22 July 1930.

Prof. Dr W. Weinberg,
Hohenstaufenstrasse, 7,
STUTTGART,
Germany.

Dear Prof. Weinberg,

I am very glad to hear from you again, and hope that your interesting researches are making good progress.

The first point raised by your letter concerning the variance of p estimated from the formula

$$\frac{\hat{p}}{1-\hat{q}^k} = \frac{\bar{x}}{k}$$

which is derived from the hypothesis that the frequency of observation of families with x defectives out of k is

$$P_x = \frac{k!}{(k-x)!\,x!}\, p^x (1-p)^{k-x} \div \left(1-(1-p)^k\right)$$

$x = 1, 2, \dots, k.$

If n_x is the number of such families observed I maximise

$$\sum n_x \log P_x$$

for variations of p, and obtain

$$\sum n_x \left\{ \frac{x}{p} - \frac{k-x}{1-p} - \frac{kq^{k-1}}{1-q^k} \right\} = 0 \qquad \text{1.}$$

or

$$\sum n_x\left(x - pk - \frac{pkq^k}{1-q^k}\right) = 0$$

or writing

$$\sum n_x x = \bar{x}\sum n_x$$

we have the solution

$$\bar{x} = k\left\{p + \frac{pq^k}{1-q^k}\right\} = \frac{kp}{1-q^k}$$

To find the variance of p so estimated, I differentiate (I) with respect to p, and equate the result to

$$-\frac{1}{V(p)}, \text{ putting } n_x = n_k P_x.$$

This gives

$$-\sum n_x\left\{\frac{x}{p^2} + \frac{k-x}{q^2} - \frac{k(k-1)q^{k-2}}{1-q^k} - \frac{k^2q^{2(k-1)}}{(1-q^k)^2}\right\}$$

$$= -n_k\left\{\frac{q^2-p^2}{p^2q^2}\bar{x} + \frac{k}{q^2} - \frac{kq^{k-2}}{(1-q^k)^2}(k-1+q^k)\right\}$$

$$= -n_k\left\{\frac{(q^2-p^2)p}{p^2q^2} + \frac{(1-q^k)}{q^2} - \frac{q^{k-2}}{1-q^k}(k-1+q^k)\right\}\frac{k}{1-q^k}$$

$$= -n_k\left\{\frac{1-pq^{k-1}}{pq} - \frac{q^{k-2}}{1-q^k}(k-1+q^k)\right\}\frac{k}{1-q^k}$$

$$= -\frac{kn_k}{pq(1-q^k)^2}\left\{1-kq^{k-1}+(k-1)q^k\right\}$$

giving the variance formula required.

Many thanks for pointing out the mistake on p. 325

of my "Mathematical Foundations". I expect you have noticed on p. 329 that x is printed for n on line 13.

I do not think the formula

$$p^{x-\frac{1}{2}}(1-p)^{y-\frac{1}{2}}\,dp$$

is any more wrong than $p^{x}(1-p)^{y}\,dp$; the point is that both are arbitrary, and each may be justified by the appropriate _a priori_ assumption.

The paper is an attempt to lay the foundations of a theory free from all assumptions of this kind.

Yours sincerely,

R. A. Fisher