1 May 1934.

W.C. Cochran, Esq.,
St. John's College,
Cambridge.

Dear Mr. Cochran,

I was interested to see, from your paper in the current Proc. Camb. Phil. Soc., that you have been working at the Analysis of Co-variance. I wonder, however, why you have not had brought to your notice the exact use of the $z$ test in cases where we wish to retain in the test all the $p-1$ degrees of freedom between $p$ treatments.

On page 188 you discuss a test for sums of squares between treatments corrected for regression by means of the coefficient derived from the residual co-variance, in which the one degree of freedom representing the difference in regression between treatments and error is inflated by the sampling errors of estimation; just as in simple regression the variance of our estimate of the mean of $y$ for a given value of $x$ is the sum of the parts $\sigma^2_\alpha$ and $(x-\bar{x})^2 \sigma^2_\beta$. Scientifically, there is no ground for giving more than normal weight to this particular component of the treatment contrasts, and the sum of squares may be deflated simply by removing the appropriate
fraction of the particular square that it has got too much of.

\[ S \left( \frac{1}{n} - \beta x \right)^2 \]

Personally, I do this by calculating the reduced sum of squares for the compound entry, treatment + error, and subtracting the corresponding reduced sum of squares for error only, the difference being less by the required amount than for treatments.

The situation is exactly analogous to what occurs when the experiment is so designed that certain treatment contrasts are partially confounded, and so are capable of estimation with reduced precision. The magnitude of the coefficient with which the squares of such components enter into the sum of squares for the purpose of testing significance is reduced accordingly; as it is, to take a simpler example of the same thing, when we take the variance between the means of groups having different numbers of members.

I thought your paper very good.

Yours sincerely,