesirable aeroelastic phenomena often involve the addition of mass and/or limiting the achievable performance of the aircraft. However, active control methods allow both for the suppression of undesirable aeroelastic phenomena, and for utilisation of desirable aeroelastic phenomena using actuators, thus increasing performance without the associated weight penalty of passive systems.

The work presented in this thesis involves the design, implementation and experimental validation of novel active controllers to suppress undesirable aeroelastic phenomena over a range of airspeeds. The controllers are constructed using a Linear Parameter Varying (LPV) framework, where the plant and controllers can be represented as linear systems which are functions of a parameter, in this case airspeed. The LPV controllers are constructed using Linear Matrix Inequalities (LMIs), which are convex optimisation problems that can be used to represent many linear control objectives. Using LMIs, these LPV controllers can be constructed such that they self-schedule with airspeed and provide upper performance bounds during the design process.

The aeroelastic phenomena being suppressed by these controllers are Limit-Cycle Oscillations (LCOs), which are a form of utter with the aeroelastic instability bounded by a structural nonlinearity in the aeroelastic system. In this work, the aeroelastic system used is the Nonlinear Aeroelastic Test Apparatus (NATA), an experimental aeroelastic test platform located at Texas A&M University.

Three and four degree-of-freedom dynamic models were derived for the NATA, which include second-order servo motor dynamics. These servo motor dynamics are often neglected in literature but are suf ciently slow that their dynamics are signi cant to the controlled response of the NATA. The dynamic model also incorporates quasi-steady aerodynamics, which are accurate for low Strouhal numbers calculated from the oscillation frequency of the wing. Is it shown how the dynamics of the NATA can be represented in LPV form, with a quadratic dependence on airspeed and linear dependence on torsional stiffness.

Using a variety of techniques the parameters of the NATA are identi ed, and shown through nonlinear simulation to provide excellent agreement with experimental results. It is also argued that structural nonlinearity, in the way of a nonlinear torsional spring connecting the wing section to the base, generally improves stability due to its largely quadratic stiffness function, and hence in many instances it is safe to linearise this nonlinearity when designing a controller.

Using a $\mathcal{H}_2$ generalised control problem representation of a Linear Quadratic Regulator (LQR) state-feedback controller, LPV synthesis LMIs are constructed using a standard transformation which render the LMIs af ne in the transformed controller and Lyapunov matrices. These matrices have the same quadratic dependence on airspeed as the NATA model. To reduce conservatism the parameter space of airspeed versus airspeed squared is gridded into triangular convex hulls over the true parameter curve, and the LMIs are numerically optimised to give an upper bound on the $\mathcal{H}_2$ norm across the design airspeed. The resulting state-feedback controller is constructed from the transformed controller and Lyapunov matrices, and can be solved symbolically as a function of airspeed, however it forms a high-order rational function of airspeed, hence it is quicker to solve for the controller gains numerically on-line.

The controller is analysed for the classical measures of robustness, namely gain and phase margins, and maximum sensitivity. While not providing the guarantees of these measures that a conventional LQR controller provides, the controller is shown to be suf ciently robust across the airspeed design range.

Experimental results for this controller were performed, and the results show excellent LCO suppression and disturbance rejection, the results from which are published in Prime et al. (2010).

Following the above work based on a scalar performance index, the upper bound on the $\mathcal{H}_2$ norm is allowed to vary with airspeed using

the same quadratic dependence on airspeed as the NATA model, and the transformed controller and Lyapunov matrices. A simple method of solving the LMIs is shown such that the LPV $\mathcal{H}_2$ upper bound is as close to optimal as possible, and using this method a new controller is synthesised.

This new controller is compared against the LPV LQR controller with the scalar performance index, and is shown to be closer to optimal across the airspeed design range. Nonlinear simulations of the controlled NATA using this new controller are then presented.

Based on Prime et al. (2008), a Linear Fractional Transformation (LFT) is applied to the NATA model to render the dynamics dependent upon the feedback of the linear value of airspeed. This allows the LMIs to be constructed at only two points, the extreme values of the linear design airspeed, rather than gridding over the parameter space as was performed above.

An output-feedback controller that itself depends upon the feedback of a function that is linearly dependent upon airspeed is constructed using an induced $\mathcal{L}_2$ loop-shaping framework. The induced $\mathcal{L}_2$ performance objective is based upon a Glover-McFarlane $\mathcal{H}_\infty$ loop-shaping process where the NATA singular values are shaped using pre- and post- lters, and minimising the induced $\mathcal{L}_2$ norm from both the input and output to both the input and output.

An LFT controller is synthesised, and simulations are performed showing the suppression of LCOs.

Declarations

Originality

This work contains no material which has been accepted for the award of any other degree of diploma in any university or other tertiary institution. To the best of my knowledge and belief, this work contains no material previously published or written by another person, except where due reference has been made in the text.

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Zebb D. Prime

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