Dear Penrose,

Thanks for your final data. I have, on the sheet marked A, inverted the matrix \((A_{-}F)\). This is effectively to throw all the discrepancies in the totals into the co-variances \(\frac{q}{C}\), where in any case they make a little difference. Multiplying any column of this inverted table by the series of co-variances of \(q\) with \((A_{-}F)\) I get the values which I have written in under these co-variances and which are really the sampling regressions of numbers of Mongols of given birth-rank group on numbers in given age group. Multiplying these by the co-variances and adding, one has the amount of variance in the number of Mongols of given birth-rank, which is removed by fixing the age totals.

The method of solving 7 simultaneous equations can only give a definite solution by virtue of the discrepancies in the totals, but actually the differences in your values, e.g. \(\alpha_{A_{-}} \sim \alpha\) ought to check with the regressions and if the co-variance of \(q\) with \(A_{-}\) added to zero, a constant increment added to all the regressions would make no difference. Actually, though, my deduction from the variance seems to be less than yours.
I am tolerably sure of the accuracy of the inverted table, which should have the property that any column multiplied by the corresponding column of the variance and co-variance tables should give products adding to unity, but with any other column should give zero. I have not been so careful to check the calculations beyond this, though the totals of the regressions seem to check to zero, as nearly as one would expect from the co-variances of your table.

When different children in the same family come in the same group for birth-rank, the variance for both children must come into the variance for that group, together with twice their co-variance, which, of course, is negative. I mention this, as it is not quite clear from your note that both co-variance entries have been deducted.

The agreement with theory seems to be admirable.

I am returning your papers, herewith.

Yours sincerely,