Random Access Schemes for Multichannel
Communication and Multipacket Reception
in Wireless Networks

by

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Abstract

Random access schemes have used advanced capabilities of the physical layer to achieve reliable data transmissions over wireless communication channels. These capabilities include multichannel communication and multipacket reception. Incorporating the advanced capabilities into the access schemes, as a cross-layer design, is a challenging task because a more sophisticated approach is required to interface the physical layer and the medium access control (MAC) layer.

This thesis presents development of research into the efficient random access schemes that provide a better set of cross-layer design approaches by taking into account the capabilities of multichannel communication and multipacket reception. The consideration is to propose multichannel random access schemes that use a channel outage concept of fading and interference. The system performance of the proposed schemes is then analysed. By considering imperfect channel information, a random backoff access scheme that operates with a channel sensing policy is developed. The sensing and access problem is formulated as a partially observable Markov decision process, and is solved with simple and efficient heuristic approaches. A new joint random access scheme that resolves packet collisions in the time and frequency domains is then proposed to enable effective uplink access. The joint scheme cooperates with a sensing method in which users are partially aware of channel conditions. With multipacket reception (MPR) capability, a new MAC protocol is developed by adopting a distributed access mechanism to support a wireless network in which MPR capable nodes coexist with non-MPR nodes.
Statement of Originality

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Hyukjin LEE

April 2011
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<td>PHY</td>
<td>Physical Layer</td>
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<td>MAC</td>
<td>Medium Access Control</td>
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<td>MPR</td>
<td>Multipacket Reception</td>
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<tr>
<td>FDM</td>
<td>Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>SIMO</td>
<td>Single Input Multiple Output</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
</tr>
<tr>
<td>RTS/CTS</td>
<td>Request-To-Send/Clear-To-Send</td>
</tr>
<tr>
<td>LOS</td>
<td>Line of Sight</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
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<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<td>UL-MAP</td>
<td>Uplink Map</td>
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<td>PRMA</td>
<td>Packet Reservation Multiple Access</td>
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<td>CSMA/CA</td>
<td>Carrier Sense Multiple Access with Collision Avoidance</td>
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<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
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<td>CSI</td>
<td>Channel State Information</td>
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<td>Abbreviation</td>
<td>Definition</td>
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<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
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<tr>
<td>GSM</td>
<td>Global System for Mobile Communications Protocol</td>
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<td>Wi-Fi</td>
<td>Wireless Fidelity</td>
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<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>POMDP</td>
<td>Partially Observable Markov Decision Process</td>
</tr>
<tr>
<td>NAV</td>
<td>Network Allocation Vector</td>
</tr>
<tr>
<td>SIFS</td>
<td>Short Interframe Space</td>
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<tr>
<td>DIFS</td>
<td>Distributed Interframe Space</td>
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<tr>
<td>ACK</td>
<td>Acknowledge</td>
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<td>NUM</td>
<td>Network Utility Maximisation</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Key</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Key</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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List of Symbols

$K$  Number of users
$N$  Number of channels
$q_{n,i}$  Channel outage probability of user $i$ over channel $n$
$p_{n,i}$  transmission probability of user $i$ over channel $n$
$p_{\text{max}}$  Maximum transmission probability
$p_c$  Conditional collision probability
$\beta$  A reducing factor
$S$  System throughput
$G$  Offered channel traffic
$\tau$  Stationary transmission probability
$P_{i,j}$  One-step transition probability from state $i$ to state $j$
$P(i, j|m, h)$  One-step transition probability from state $\{m, h\}$ to state $\{i, j\}$
$\pi$  Stationary probability of staying at state $i$
$\pi_{i,j}$  Stationary probability of staying at state $\{i, j\}$
$S(t)$  Network state at slot $t$
$A(t)$  Action state at slot $t$
$r(S(t), A(t))$  Reward function of $S(t)$ and $A(t)$ at slot $t$
$\Lambda(t)$  A belief vector at slot $t$
$V_t(\Lambda(t))$  Maximum expected remaining reward at slot $t$
$\Omega_i(t)$  Probability distribution of user $i$’s channel conditions at slot $t$
List of Symbols

\( p_t^d \)  Packet transmission probability of direct-links
\( p_t^u \)  Packet transmission probability of up-links
\( P_{tr} \)  Probability that there is at least one packet transmission among all users
\( P_{us}^u \)  Conditional probability that multiple up-links are successfully established
\( P_{ds}^d \)  Conditional probability that multiple direct-links are successfully established
\( P_{idle} \)  Probability of an idle transmission
\( P_{coll} \)  Probability of a collision
\( P_{suc}\)  Probability of a successful transmission
\( T_s \)  Time duration for a successful transmission
\( T_c \)  Time duration for a collision
\( W^d \)  Contention window size for direct-links
\( W^u \)  Contention window size for up-links
\( S_{opt} \)  Optimal throughput
\( S_{max} \)  Maximum throughput
List of Publications


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Chapter 1

Introduction

Random access schemes have received increasing attention for enabling effective uplink access in next generation wireless communication systems such as WiMAX and 4G [1, 2]. When designing the random access schemes in the medium access control (MAC) layer, incorporating capabilities of the physical layer is important for achieving reliable data transmissions over wireless communication channels [3].

Recent development of advanced signal processing techniques provides more flexible and reliable capabilities, changing the underlying characteristics of the physical layer. These capabilities include multichannel communication and multipacket reception. The simultaneous communication of multiple channels is made possible by applying advanced multiplexing techniques [1, 2, 4–6], and the simultaneous reception of multiple packets is made feasible by using signal separation techniques [7–9].

