Dear editor,

I think the dictionary has a word for this.

Consider a sequence of positive numbers $n_1, n_2, \ldots, n_N$ and a function $f(x)$ defined as

$$f(x) = \frac{1}{x} \left( \frac{1}{n_1^x} + \frac{1}{n_2^x} + \cdots + \frac{1}{n_N^x} \right)$$

Show that the sum $\sum \frac{n_i - m}{n_i^{x+1} \log n_i}$ converges to $0$ as $x \to \infty$.

Now, consider the function

$$g(x) = \frac{\prod_{i=1}^N (n_i + x)}{(n_i + x)^{n_i - 1}}$$

of which the maximum is $g_0$, while the minimum is $g_1$.

$$g_0 = \frac{m^N}{(m+1)(m+2)}$$

Hence, the maximum is achieved when $x = m/2$.

$$\int = \sum \frac{n_i - m}{n_i^{x+1} \log n_i}$$

In the limit, we have

$$\lim_{x \to \infty} \frac{\prod_{i=1}^N (n_i + x)}{(n_i + x)^{n_i - 1}} = \sum \frac{1}{n_i^{x+1}}$$

Thus, the maximum is achieved when $x = m/2$.

$$\frac{m^N}{m+3} \sum \frac{1}{n_i^{x+1}}$$

Using

$$\frac{dX}{dx} = \sum \frac{1}{n_i^{x+1}} \cdot \frac{1 - \frac{x}{n_i}}{1 + \frac{x}{n_i}}$$

we have

$$\sum \frac{n_i - m}{n_i^{x+1} \log n_i} = \frac{m^N}{m+3} \sum \frac{1}{n_i^{x+1}}$$

Now,

$$\frac{\partial X}{\partial m} = \frac{-X}{\partial X/\partial m}$$

Thus,

$$V(m) = \left( \sum \frac{n_i - m}{n_i^{x+1} \log n_i} \right)^2$$