18th April, 1955.

My dear David,

I only mentioned the case of n=1 or 2 in order to raise the question in your mind as to the relevance of the expression \( \frac{1}{\sigma^2} \). I won’t admit at all that one knows nothing about an unknown variance when one has one or two degrees of freedom from which to estimate it, in fact I should assert that one can derive in either of these cases an explicit fiducial distribution of the unknown variance.

With Barnard’s help I have been wrestling with the relationship between axiomatic assertion of ignorance, inductive reasoning, a certain inference etc., and in case you are interested I enclose two little sections from two different chapters of the book I am putting together.

Sincerely yours,

Encs.
The governing characteristic of inductive reasoning is that it is always used to arrive at statements of uncertainty, and that logical situations are recognizable in which different types or degrees of uncertainty require to find rigorous expression. It has been thought that the Theory of Mathematical Probability, in spite of the fact that a probability statement is in reality a statement of a specific type of uncertainty, could be included in strictly deductive processes. This has seemed possible largely because many mathematical treatises have adopted a formal and abstract treatment in which the element of uncertainty is inoperative, just because applications to the real world are avoided.

The logical characteristic, which has been too much overlooked, of all inferences involving uncertainty is that the rigorous specification of the nature and extent of the uncertainty by which it is qualified must in general involve the whole of the data, quantitative and qualitative, on which it is based.

As soon as it is regarded realistically it is seen that the concept of Mathematical Probability shows this requirement. In a statement of probability the predicand, which may be conceived as an object, or as a proposition, is asserted to be one of a set of a number however large of like entities of which a known proportion, $P$, have some relevant characteristic, not possessed by the remainder. It is further asserted that no subset of the entire set having a different proportion can be recognized.
If, therefore, any portion of the data were to allow of the recognition of such a sub-set, to which the predicand belongs, a different probability would be asserted using the smallest such sub-set recognizable.

Which can be known only by an exhaustive examination of the data, when no further sub-set is recognizable, the predicand is spoken of as a random member of the ultimate set to which it belongs. An imagined process of sampling in which a succession of predicands are identified may be used to illustrate the relation between the proportion expected to be observed in the sample, and the primary proportion required to specify the set, now to be identified with the population sampled. Rather unsatisfactory attempts have been made to define the probability by reference to the supposed limit of such a random sampling process.

Difficulty has sometimes been expressed when the reference set, or the population sampled, is said to be infinite. The definition and consequent calculations can, however, be applied to any finite set however large, and the limit of these results, where the number in the set is increased indefinitely is all that is meant by the results of sampling from an infinite population. The clarity of the subject has suffered from attempts to conceive of the "limit" of some physical process to be repeated indefinitely instead of the ordinary mathematical limit of an expression where some element is increased indefinitely.

The following sections are intended to illustrate the kinds of reasoning, and concurrent mathematical operations, appropriate to various types of uncertainty.
Various writers, including Sir Harold Jeffreys and A. Kolmogoroff, recognizing the rational cogency of the fiducial form of argument, and the difficulty of rendering it coherent with the customary forms of statement used in mathematical probability, have proposed the introduction of new axioms to bridge what was felt to be a gap. The treatment in this book involves no new axiom; it does however rely on a property inherent in the semantics of the word "probability", though not required explicitly so long as the applicability in the real world of the logical relationship denoted is not in question. Purely abstract studies of the formal mathematics of probability can, in fact, be developed without reference to this aspect of the word's meaning. It is not, of course, denied that mathematical definition may and as it often does, often does have axiomatic implications. The distinction should be made, in this case, that the completion of the word's definition specifies the nature and extent, not of the knowledge, but explicitly of the ignorance, required in the logical situation envisaged, and that as long as it is assumed, as in purely deductive reasoning is proper, that valid deductions can be drawn from every subset of the axiomatic material available, it can be argued, as by Venn, that "His ignorance affects himself only, and corresponds to no distinction in the things". Mathematical probability, however, as conceived by the early writers, was applicable to the real world, and to make it available not only
in deductive, but also in inductive reasoning a more complete definition is required. The subject of a statement of probability must not only belong to a measurable set, of which a known fraction fulfills a certain condition, but also subset to which it belongs, and which is characterized by a different fraction, must be recognizable.