April 26, 1940

Dear Professor Fréchet,

Looking at the P.S. of your letter of April 24th in order to give you a quick reply, I see that the paradox is partly my own fault. I had sent the solution of the distribution of

$$\frac{S(y_y)}{x-y}$$

where $x$ is distributed normally about zero, without noticing that your problem with $S(t) = 0$ introduces a restriction which diminishes the degrees of freedom by one. For the common form of the analysis of variance we then have

\[
\begin{align*}
\text{sum of squares} & \quad \text{mean sum of squares} \\
\eta-2 \frac{S(y_y)}{x-y} & \quad \frac{S(y_y)}{x-y} \\
\eta-1 & \quad \frac{S(y_y)}{x-y}
\end{align*}
\]

so that the ratio you enquire about may be equated to

$$\frac{S(y_y)}{S(y_y)} \left( \frac{S(y_y)}{A} \right) \approx \frac{t/n-1}{\sqrt{n-2+t^2}}$$

the distribution of which is equally derived from that of $t$, being that of the sine instead of the tangent of an arbitrary angle. $t$ has now, of course, $n-2$ degrees of freedom.

Yours sincerely,