Dear Dr. Fisher,

Many thanks for your note, which helps considerably to clear up my difficulties. I am enclosing your proof of the anomalous result \[ V(m) = \left( \frac{n+1}{n+1} \right) \frac{t}{\sqrt{s^2}} \], which I assume you now agree to be wrong. The anomaly arises, I think, from the fact that in the proof I enclose you have treated \( t \) as Student's \( t \) while keeping \( s \) fixed, i.e. ignoring the correlation between \( t \) and \( s \), whereas in the proof you have sent me you use the correct distribution of \((x-m)\) for fixed \( s^2 = \frac{n s^2 + (x-m)^2}{n+1} \). The latter proof follows the lines of the proof on p. 562 of Bartlett's Proc. Camb. Phil. Soc. paper "The Information available in small samples" Vol. 32. Oct. 1936.

To turn to the case you discuss in your paper "The Fiducial Argument in Statistical Inference" i.e. the distribution of \( \epsilon = \sum_{i=1}^{n} \frac{t^*}{\sigma} - \epsilon \), do you consider that the correct fiducial distribution is obtained by
treating $A_1, A_2$ as fixed numbers and regarding the t's as following Student's t-distribution? This is what Behrens did and is, I think, the same type of procedure as led to the anomaly in the proof I am enclosing. I understand that Yates has worked out the case $A_1 = A_2$ with 6 degrees of freedom and believes that here also the method gives what seems to us the anomalous result that the case $A_1 = A_2$ has wider 5 per cent fiducial limits of $\epsilon$ than the case $A_1 / A_2 = 0$ or $\infty$. I understand that you suggested to Fairfield Smith that as a first approximation to the exact test he should look up $t = \frac{\epsilon}{\sqrt{A_1^2 + A_2^2}}$ with a number of degrees of freedom equal to $\frac{m (A_1^2 + A_2^2)}{A_1^4 + A_2^4}$.

This approximation gives smaller 5 per cent fiducial limits of $\epsilon$ for $A_1 = A_2$ than for $A_1 / A_2 = 0$ or $\infty$.

I am, as you see, still in doubt about Behren's treatment of this case and would welcome your opinion.

Yours sincerely,