Since successful data transmissions controlled in the MAC layer rely on a capability of the physical layer, the design of new random access schemes must incorporate the advanced capabilities of the multichannel communication and the multipacket reception. Fundamentally, such a cross-layer design is a challenging task because we need a more sophisticated approach that interfaces the physical layer and the MAC layer [11, 12].

In this thesis, we will develop efficient random access schemes that provide a better set of cross-layer design approaches by taking into account the capabilities of multichannel communication and multipacket reception. These random access schemes are suitable for handling initial access, bursty traffic, and short packets in uplink communications.
1.1 Research Challenges in Random Access

Traditionally, random access schemes have been designed with the minimum characteristics of the physical layer [10, 13–15]. Such physical layer characteristics can be modelled by using a simple collision model that represents a successful data transmission. In the collision model, if multiple packets are transmitted simultaneously, the packets cannot be received (i.e., a collision occurs). Only a single transmission is received successfully without a failure through an error-free channel.

Based on the idealised collision model, conventional random access schemes have mainly focused on collision resolution or collision avoidance techniques [10]. However, these conventional approaches have the difficulty of extending the collision model to more elaborate models that incorporate the multichannel communication and multipacket reception capabilities. Thus, an entirely new way of treating random access is required [9].

This thesis addresses new challenges when adopting the multichannel communication and multipacket reception capabilities. One challenge is to exploit time-varying conditions of wireless channels in the multichannel communication. The other is to use the multipacket reception (MPR) capability for heterogenous networks in which MPR-enabled nodes co-operate with non-MPR nodes.

1.1.1 Time-varying multiple channels

Wireless channels have a time-varying nature which is difficult to characterise when designing wireless systems. Since the channels experience noise, fading, and interference, their conditions change with time, and it is difficult to communicate without a link breakage under the time-varying conditions.

As a cross-layer design approach, exploiting the time-varying multiple channels is a key consideration in random access schemes for improving system performance [9, 16–20]. The cross-layer approach considers channel fluctuations at the physical layer that provide valuable information to access channels. For example, when a channel is in a deep fade, a user seldom transmits a packet with the current channel state, and thus waits for a better channel state.

To develop an efficient random access scheme, we should consider a combined prob-
lem of exploiting channel conditions and mitigating packet collisions. A number of ran-
dom access schemes have been developed, mainly focusing on mitigating the packet col-
lisions [5, 6, 32, 74, 78–80] or exploiting the channel conditions [72, 73, 92]. In collision-
aware access schemes, users have no knowledge of channel conditions, and are likely to
access bad channels, leading to performance degradation. Similarly, channel-aware ac-
cess schemes with no collision resolution can cause more collisions, although they can
access good channels. Further development of the access schemes is thus required to
improve system performance by designing a more advanced collision resolution scheme
with a channel condition sensing capability.

1.1.2 Coexistence of MPR and non-MPR capabilities

The multipacket reception (MPR) allows simultaneous transmissions over a channel with
advanced signal processing techniques. Such MPR capability can be modelled to rep-
resent a multiuser physical layer for random access by using conditional probabilities
instead of deterministic failure in the conventional single packet reception. In the MPR
model, a successful reception can be assumed if a user’s signal to interference and noise
ratio (SINR) exceeds a certain threshold.

The MPR capability has the potential to improve network performance, while it presents
new challenges by allowing MPR at the physical layer [8]. In the conventional collision
model, the outcome of a particular transmission is simple and it can be a success, a col-
lision, or idle. In contrast, with the outcome of a transmission with MPR, a high level of
uncertainty exists. For example, in the conventional collision model, if a packet is suc-
cessfully received, it implies that no one else transmitted. In the MPR model, the success-
ful reception of multiple packets does not imply that a particular user has not transmitted
because of the probabilistic modelling of MPR. Such uncertainty makes the conventional
collision resolution methods difficult to apply for MPR channels.

In order to improve network performance, a number of MPR access schemes have
been proposed in various contexts [21–23, 64–68, 103]. However, most existing schemes
assume the existence of a central coordinator for packet transmissions. These MPR
schemes are hardly applicable to distributed wireless networks such as ad hoc networks.

Recently, some MPR schemes [66, 67] considered a distributed design by adopting
the request-to-send (RTS)/ clear-to-send (CTS) mechanism [28, 29]. In [68], alternative backoff schemes were proposed to resolve unfairness of accessing a channel when distributed nodes enable the MPR capability. Although all the existing MPR schemes have been developed to exploit the MPR capability for a better performance, MPR schemes are not yet considered to co-operate with the conventional non-MPR schemes. Such a coexistence scenario is feasible in distributed wireless networks.

1.2 Contribution of the Thesis

The original contributions of the thesis made to random access schemes lie both in wireless system design and applications. The originality and breadth of these contributions is evidenced by the list of publications on page xxv.

The work in this thesis is based on the premise that new innovative access schemes must take into account the flexible and reliable capabilities in the physical layer: the multichannel communication and the multipacket reception.

In the thesis, we identify several prominent access problems when incorporating the advanced capabilities for wireless networks: (i) simultaneous usage of multiple channels which have time-varying channel conditions with limited information, and (ii) coexistence of MPR-enable schemes with conventional non-MPR schemes in a heterogeneous network.

Associated with the problems, we present several solutions with (i) multichannel-aware collision resolution that uses random backoff and channel sensing methods, and (ii) refined MPR access schemes that extend a distributed method to support the coexistence.

Figure 1.1 shows the research problems and solutions for random access with the multichannel communication and multipacket reception capabilities.

The primary contributions of this thesis are as follows:

- We develop efficient random access schemes that allow simultaneous usage of multiple channels under time-varying conditions. These schemes are proposed through rigorous Markov analysis (Chapter 3).

- We design multichannel random access schemes that cooperate with channel sensing methods when multiple channel conditions are partially provided. More realis-
Figure 1.1: The research problems and solutions for random access with the multichannel communication and multipacket reception capabilities.

