13 February 1931.

Dr. Graham,
Fisheries Laboratory,
LOWESTOFT.

Dear Graham,

I have just looked through the cod paper, and wondered if it would be worth while putting out an automatic method of smoothing due to Shepard as applied to one of your hauls. I have taken the first without grouping. The three columns following the frequencies are the 2nd., 4th. and 6th. differences. E.g. opposite 11 comes -2 because $6 + 14 - 2(11) = -2$, 6 and 14 being the numbers before and after 11. The process is repeated to get the other two columns. Then a formula which suits your data at least for the smaller fish is

\[(21 - 9\delta_6 - 2\delta_8)\]

applied as an operator to each observation. For the 20cm. class one then has

\[
\begin{align*}
+21 \times 11 &= +231 \\
-9 \times 5 &= -45 \\
+2 \times 23 &= +46 \\
\hline
&= 232
\end{align*}
\]
giving a smoother value 232 which you can if you like divide by 21, to get back to the old numbers. Without doing this though the modes show up pretty well, at 22 cm, for example and again at 37 and 45.

The result is rather like 5 point grouping only instead of replacing \( d \) by

\[
\frac{b + c + d + e + f}{10}
\]

you replace it by

\[
-2a + 3b + 6c + 7d + 6e + 3f - 2g
\]

which stops you flattening out the modes in the way that plain grouping tends to do.

Obviously no mode scoring less than 21 can be any good, and the mode at 45 depending on about 3 fish in all is fairly conjectural, but with an objective method one is in a position to say whether the successive hauls really tell the same story.

In locating the mode near 261, 295, 274 I take the differences 34 and 21 and divide their differences by twice their sum giving \( \frac{13}{110} \) as the excess over 22 cm.

The only technical criticism of your paper is that the tests of goodness of fit are probably not worth much, but no one will mind that.

Yours sincerely,