Dear Dr. Fisher:

I must apologize for an error I made in a letter to you about a month ago. In discussing the variance of optimum statistics, I believe & gave the example of the curve 
\[ f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2}} \]
comparing the variance of \( \bar{m} \), the optimum estimate of the mean of the distribution, with that of \( \bar{x} \), the mean of the sample. By an error, I obtained 
\[ \sigma_{\bar{m}}^2 < \sigma_{\bar{x}}^2 \text{ when } p > \frac{\pi}{2}. \]
Now, however, I find that 
\[ \sigma_{\bar{m}}^2 = \frac{p-1}{np(\pi p+1)} \]
(in the limit for large samples), while 
\[ \sigma_{\bar{x}}^2 = \frac{1}{np(2\pi p+3)} \]
Hence, \( \sigma_{\bar{m}}^2 < \sigma_{\bar{x}}^2 \) provided \( p > \frac{\pi}{2} \); or rather, provided \( p > 1 \), since for \( \frac{-\pi}{2} < p < 1 \) the formula for \( \sigma_{\bar{m}}^2 \) gives negative values. However, for \( -1 < p < \frac{-\pi}{2} \), if we may trust the formulae, \( \bar{x} \) has the smaller variance; but in this case \( \bar{m} \) is probably not given accurately by 
\[ \frac{p-1}{np(\pi p+1)} \]
even for large samples.

The general question of the exact circumstances in which optimum statistics have minimum variance — or minimum mean square deviation from the true value, which for the rectangular case is directly the variance — is extremely interesting. That the property is not perfectly general seems clear from a consideration of some of the distributions having discontinuities; and also from the fact that, if the true value were known, a system of estimation could be devised which would give it with arbitrarily small variance; and such a system of estimation might happen...
to be adopted even if the true value were unknown.

I have two students working on the optimum estimates of $m$ for the above curve and for the Type III case you treated. Failing to get anything of consequence for small samples by purely mathematical methods, they will probably soon resort to experiment.

Cordially yours,

Harold Hotelling