- We develop an enhanced MAC protocol that exploits the multipacket reception capability to support coexistence of MPR-enable and non-MPR nodes. This thesis is the first to address the coexistence problem when using the MPR capability (Chapter 6).

- We analyse system performance by using Markov models, and optimise its performance by adjusting system parameters based on the derived models (Chapters 3, 4, 5, and 6).

1.3 Thesis Organisation

Chapter 1 introduces the challenging research problems that occur because of the advent of advanced signal processing techniques, and states our contribution of the thesis motivated by the challenges.

In Chapter 2, we discuss the technical background of this research by outlining the capabilities of multichannel communication and multipacket reception. We provide a sufficient literature review on existing random access schemes associated with the advanced capabilities.

Chapter 3 addresses a channel outage concept that represents channel fading and in-
interference for an efficient design of medium access protocols. Under multichannel outage environments, we analyse the system throughput of Aloha-type access protocols and propose multichannel outage-aware access protocols to improve throughput. Furthermore, we study throughput performance by applying game-theoretic access approaches with channel outage effects.

In Chapter 4, we propose a random backoff access scheme that operates with a channel sensing policy under imperfect and time-varying multichannel conditions. The sensing and access problem is formulated as a Partially Observable Markov Decision Process (POMDP) and is solved with simple and efficient heuristic approaches.

In Chapter 5, a new joint random access scheme is proposed to enable effective uplink access when users are partially aware of channel conditions in wireless networks. The access scheme mitigates packet collisions with a joint backoff in the time and frequency domains, cooperating with a sensing method that exploits the channel conditions. The analysis of the joint access performance is facilitated by a new Markov model that provides a closed-form throughput expression.

In Chapter 6, we develop a new MAC protocol that uses multipacket reception (MPR) capability to achieve better throughput than conventional MAC protocols. By adopting a request-to-send/clear-to-send mechanism in IEEE 802.11 MAC standards, the proposed MPR MAC protocol improves throughput when a wireless network operates with MPR capable nodes and non-MPR nodes. We also analyse the system throughput of the coexistence of different link characteristics of nodes, and optimise its throughput by adjusting contention window sizes with respect to certain throughput requirements of the nodes.

Chapter 7 summarises the research performed and suggests some future research directions in connection with our current research work. In the future work, the proposed schemes of the previous chapters will be extended from the perspective of cross-layer modelling and optimisation for wireless networks.
Chapter 2

Background and Related Work

This chapter outlines the capabilities of multichannel communication and multipacket reception. It discusses how such capabilities are incorporated into the existing random access schemes through the IEEE medium access control standards. The chapter serves to provide a sufficient background for the research of developing new random access schemes presented in this thesis.
2.1 Multichannel Communication

Multichannel communication capability is to allow simultaneous usage of multiple channels by applying multiplexing techniques. Two common multiplexing techniques are frequency division multiplexing (FDM) and orthogonal FDM (OFDM).

In an FDM system, each communication channel uses a single carrier allocated to a unique frequency range. Since these channels are non-overlapping, multiple users can operate concurrently by simply using different frequency channels. Figure 2.1 illustrates the basic idea behind FDM.

In an OFDM system, a single channel utilises multiple subcarriers that have a narrow bandwidth. These sub-carriers are able to partially overlap without interfering adjacent subcarriers. Figure 2.2 shows the concept of overlapping subcarriers behind OFDM. Because of the overlapping usage of the spectrum, the OFDM system is more efficient than the FDM system.

In this section, we will discuss the principles of OFDM for multichannel communication and focus on its recent development in multichannel systems.

2.1.1 Orthogonal frequency division multiplexing

OFDM is a subset of frequency division multiplexing. It uses multiple orthogonal subcarriers to transmit signals. Each subcarrier is allocated with a narrow bandwidth so that its frequency response characteristics are nearly ideal, i.e., each subcarrier experiences flat fading. This simplifies a system design because the flat fading can be compensated by using a simple equalizer in the frequency domain [24]. An OFDM system offers a better performance because the sum of data rates of all overlapped subcarriers is higher than that of a single carrier used in an FDM system.

In communication systems, there are two important considerations to guarantee successful signal transmission [25]: (i) adjacent channel interference and (ii) inter symbol interference.

The adjacent channel interference occurs when modulation of each sinusoid carrier is applied without any filtering. The modulation causes constant transitions in its shape and amplitude, and this results in high channel power outside of a transmitted bandwidth,
interfering adjacent channels in the frequency domain.

An OFDM system avoids the adjacent channel interference by limiting channel bandwidth through a pulse shaping filter. In general, the filter is a sinc-shaped pulse in the time domain and it appears as a square wave in the frequency domain. Hence, the sinc-shaped pulse effectively eliminates spectral leakage and limits channel bandwidth by using a smaller portion of the frequency domain. Some commonly used sinc-shaped pulses are the raised cosine filter, the root raised cosine filter, and the Gaussian filter.

The inter symbol interference is the other important consideration caused by multipath fading when signals are transmitted over long distance through various mediums. The characteristic of the physical environment makes some symbols delay beyond their given time interval. As a result, the delayed symbols can interfere with the following or preceding transmitted symbols.

In an OFDM system, the inter symbol interference can be effectively mitigated by using a cyclic prefix and optimising the length of the cyclic prefix. The cyclic prefix appends the first section of a symbol to the end of the symbol to remove multi-path channel
reflections of the original signal. This repetition avoids the interference with the subsequent symbol. In addition, when the cyclic prefix is sufficiently long (compared with the spread of the channel), the inter symbol interference can be fully mitigated.

