Professor R.A. Fisher,
Department of Genetics,
Whittingehame Lodge,

My dear Professor,

Suppose that samples from two normal
distributions of equal mean gives sample
means $\overline{x}_1$, $\overline{x}_2$, and variances of those means
$s_1^2$, $s_2^2$. Then

$$\overline{x}_1 - \mu = s_1 t_1,$$

$$\overline{x}_2 - \mu = s_2 t_2.$$

If we wish to estimate $\mu$ from the combined
evidence of the two samples, and to assign
fiducial limits to our estimate, I think we
can legitimately take

$$\overline{x} = \frac{1}{2}(\overline{x}_1 + \overline{x}_2)$$

and use the Fisher-Behrens distribution to
give the limits.

We might hope, however, that a weighted
mean, $\overline{x}_0$, defined by

$$\overline{x}_0 \left( \frac{1}{s_1^2} + \frac{1}{s_2^2} \right) = \frac{\overline{x}_1}{s_1^2} + \frac{\overline{x}_2}{s_2^2}$$

would be more precise. Now

$$\overline{x}_0 - \mu = \left( \frac{t_1}{s_1^2} + \frac{t_2}{s_2^2} \right) \left( \frac{1}{s_1^2} + \frac{1}{s_2^2} \right)^{-1}.$$

Do you see any objection to the use of the
Fisher-Behrens argument on this equation?
We can integrate
\[
\left( \frac{t_1}{s_1} + \frac{t_2}{s_2} \right)
\]
over the region for which it exceeds \( a \left( \frac{1}{s_1^k} + \frac{1}{s_2^k} \right)^k \),

and it seems to me that the logic should be just as good as in the case you have discussed.
I am primarily anxious to be reassured that I am not falling into heresy, before I proceed further. However, I have looked at the next step, in the light of your 1941 paper (Ann. Eugen. 11, p.141). If we modify your transformation at the top of p. 152, writing

\[
t_1 = x \cos \theta + y \sin \theta, \\
t_2 = -x \sin \theta + y \cos \theta,
\]

and \( s_1/s_2 = \cot \theta \),

I think that the same integral is obtained. If this is so, the Fisher-Sukhatme tables complete the solution to the problem, but with a modified definition of \( \theta \). If I have slipped on this last step, presumably the method you used could be adapted to the formation of new tables.

This idea seems very elementary. Is it new? Its usefulness is clear. For example, it gives the complete solution to the estimation problem in an incomplete block experiment, when a final set of means is formed by combination of inter- and intra-block estimates. A weighted mean of \( k \) constituents could be tackled in the same way, but \((k-1)\) ancillary \( \theta \)-statistics would be needed, and the preparation of tables would be an unpleasant task.

Sincerely,

D.J. Finney