Dear Mr Lyle,

The problem of five drivers and five machines is analogous in form to a good many types of experiment in common use. Taking a succession of five time intervals only, it might be worth while subdividing the total a step further as follows:

\[
\begin{array}{l}
\text{degrees of freedom} \\
\text{Between machines} & 4 \\
\text{Between drivers} & 4 \\
\text{Between periods} & 4 \\
\text{Remainder} & \frac{12}{24} \\
\text{Total} & 24 \\
\end{array}
\]

as in the Latin Square. The periods might well have an effect due to changes of temperature, or fatigue, or some such factor. If the experiment were repeated on ten different days, you would have, of course, nine degrees of freedom for days, and the remaining elements could be divided, making a full division, as follows:

\[
\begin{array}{l}
\text{degrees of freedom} \\
\text{Machines} & 4 \\
\text{Drivers} & 4 \\
\text{Periods} & 4 \\
\text{Machines by days} & 36 \\
\text{Drivers by days} & 36 \\
\text{Periods by days} & 36 \\
\text{Remainder} & 120 \\
\text{Total} & 240 \\
\end{array}
\]
The elements of your analyses are the same as these, only to some extent grouped together, and probably the sub-division I have made, though logically distinguishable, might really be found to be drawn from the same causes, and therefore need not be distinguished in practice. However, it will probably be worth while to look at the results of the full sub-division, e.g., not only might days and periods within the day contain special causes of variation, but, if the effective point were related to temperature, the effect might well be different on warm and on cold days. In this case period by days would also include special causes not affecting general error derived from the remainder, and some might obviously be true of drivers by days and of machines by days, if, for example, the machine is re-assembled for each days work.

I think the question of randomisation only comes in with regard to the applicability of the results, e.g., if you only take five drivers out of 50 possible and wish to apply the results to the whole 50, the five tested ought to be chosen at random. The same applies to machines. As to periods of work, or time of year, I presume you would prefer to explore these, so far as necessary, methodically rather than interpret your results at any one season as a sample of the year.

Yours sincerely,