Orthogonality of subcarriers is a key principle to mitigate the inter symbol interference [26]. Each subcarrier has a sinusoidal waveform,

\[ x_k(t) = \sin 2\pi f_k t, \quad k = 0, 1, \ldots, K - 1, \]  

(2.1)

where \( f_k \) is the mid-frequency in the \( k \)th subcarrier. The subcarriers can be orthogonal over the symbol interval \( T \) by selecting the symbol rate on each subcarrier to be equal to the separation of adjacent subcarriers, showing that

\[ \int_0^T \sin(2\pi f_k t + \phi_k) \sin(2\pi f_j t + \phi_j) dt = 0. \]  

(2.2)

The relative phase relationship between subcarriers (i.e., \( \phi_k \) and \( \phi_j \)) is independent from their orthogonality. This allows the subcarriers to be partially overlapped in which the maximum power of each subcarrier is allocated to the minimum power of each adjacent subcarrier.

The orthogonal allocation of subcarriers is done by applying a pulse shaping filter. With OFDM systems, a sinc-shaped pulse is applied in the frequency domain of each subcarrier. As a result, each subcarrier remains orthogonal to one another. Note that with FDM systems, a sinc-shaped pulse is applied in the time domain by attenuating the beginning and ending portions of the symbol period. Figure 2.3 shows the orthogonality of subcarriers in the frequency domain of an OFDM system [27]. Each subcarrier is represented by a different peak of a pulse shaping waveform and its peak is located with the zero crossing of all subcarriers.

OFDM is widely used. Compared with FDM, its attractions are simpler design, greater spectral efficiency and lower inter symbol interference.

### 2.1.2 Multichannel systems of OFDM

Many emerging multichannel systems are OFDM-based. They include Wi-Fi for wireless local area networks, WiMAX and long term evolution (LTE) for wireless metropolitan area networks, and digital video broadcast (DVB) for wireless wide area networks [1,
Wi-Fi supports multichannel systems by using the wireless technology based on the IEEE 802.11 standards [28–30]. The IEEE 802.11a and IEEE 802.11g implementations specifically use OFDM techniques, providing 12 non-overlapping channels in the 5 GHz band and 3 non-overlapping channels in the 2.4 GHz band, respectively. Each channel occupies 20 MHz of bandwidth for the IEEE 802.11a standard and 25 MHz of bandwidth for the IEEE 802.11g standard, and it consists of 52 subcarriers of 300 kHz each. By using different modulation schemes, e.g. BPSK, QPSK, 16-QAM, or 64-QAM\(^1\), the IEEE 802.11a/g standard can provide throughput values of up to 54 Mbps. In the emerging IEEE 802.11n standard, OFDM is extended by adopting the multiple-input multiple-output (MIMO) technology to improve throughput of the previous standards [31]. With different modulation and coding schemes, the IEEE 802.11n increases the maximum throughput from 54 Mbps to 600 Mbps. For instance, as with the MIMO-OFDM technology, the IEEE 802.11n uses four spatial streams with 64-QAM 5/6 encoding at a channel width of 40 MHz, achieving 600 Mbps [30].

WiMAX also supports multichannel systems by using the OFDM technology in the IEEE 802.16 standards [32]. The WiMAX systems are designed to provide internet access across long wireless communications links. The initial IEEE 802.16 standard operates in

\(^1\)Referred to as these symbols, BPSK denotes binary phase shift key, QPSK quadrature phase shift key, and QAM quadrature amplitude modulation
the 10-66 GHz band with line of sight (LOS) connectivity, while the IEEE 802.16a standard works in the 2-11 GHz band for non-LOS (NLOS) communications. The current version of the standard (IEEE 802.16e) includes both LOS and NLOS communication in the 10-66 GHz and 2-11 GHz bands, respectively. WiMAX in LOS communications provides a 36-135 Mbps data rate. In NLOS communications, it can increase data throughput of up to 75 Mbps. In WiMAX, each OFDM channel consists of 128 to 2048 subcarriers and can occupy bandwidths from 1.25 MHz to 20 MHz. In addition, each of these subcarriers is modulated using BPSK, QPSK, 16-QAM, or 64-QAM modulation, depending on the requirements of the physical channel. Since the OFDM technology is more resilient to multi-path symbol interference, the IEEE 802.16 standards fully utilise the OFDM technology to transmit data over long distances of up to 50 km.

In Table 2.1, we summarise the characteristics of OFDM-based MAC standards associated with Wi-Fi and WiMAX systems. The parameters are frequency band, channel bandwidth, channel numbers, modulation, radio technology, and maximum data rate. For comparison purpose, we also provide other MAC standards that provide multiple channels with different physical layer technology used in IEEE 802.15.1 Bluetooth [33] and IEEE 802.15.4 Zigbee [34] for wireless personal area networks.

### 2.2 Multipacket Reception

Multipacket reception (MPR) is a capability through which a receiver can detect multiple packets transmitted simultaneously from different users over a channel. This MPR concept was first addressed by S. Ghez et al [35]. Its motivation comes from the need to adapt variations of wireless channels to advanced signal processing at the physical layer and effective access schemes at the MAC layer. The idea of cross-layer design is thus required for a successful implementation of MPR.

