Dec. 19, 1935:

Dear Dr. Fieker,

Professor Whittaker has informed me that you had asked for certain of my affinities. I have sent these this evening under a separate cover. One or two of them you may already have. The one on least squares and orthogonal polynomials might have been improved had I noticed in due time a slight numerical simplification, namely that in the working tables each column has usually an integer H.C.F., > 1, which may be cancelled throughout, its square being cancelled from the divisor at the foot of the column; so that, e.g. the working table for $n = 10$, p. 62, reduces to

\[
\begin{array}{ccccccc}
1 & -9 & 6 & -42 & 18 & -6 \\
2 & -4 & 56 & -40 & 20 \\
1 & -16 & 45 & -35 & 10 & -30 \\
10 & +30 & 40 & & & & \\
10 & -30 & & & & & \\
\end{array}
\]

\[
\begin{array}{cccccc}
10 & 330 & 132 & 8580 & 2860 & 780 \\
& & & & & \\
\end{array}
\]

decidedly smaller numbers. Of course on the machine it would not
make much difference.

There is another aspect in the press, and I shall send it also when it comes out. Its interest is mainly theoretical.

The principal result is that the bivariate correlation function in fourfold sampling with replacement has the form

\[ \phi(x, y; \delta) = \phi(x, y; 0) \left[ 1 + \frac{\delta}{(x+y+\delta)_{(x+y)}} G(x; h) G(y; h') \right] \]

\[ + \frac{\delta^2}{(2x+y)(2y+x)} G(x; h) G(y; h') \cdots \]

where \( \phi(x, y; 0) \) is the corresponding uncorrelated function for the same marginal \( h, h', f, f' \), and the \( G \)'s are polynomials

\[ G(x) = x^{(r)} - \sum_{r} P(n-r+1) x^{(n)} + \sum_{r} \frac{P(n-r+2)}{r!} x^{(n-r-2)} \]

\[ + (-1)^{r} x^{(n-2r)}, \]

with \( x^{(r)} = x(x-1) \cdots (x-r+1) \); having orthogonal properties

\[ \sum_{0}^{\infty} (x)^{n} \chi_{m} x^{m} G_{r}(x) G_{s}(x) = 0, \quad r \neq s \]

\[ \sum_{0}^{\infty} (x)^{n} \chi_{m} x^{m} G_{r}(x) = \pm n^{(m)}(x), \quad r = s \]

Regressions and higher moment arrays come out readily from the orthogonal properties. If replacement is not allowed,
There are corresponding polynomials

\[ T_n(x) = x^n - r \cdot \frac{(n+1)(N^2 - n^2)}{N-2n+2} x^{n-1} + \cdots \]

which possess analogous properties in respect of the hypergeometric frequency function, and lead to a similar fourfold correlation function.

I do not know whether you will have any opportunities to share if your own papers (in Phil. Trans. and Camb. Phil. Soc.) on the foundations of theoretical statistics; but if by chance you were able to spare me a copy, I should be very grateful indeed.

I am, Yours sincerely,

A. C. Aitken
December 30th, 1935

Dear Dr Aitken,

Thanks for your letter. I was very glad to get your offprints, some of which I had not before read.

About offprints of mine, I should like you to have a selection of all which would be useful to you, and I have many left though some of the earlier ones are exhausted. I believe the best plan would be for you to look through the bibliography printed in the latest 5th edition of "Statistical Methods" and send me a list of those which you would like to have.

Yours sincerely,