Dear Hope-Jones,

I am sorry I used the word matrix. It is totally unnecessary for the purpose, for there are no ideas required beyond those of simple simultaneous equations. Indeed I do not use the word in Statistical Methods through an appreciation of its paralyzing effect.

The exponent in the general expression for frequency for normal distributions in several variates is of course quadratic in these variates. If the variates are numbered from 1 upwards, we may as well use the same numbers in the coefficients and write the quadratic expression

$$a_{11}x_1^2 + 2a_{12}x_1x_2 + \ldots$$

Then out of sheer mental tidiness you will probably think of the aggregate or herd of coefficients as arranged ordainly in

$$\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}$$
Corresponding with any such aggregate there are of course other sets of related quantities. For example you can set out the equations

\[
\begin{align*}
\alpha_{11} 1 + \alpha_{12} 2 + \alpha_{13} 3 & = 1, 0, 0 \\
\alpha_{21} 1 + \alpha_{22} 2 + \alpha_{23} 3 & = 0, 1, 0 \\
\alpha_{31} 1 + \alpha_{32} 2 + \alpha_{33} 3 & = 0, 0, 1 \\
\end{align*}
\]

and so on, and write the solutions

\[
\begin{align*}
+ = 0, c, c, c, c, c, c, c \\
* = c, c, c, c, c, c, 0, 0 \\
+ = 0, c, c, c
\end{align*}
\]

Then the heed of \(\hat{A}^\prime\)'s which are similarly ordered to the \(\alpha^\prime\)'s are the coefficients in the quadratic expression in the exponent. The \(\alpha^\prime\)'s are in a sense the reciprocal of the heed of the \(\alpha^\prime\)'s. The point is that if the \(\alpha^\prime\)'s are the coefficients in the quadratic expression in the exponent, then the \(\alpha^\prime\)'s will be the variances and covariances of the several variates, or -- for that matter -- \textit{vice versa}. 
The point about regressions is that, whether or not the relationship between \(y\) and \(x\) is reciprocal, i.e., in Plato's heaven of ideas, yet in this vale of tears our interest in the relationship between two variates is always unsymmetrical. We may think of \(x\) as a cause or condition of \(y\). For we may be interested in \(x\) as a means of predicting \(y\); \(x\) may belong to the past and \(y\) to the future. The mathematical consequences of this aspect of our interest is that, while we are much concerned with the distribution of \(y\) given \(x\), we are chiefly unconcerned with the distribution of \(x\) per se. We can study the change of height with age, even though the ages available are arbitrarily restricted, as for example to school age, or governed by past changes in the birth-rate, so long as the selections or restrictions which have affected the data have done so only through the variate Age and not through the variate Height. A second fundamental and controlling difference between the variates is that, so long as the
observations on height are unbiased, they may well be inaccurate without affecting the method or the nature of our inference, whereas if the observations of age were inaccurate the scope of our inference shrinks so that all we can get is a method or formula for predicting height from possibly inaccurately observed age, and must not deceive ourselves into thinking that this formula, which is the best we can get for its purpose, will represent a true relation or incorporate a true average growth-rate.

I stress these differences because the lure of m-dimensional polyhedrometry, based on the symmetrical aspect of multivariate covariation, has really been responsible for a large part of the fallacious use of the correlation-coefficient, total, partial or multiple, in statistical work in the past.

In the particular case you were concerned with, the simultaneous regression of offspring on measurements of two parents, \( b_1 \) and \( b_2 \) represent the increase in the average of the offspring corresponding with unit increase in the actual
values of the parents selected, other things being equal in each case, i.e. if I know how much an extra gallon of milk from a heifer will yet in calf will be worth to me, I could use the regression to indicate how much more one should be willing to pay for the calf from a mother with high record than for the calf from a different mother; or to predict the response of a herd to the particular degree or intensity of selection to which it might be worth while to expose it. Such a coefficient of response to selection, which was what Galton was thinking of originally before the correlation-coefficient was injected, is I suggest more directly relevant to human interest than the abstract correlation-coefficient. Neither, of course, should be thought of as analogous to the fundamental constants of physics, for they will both depend on the material and its circumstances.

Sorry you don't see your way to stop a week-end with me at once. You might bear the possibility in mind in case it ever seems convenient.

Yours sincerely,
On a separate page, because I shall show him my copy of the main letter, a note on my pupil H.P.F. Swinnerton-Dyer, whom you are likely to hear of again. He has won the first mathematical prize of the School before the age of 16: I doubt if a boy under 17 has ever won it before. Keynes won it at 17 in 1901, and I did in 1902. Also before he was 16 he had written an article which has since been printed in the Journal of the London Mathematical Society, improving on Euler’s general solution of $a^4 + b^4 = c^4 + d^4$. He is not leaving the School for more than a year yet; but I am giving him up because he has outgrown me, and passing him over to a younger man who knows the modern stuff which I never even heard of at Cambridge.

Another P.S. I see your Secretary is K. Williams. I wonder if she is Kathleen W., formerly Sec. of the Royal Astronomical, of whom I heard last from my son when he joined it. I knew her as a girl, when she was a remarkably fine pianist. At one time I was deeply attached to her sister; but I have heard nothing of her since I left Chesham, where they lived.
\[
\begin{align*}
a_{11}v_1 + 2a_{12}v_1v_2 + \cdots \\
\vdots \\
a_{12}v_2 \\
\vdots \\
a_{17}v_7 \\
\vdots
\end{align*}
\]

Equations:

\[
\begin{align*}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 1 \\
a_{12}x_1 + a_{22}x_2 + a_{23}x_3 &= 0 \\
a_{13}x_1 + a_{13}x_2 + a_{33}x_3 &= 0
\end{align*}
\]

Solutions:

\[
\begin{align*}
x_1 &= \xi_{11} + \xi_{12} + \xi_{13} \\
x_2 &= \xi_{21} + \xi_{22} + \xi_{23} \\
x_3 &= \xi_{31} + \xi_{32} + \xi_{33}
\end{align*}
\]