This section will address the principles of MPR and describe MPR capable systems for uplink transmissions.
Table 2.1: Comparison of the IEEE MAC standards for the multichannel systems

<table>
<thead>
<tr>
<th>Std</th>
<th>Band</th>
<th>Bandwidth</th>
<th>No.of channels</th>
<th>Modulation</th>
<th>Radio technology</th>
<th>Max. data rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 802.11b</td>
<td>2.4 GHz</td>
<td>25 MHz</td>
<td>14 (3 non-overlapping)</td>
<td>BPSK, QPSK, CCK</td>
<td>DSSS</td>
<td>11 Mbps</td>
</tr>
<tr>
<td>IEEE 802.11a</td>
<td>5 GHz</td>
<td>20 MHz</td>
<td>12 non-overlapping</td>
<td>BPSK, QPSK, M-QAM</td>
<td>OFDM</td>
<td>54 Mbps</td>
</tr>
<tr>
<td>IEEE 802.11g</td>
<td>2.4 GHz</td>
<td>25 MHz</td>
<td>14 (3 non-overlapping)</td>
<td>BPSK, QPSK, M-QAM</td>
<td>OFDM</td>
<td>54 Mbps</td>
</tr>
<tr>
<td>IEEE 802.11n</td>
<td>2.4/5 GHz</td>
<td>20/40 MHz</td>
<td>14 (3 non-overlapping)</td>
<td>BPSK, QPSK, 16-QAM, 64-QAM</td>
<td>OFDM</td>
<td>600 Mbps</td>
</tr>
<tr>
<td>IEEE 802.16</td>
<td>10-66 GHz</td>
<td>20,25, 28 MHz</td>
<td>various numbers</td>
<td>64-QAM, 256-QAM</td>
<td>OFDM</td>
<td>36-135 Mbps</td>
</tr>
<tr>
<td>IEEE 802.16a</td>
<td>2-11 GHz</td>
<td>20,25, 28 MHz</td>
<td>various numbers</td>
<td>64-QAM, 256-QAM</td>
<td>OFDM</td>
<td>75 Mbps</td>
</tr>
<tr>
<td>IEEE 802.16e</td>
<td>2-66 GHz</td>
<td>1.25 - 20 MHz</td>
<td>various numbers</td>
<td>64-QAM, 256-QAM</td>
<td>OFDM</td>
<td>350 Mbps</td>
</tr>
<tr>
<td>IEEE 802.15.1</td>
<td>2.4 GHz</td>
<td>1 MHz</td>
<td>79</td>
<td>GFSK</td>
<td>FHSS</td>
<td>1 Mbps</td>
</tr>
<tr>
<td>IEEE 802.15.4</td>
<td>868/915 MHz</td>
<td>2 MHz</td>
<td>16</td>
<td>BPSK, QPSK</td>
<td>DSSS</td>
<td>250 kbps</td>
</tr>
</tbody>
</table>
2.2.1 The multipacket reception principles

The MPR capability relies on the ability of separating intended signals from observed signals. This signal separation can be achieved by exploiting temporal, spectral, or spatial diversity in transmission and reception [8].

Temporal and spectral diversity is employed by designing waveforms carefully at the physical layer. The key to signal separation is to allocate a specific signature code to each user. For this separation approach, multiuser detection techniques are commonly used in code division multiple access. These techniques can apply training symbols in a data stream to offer the ability to track one or a group of users [7], or employ structures of a channel and characteristics of input sources without training symbols [21, 36, 37].

Spatial diversity is another aspect to separate signals [3]. A proper implementation of antenna arrays allows the signal separation. There are two main techniques for exploiting transmit antenna arrays: space-time coding and transmit beamforming. These two strategies are based on different assumptions regarding channel feedback available at the transmitter. Space-time coding requires no feedback, whereas conventional beamforming requires accurate feedback.

In general, a MPR channel model that incorporates temporal, spectral, and spatial diversities can be obtained from a channel model in a multiple-input and multiple-output (MIMO) system [8, 35]. The MPR channel model in [35] is a symmetrical model with indistinguishable users analogous to the classical urn model with indistinguishable balls. This model does not differentiate users that may have their own access rates. A more generalised MPR model has been developed to support the user diversities [8]. With the MPR channel model, a basic problem of the signal separation is to design a channel estimator that allows a receiver to extract its intended packets. The channel estimator design strongly depends on the knowledge of the channel impulse response and the format of transmission.

Figure 2.4 illustrates a general model of the signal separation at the MPR physical layer [8]. In the model, $M$ users transmit to a receiver equipped with $N$ antenna array.

\footnote{Note that since a MIMO technology uses multiple antennas both on a transmit side and a receive side, we can apply the MIMO technology to obtain the MPR capability.}
Figure 2.4: A general model for multiuser communications and receiver multipacket reception.

elements. Let $s_i(t)$ denote a transmitted signal from the $i$th user, $i \in \{1, ..., M\}$, and $x_j(t)$ denote a received signal from the $j$th antenna array element, $j \in \{1, ..., N\}$. The received signal relies on a channel noise $n_j(t)$ at the $j$th antenna. For the signal separation, the receiver needs to detect the multiple transmitted signals by estimating them based on the multiple observed signals. The estimated signal $\hat{s}_i(t)$ of the $i$th user can be obtained with an estimator $F(z)$. The design of the estimator needs knowledge of a channel impulse response $H(z)$ that depends on the form of modulation, the transmission protocol, and the configuration of transceiver antenna array. In wireless networks, it is unrealistic to assume that the receiver has the full information of the channel impulse response. Thus, it is challenging to design an estimation technique without the knowledge of the channel impulse response [21]. Using structures of channels and characteristics of signals is desirable to develop efficient signal separation techniques [21, 36, 37].

2.2.2 Multipacket reception capable systems

The multipacket reception capability has been implemented for wireless systems by placing multiple antennas at a receive side. A typical example of MPR systems is a wireless local area network (WLAN) for uplink transmissions.

A MPR system configuration is illustrated in Figure. 2.5. An access point is mounted with multiple antennas and each user has only one antenna. This MPR system allows efficient uplink transmissions in a centralised manner for a WLAN. The uplink transmissions in the MPR system can also be accomplished by using a distributed random access
Recent MPR systems adopt combined technologies of MIMO and OFDM, aiming at achieving higher user throughput, improved signal quality and increased system capacity. These MIMO-OFDM technologies for the MPR systems are used with new modulation and coding mechanisms in the standards of Wi-Fi, WiMAX, and LTE [?, 1, 2, 31].

