The analysis of variance of \( s(s-1)/2 \) frequencies \( a_{ij} \), where \( i \) and \( j \) are unequal and unordered, and take integral values from 1 to \( s \).

In addition to the one degree of freedom for the total, which contributes
\[
\frac{2}{s(s-1)} S_{ij}^2 \frac{s(s-1)}{s(s-1)}
\]
to the sum of squares for \( s(s-1) \) degrees of freedom \( S_{ij}^2(a_{ij}) \),

we need only consider linear functions with coefficients adding to zero; \( \mu \) of these are
\[
A_i = (s-2)S_j(a_{ij}) - 2S_k(a_{ik})
\]
satisfying \( S(A_i) = 0 \),

where \( i, j \) and \( k \) are all unequal. These must jointly specify the distribution of frequency among the \( s \) suffixes, having \( s(s-1)/2 \) degrees of freedom.

The sum of the squares of the coefficients in \( A \) is
\[
(s-1)(s-2)^2 + 4(s-1)(s-2) - s(s-1)(s-2);
\]
\[
\sum_i A_i^2
\]
the sum of the squares of the \( s \) quantities \( A_i \), the sum of the coefficients of terms such as \( a_{ij} \) must be
\[
s^2(s-1)(s-2)
\]
\[
\frac{1}{s^2(s-2)} \sum_i A_i^2
\]
Hence in the expression
\[
\frac{1}{s^2(s-2)} \sum_i A_i^2
\]
the sum is \( s-1 \), equal to the number of degrees of freedom.
There will remain
\[ \frac{1}{4}s(s-1) - 1 - (s-1) = \frac{1}{4}s(s-3), \]
degrees of freedom for deviations among the frequencies compatible with fixed \( A_i \).

For these we may define
\[ B_{ij} = (s-2)(s-3) a_{ij} - (s-3)5(a_{ik}) - (s-3)5(a_{jk}) + 2s(a_{kk}) \]
in which \( i, j, k, \ell \) are all unequal.

The sum of the coefficients in \( B_{ij} \) is zero.
The sum of the products of corresponding coefficients in \( B_{ij} \) and \( A_i \) is
for \( a_{ij} \) \( (s-2)^2(s-3) \)
for \( a_{ik} \) \( -2(s-2)(s-3) \) \( (s-2) \) terms
for \( a_{jk} \) \( 2(s-3) \) \( (s-2) \) terms
for \( a_{k\ell} \) \( -4 \) \( 2(s-2)(s-3) \) terms
coming in all to zero; similarly the sum of the products for \( B_{ij} \)
and \( A_i \) is
for \( a_{ij} \) \( -2(s-2)(s-3) \)
for \( a_{ik} \) \( -(s-2)(s-3) \) \( 2 \) terms
for \( a_{ij} \) \( 2(s-3) \) \( 2(s-3) \) terms
for \( a_{k\ell} \) \( 2(s-2) \) \( (s-3) \) terms
for \( a_{k\ell} \) \( -4 \) \( 2(s-3)(s-4) \) terms
again giving a zero total, and showing that each of the components \( B \) is orthogonal to each of the components \( A \).

But the sum of the squares of the coefficients of \( B \) is
\[ (s-2)^2(s-3)^2 + 2(s-2)(s-3)^2 + 2(s-2)(s-3) \]
\[ = (s-1)(s-2)^2(s-3) \]
The sum for all the \( \frac{1}{2}s(s-1) \) expressions \( B_j \) is

\[
\frac{1}{2}s(s-1)^2(s-2)^2(s-3),
\]

so that in

\[
\frac{1}{(s-1)^2(s-2)^2} S(B^2)
\]

the sum is \( \frac{1}{2}s(s-1) \), the same as the number of degrees of freedom.

The orthogonal analysis so arrived at may be written

\[
d.f. \quad \frac{S(B^2)}{S(A^2)}
\]

\[
\begin{array}{c}
1 & \frac{2}{s(s-1)} S(a_{ij}) \\
(s-1) & S(A^2) \\
(s-2) & \frac{1}{s^2} S(B^2) \\
(s-1)^2(s-2)^2 & \frac{1}{(s-1)^2(s-2)^2} S(B^2)
\end{array}
\]

\[
\frac{\frac{1}{2}s(s-1)}{S(a_{ij})^2}
\]

2. Effect of breeding for a single generation on the self-sterility alleles of a large population.

If \( p_{ij} \) stand for the relative frequency of the genotype \((i, j)\), and

\[
P_k = \frac{1}{s} \sum \mathbf{p}_{ij}
\]

for the relative gene frequency, we may define the components

\[
A_i = (s-2)\sum \mathbf{p}_{ij} - 2\sum \mathbf{p}_{ik}(\mathbf{p}_{jk})
\]

and

\[
P_{ij} = (s-2)(s-3)\mathbf{p}_{ij} - (s-3)\sum (\mathbf{p}_{ik} + \mathbf{p}_{jk}) + 2\sum \mathbf{p}_{ik}(\mathbf{p}_{jk})
\]
and express the gene and genotype frequencies in terms of these components, as

\[ p_i = \frac{1}{s} + \frac{1}{2s} A_i \]

\[ p_{ij} = \frac{2}{s(s-1)} + \frac{1}{s(s-2)} (A_i + A_j) + \frac{1}{(s-1)(s-2)} B_{ij} \]

The genetic \textit{recessiveness} formula for one generation is

\[ p_{ij}' = \frac{p_{ij}^{km}}{p_{ij}^{km} - p_i - p_k} + \frac{p_{ij}^{km}}{4} \sum_{x=1}^{s} \frac{p_{ij}^{km}}{p_{ij}^{km} - p_x} \]

on substitution in terms of the components \( A \) and \( B \), the first term of these gives

\[ \frac{1}{4s} \frac{2 + A_j}{s-1} \sum_{x}^{s-1} \frac{2}{s-2} + \frac{1}{s} (A_i + A_j) + \frac{1}{(s-1)(s-2)} B_{ij} \]=

\[ \frac{1}{4s} \left\{ \frac{4}{s-1} + \frac{2A_i}{s-1} \right\} \frac{(s-3)A_i - A_j}{(s-1)(s-2)^2} - \frac{2s}{(s-1)(s-2)^2} B_{ij} \}

adding the same expansion with \( i \) and \( j \) interchanged we have finally

\[ p_{ij}' = \frac{2 + A_i}{s(s-1)} + \frac{s^2 - 4s + 2}{s(s-1)(s-2)^2} (A_i + A_j) - \frac{1}{(s-1)(s-2)^2} B_{ij} \]

so that all \( A_i \) are decreased in the ratio

\[ \lambda = \frac{s^2 - 4s + 2}{s(s-1)(s-2)^2} = 1 - \frac{s}{(s-1)(s-2)} \]

and all \( B_{ij} \) in the ratio

\[ \lambda = \frac{1}{s-2} \]