August 3, 1942

Dear Jackson,

I have received your 5 Airgraph letters, which were a success for speed, for they arrived in about a month, but not so much so for legibility, chiefly perhaps because my eyes are worse than ever. Here is a note on the life table relationships which may be useful for reference.

If $l_x$ is the proportion of flies surviving from emergence to age $x$, one can usefully introduce a second function

$$ \int_0^t l_x \, dt = L_x $$

Then the expectation of life at emergence is

$$ \frac{L_0}{l_0} = L_0, \text{ since } l_0 \text{ is unity; } $$

and the expectation of life at age $x$ is

$$ \frac{L_x}{l_x} $$

Now, if you have a life table population flying about, i.e., flies of age $x$ with frequency proportional to $l_x$, the death rate of a random sample of all ages will be

$$ \frac{L_0}{L_0} = \frac{1}{L_0} $$
but the death rate of the same flies one week later will be

\[ \frac{1}{L} \]

These relations will be disturbed, however, since an increasing population will have a high proportion of young flies, and a decreasing population a high proportion of old ones. But, for a stationary population, your values, p, were constant, i.e. independent of age (the early life-table actuaries made an exactly similar assumption) you would have

\[ \frac{1}{L_x} = \frac{1}{L_0} \quad e^{kx} \quad p = e^{-k} \]

whence

\[ \frac{-1}{L_x} \quad \frac{dL_x}{dx} = \frac{1}{L_0} \quad e^{kx} \]

or \(-\log e^{L_x} = \frac{1}{kL_0} \quad e^{kx} + \text{constant}\)

and

\[ L_x = Ce^{-\frac{1}{kL_0} e^{kx}} \]

and \(l_x\) is the same thing multiplied by \(\frac{1}{L_0} \quad e^{kx}\)

Putting \(x = 0\) and \(L_x = l\), this comes to

\[ \frac{c}{L_0} \quad e^{-\frac{1}{R_k} l} - 1 \quad \text{or} \quad C = L_0 \quad e^{\frac{1}{R_k} l} \]

So

\[ l_x = e^{R_k} \quad e^{-\frac{1}{kL_0} (e^{R_k} - 1)} \]

the death rate at any age \(x\) is thus

\[ \mu_x = -\frac{d}{dx} \log e^{l_x} = -k + \frac{1}{L_0} \quad e^{R_k} \]

from which it appears that, as \(\mu_x\) cannot be \(\frac{1}{kL_0}\), the factor must be not less than unity.
This gives an eminently reasonable life table, e.g., it fits the human data over a considerable range of ages, so that I do not think you ought to say that there is any theoretical reason to think that \( p_x \) rises. Of course your data may show it to rise, but that is another matter.

I am not quite clear what you mean by the span of life as contrasted with the mean life, perhaps it is the mean expectation of a random sample of all ages. I think this does not matter as in any case it can be expressed in terms of the two constants.

A point I am not clear about in letter 2 is, if you take 5 successive values of \( y \), and sum them to make \( Y \), which particular values of \( y_s \) do you add to make \( Y_s \) ?

I should be immensely interested to have these ideas developed in a paper for the Annals. The whole problem of the interpretation of re-capture data is so intricate and of such immense research importance that the very existence extensive experience you have gathered with tsetse flies ought, at all costs, to be put on record.

Yours sincerely,