2.3 Random Access Schemes

Random access schemes dynamically allocate radio resources to multiple users. These random access schemes are particularly required for bursty data or distributed traffic. A number of solutions have been proposed to solve the problem of how to efficiently allow many users to transmit their randomly arriving data.

Random access schemes can be classified into conventional schemes and recent schemes as shown in Figure 2.6. The conventional schemes have been designed to achieve collision resolution and system stability in a single channel environment, while the more recent schemes have been developed with respect to new design issues: multichannel communication, multipacket reception, and multiuser diversity. We will briefly summarise a literature review of existing random access schemes in the following subsections.
2.3.1 Conventional random access schemes

Conventional random access schemes operate in a single channel environment. When multiple packets are transmitted simultaneously over a single channel, the conventional random access schemes cannot receive the packets. This unsuccessful transmission is regarded as a packet collision.

The conventional random access schemes perform well for delivering short packets [10]. In addition, the random access schemes can handle bursty data traffic with low collisions, while reservation protocols such as packet reservation multiple access (PRMA) [38, 39] are adequate for periodic voice traffic.

Well-known random access schemes include Aloha (or slotted Aloha) and carrier sense multiple access with collision avoidance (CSMA/CA). In Aloha, users simply transmit packets at will without regard to other users. If the packet is not acknowledged by the receiver after some period, it is assumed lost and is retransmitted. When the intensity of the traffic increases, this scheme becomes inefficient and delay prone because most transmissions result in collisions. CSMA/CA commonly used in wireless LANs improves upon Aloha through carrier sensing, in which users listen to the channel before transmitting in order to not cause avoidable collisions. Although a CSMA/CA-based scheme is well applied for unlicensed band systems, it is not suitable for cellular networks that use licensed bands because the carrier sensing leads to channel inefficiency. For example, the cellular systems such as GSM, CDMA2000, LTE, and WiMAX have used an Aloha type of solution to transmit bursty packets for controlling transmit power, data rate, and cod-
ing rate. Since Aloha has a simple transmission structure compared with CSMA/CA, it can effectively cooperate with more complex access or scheduling schemes in the cellular systems.

From a historical perspective, the conventional random access schemes have been designed with respect to the following two issues: collision resolution and stability.

The collision resolution aims to mitigate any simultaneous transmissions that cause irrecoverable failure, improving system throughput. For an efficient collision resolution, tree algorithms were first developed in 1979 by splitting user transmissions based on the sequence of collision events [14]. The development of the tree algorithms triggered an intensive search for random access schemes that achieve the maximum throughput. Further developments were made by the idea of using classical group testing in which users who have packets can be statistically identified by a sequence of tests [15, 40].

The stability is the other important issue to avoid buffer overflow by reducing the data arrival rate to some equilibrium point. Typically, when the number of users increases, their transmission probability must be reduced to make the system stable. In order to find a stability region of Aloha random access, a number of studies were made for stability analysis [41, 42]. However, the conventional approaches for collision resolution and stability may not be suitable to incorporate more advanced new issues and thus we may consider a different new approach to develop random access [9].

2.3.2 Recent random access schemes

With the advent of advanced techniques, recent random access schemes have been developed, focusing on: (i) multichannel communication, (ii) multipacket reception, and (iii) multiuser diversity. These new capabilities have been incorporated with the collision resolution and stability issues of the conventional random access schemes.

The multichannel communication allows a more flexible opportunity to resolve packet collisions and to stabilize a system by transmitting over multiple channels, compared with a single channel system. Designing random access schemes that efficiently utilize multiple channels is a key consideration for improving system performance.

The multipacket reception reduces packet collisions because the collided packets can be successfully detected with certain signal processing techniques. Such capability thus
improves system performance with stability. For exploiting the MPR capability, random access schemes have to be designed with a new collision model which has a high level of uncertainty of reception probabilities.

The multiuser diversity is obtained from exploiting different users’ channel state information. Such channel exploitation allows users to adjust their transmission probabilities over preferable channels. In order to utilise the multiuser diversity fully, the collision resolution and stability also need to be taken into account together. However, in general, finding an optimal solution for the multiuser diversity is difficult because of insufficient channel information.

A brief summary of the recent random access schemes is presented with the multichannel communication, the multipacket reception, and the multiuser diversity as shown in Figure 2.6.

**Multichannel random access schemes**

Earlier work has focused on multichannel slotted Aloha [43–46]. In [43, 46], the performance of the multichannel slotted Aloha was analysed for multichannel satellite communication with fixed bandwidth per channel. The multichannel slotted Aloha has also been modified to reduce the number of connections [44] and to utilise a reservation concept [45].

Emerging work on multichannel random access considers the use of random access schemes for power ramping algorithms and the hand-over process of mobile users in the IEEE 802.16 standards [5, 6, 47, 48]. Figure 2.7 illustrates a frame structure in the IEEE 802.16 orthogonal frequency division multiple access (OFDMA) standard [6]. This frame structure is used for initial ranging and bandwidth request through multichannel random access schemes. In the OFDMA frame structure, each user should send a short request packet to the base station by using a random access scheme in the uplink (UL) subframe. After the base station receives the request packets successfully, it allocates subchannels to the requests, and broadcasts the allocation on uplink map (UL-MAP) in the downlink (DL) subframe. Each user can access the reserved sub-channels (i.e., UL burst channels) in the next uplink subframe.

In order to reduce excessive amounts of access delays, a fast retrial scheme based on
slotted Aloha was proposed by exploiting the structure of OFDMA in [5]. This scheme resolves collisions by randomly accessing subchannels for retransmission, instead of deferring access time in conventional access schemes. In [6], resource allocation in an OFDMA system is considered to partition the overall resources into two portions: one for Aloha-based random access and the other for connection-oriented access. With a truncated binary backoff access and time-division OFDMA, the limited radio resources have been allocated optimally in both the time and frequency domains. Only the truncated binary backoff in OFDMA was also analysed in [47] similar to that in IEEE 802.11. In [48], a multichannel CSMA/CA scheme was developed to resolve collisions by transmitting simultaneously on different subchannels at different slot times. System efficiency of the CSMA/CA-based scheme is improved by exploiting the OFDMA system features.

Multipacket reception related schemes

Multipacket reception on a Aloha scheme was first studied in [35, 49], deriving the maximum stable throughput of Aloha channel. The collision Aloha channel was modelled to determine the number of successfully received packets in each slot as a random variable. In addition, asymptotic performance on MPR was analysed on the collision resolution.
techniques and the capture probability with infinite number of users in [50, 51].

A variety of extensions on MPR models and analysis have been made in [8, 22, 23, 52–60]. In particular, the channel model of [35] was improved for retransmission control schemes by utilizing additional feedback [54]. The stability on a MPR model was studied based on a game theoretic analysis, assuming perfect information is available [55]. Furthermore, Zhao and Tong designed new MAC protocols to maximise throughput by taking into account channel history and quality of service constraints [22, 23]. An extension of a CSMA protocol in multipacket reception networks was analysed, assuming that nodes can sense a power level in a channel and the number of packets currently being transmitted over the channel [56]. A spatial distribution function of nodes was considered to provide the stability of Aloha protocols in a multipacket reception channel [57]. A signal-to-noise ratio model was also used to represent capture effects under single packet reception and multiple packet reception [58]. In order to simplify a MPR model, a simple power-aware algorithm was proposed, operating with local channel knowledge and received signal strength measurements [59]. Moreover, a new random access scheme that contents a MPR channel was developed based on the optimal stopping theory to stop the contention process and to restart data transmission [60].

Recent development of MPR schemes is also extended for designing distributed schemes that can be used in WLANs, ad hoc networks, or wireless sensor networks [61–68]. The slotted Aloha on multihop MPR networks was proposed and its performance was analysed [61]. The RTS/CTS mechanism, one of well-known distributed schemes in IEEE 802.11 standards [28, 29], was extended to incorporate the MPR capability [66, 67]. In [68], the existing backoff scheme was further enhanced to resolve unfairness to access a channel when enabling the MPR capability with spatially distributed nodes. Showing the performance improvement of WLANs with the MPR capability, a cross-layer protocol was designed to incorporate advanced signal processing techniques [62, 64]. In [65], new distributed MPR MAC protocols were developed to maximise system throughput and minimise energy efficiency subject to certain throughput constraints in wireless sensor networks. The utilisation of the MPR capability was taken into account for distributed vehicular networks that have no fixed paths and lack of continuous network connectivity [63].
Multiuser diversity related schemes

Multiuser diversity can be exploited by transmitting users’ packets when their channel conditions are favorable. In such channel exploitation, centralised schemes can be applicable with full channel state information, while decentralised schemes can operate with local channel information. These multiuser diversity related schemes have been developed for wireless systems with a single channel or multiple channels.

For a single channel communication system, channel-aware Aloha was deployed to use only local knowledge of channel states [69, 70]. It achieves the same system throughput of a centralised scheduler by using a splitting algorithm in which the splitting sequence depends the users’ channel gain. In [71], transmission control schemes were developed as a variant of slotted Aloha protocol, showing that the transmission control effect is equivalent to changing the probability distribution of the channel state.

The multiuser diversity has also been applied to multichannel systems. In [72], an opportunistic multichannel Aloha was proposed for uplink transmissions in OFDMA wireless networks. It maps from a user’s channel state information to its transmission probability and subcarrier allocation. Using the extreme-value theory, the proposed scheme was shown to be asymptotically order optimal. The stability and throughput performance of the opportunistic multichannel Aloha was analysed, providing stability conditions and upper bounds on average queue sizes [73]. By taking into account different channel statistics, throughput optimisation was formulated as a convex problem and was solved with a simple binary-like access strategy. An efficient multichannel random access was also proposed with implicit message transmission to make communications more reliable in a contention based environment [74]. By taking into account local channel state and traffic information, decentralised optimisation for multichannel random access was developed in [75]. This scheme adjusts a transmission probability based on local channel state information collected from neighbours and allocates power for each traffic flow on each subchannel. Since the scheme exploits both multiuser diversity and traffic spatial distribution, it outperforms existing channel-aware Aloha.
2.4 Summary

The capability of multichannel communication is to allow simultaneous usage of multiple channels by applying multiplexing techniques. The capability of multipacket reception is to detect multiple intended packets from their observations by using signal separation techniques. These advanced capabilities have been incorporated into the existing random access schemes through the IEEE medium access control standards. In order to improve system performance, new random access schemes must take into account the conventional collision and stability issues as well as the advanced capabilities.
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Chapter 3

Multichannel Outage-aware Access Schemes for Wireless Networks

This chapter presents the throughput analysis of Aloha-type access schemes based on multichannel outage models. Using the results from the analysis, the following two multichannel outage-aware access schemes are proposed to improve throughput: (i) frequency-domain backoff with decentralised channel selection, and (ii) time-domain backoff with centralised channel allocation. In addition, the throughput analysis is studied with game-theoretic access approaches that take channel outage into account. Simulation results show the impacts of the outage-aware access schemes and the game-theoretic approaches on system throughput with respect to various channel numbers and outage probabilities.
3.1 Introduction

Major impairments of wireless channels are fading and interference that can result in degradation of wireless communications [76]. Channel outage can happen due to fading and interference and should be taken into consideration for an efficient design of medium access schemes. The channel outage probability is the probability of preventing channel access, and depends on fading and interference characteristics.

A number of medium access schemes have been developed to access multiple channels that become readily available in wireless networks\(^1\). Such multiple channel capability offers better access opportunity for users, while the channels are subject to the channel outage caused by time-varying fading and interference. Hence, to develop an efficient access schemes, both the multichannel capability and the channel outage should be considered.

Conventional access schemes that exploit the multichannel capability mainly focus on mitigating packet collisions for improving system performance. The packet collisions can be resolved by delaying access time as in a conventional backoff access scheme (i.e., the time-domain backoff scheme) [6, 32, 74], or by accessing different channels in the manner of a hopping access scheme (i.e., the frequency-domain backoff scheme) [5, 78–80]. Although these access schemes provide efficient collision resolution under multichannel environments, they have not fully exploited the channel outage. System throughput can be improved by transmitting packets through relatively favourable channels with low outage probability.

In this chapter, we consider Aloha-type access schemes for uplink communications. First, we analyse system throughput of the Aloha-type access schemes based on multichannel outage models. The throughput analysis is facilitated with a Markov model for the access scheme procedures. Using the results from the analysis, we then propose (i) a frequency-domain backoff scheme with an outage-aware channel set that selects a favourable channel in a distributed manner, and (ii) a time-domain backoff scheme with a lowest-outage increasing heuristic that allocates a channel in a centralised manner. In

---

\(^1\)Current wireless standards support multiple non-overlapping channels for wireless communication; IEEE 802.11a [29] and IEEE 802.15.3 [77] provide 12 and 13 non-overlapping channels, respectively.
addition, we study throughput performance by applying game-theoretic approaches that take optimal outage-aware access behaviours of selfish users into account.

### 3.2 System Model and Aloha-type Access Schemes

We consider an uplink wireless system that consists of a base station and $K$ wireless users. The base station can communicate with the users through $N$ wireless channels simultaneously. Figure 3.1 shows the relation between the base station, the users, and the channels for uplink communications.

The network operating environment is one of random access for the users. Aloha-type access is used because of its channel efficiency [5,72,81,82]. CSMA/CA-type access [79, 83] is not used because it is less efficient because of the additional time needed for carrier sensing.

#### 3.2.1 Slotted Aloha scheme

The slotted Aloha scheme has been widely used for random access because of its simple implementation.

In the slotted Aloha scheme, each user transmits a packet at a discrete slot time if its selected probability is less than a fixed transmission probability [76]. Otherwise, the user waits until the next slot time comes. This fixed probability approach incurs more collisions when the number of contentions increases.
3.2.2 Persistence slotted Aloha scheme

The persistence slotted Aloha scheme has been implemented to resolve collisions effectively by adjusting its transmission probability.

The transmission probability is adjusted by selecting the minimum value between the maximum transmission probability and the updated transmission probability. This probability adjustment is done by an update function, i.e.,

$$\min\{p_{\text{max}}, p_{ij}(t)I_{z=0} + p_{\text{max}}I_{z=1} + p_{ij}(t)\gamma I_{z=2}\},$$  \hspace{1cm} (3.1)

where $p_{\text{max}}$ is the maximum transmission probability, $p_{ij}(t)$ is the transmission probability that user $i \in \{1, ..., K\}$ accesses channel $j \in \{1, ..., N\}$ at slot $t$, and $\gamma$ is the reducing factor between 0 and 1, i.e., $0 < \gamma < 1$. The indication function $I_z$ is 1 when the event $z$ occurs, where $z$ is 0 when no transmission has taken place, $z$ is 1 for a successful transmission, and $z$ is 2 when there is a collision.

The number of retransmissions is limited to the maximum retransmission number $m$. When the persistence slotted Aloha scheme reaches its maximum retransmission number, it does not update the transmission probability and uses the current transmission probability.

3.3 Throughput Analysis

This section derives the throughput expressions of the slotted Aloha scheme and the persistence slotted Aloha scheme.

3.3.1 Throughput of the slotted Aloha scheme

System throughput is defined as the average aggregated number of packets delivered from all users through multiple channels at each slot.

Let $p_{n,i}$ denote the transmission probability that user $i \in \{1, ..., K\}$ accesses channel $n \in \{1, ..., N\}$, and $q_{n,i}$ denote the channel outage probability of user $i$ over channel $n$.

The system throughput $S$ is

$$S = \sum_{n \in \{1, ..., N\}} \sum_{i \in \mathcal{M}_n} (1 - q_{n,i})p_{n,i} \prod_{j \in \mathcal{M}_n \setminus \{i\}} (1 - (1 - q_{n,j})p_{n,j}),$$ \hspace{1cm} (3.2)
where $\mathcal{M}_n$ is the set of users who access channel $n$. The term $(1 - q_{n,i})p_{n,i}$ is the transmission probability of user $i$ without being affected by the channel outage. The term $\prod_{j \in \mathcal{M}_n \setminus \{i\}}(1 - (1 - q_{n,j})p_{n,j})$ is the probability that other users have no transmission over channel $n$.

We assume that all $K$ users are identical and access a channel fairly over $N$ channels (i.e., $p_{n,i} = p$ and $q_{n,i} = q$ for all users and channels). With the stationary probabilities, $p$ and $q$, the system throughput is written as

$$S = N {K \choose 1} (1 - q) \frac{p}{N} \left(1 - (1 - q) \frac{p}{N}\right)^{K-1} \approx K(1 - q)p e^{-(1-q)\frac{G}{N}(K-1)}. \quad (3.3)$$

If the number of users is large, the system throughput becomes

$$S \approx (1 - q)G e^{-(1-q)\frac{G}{N}}, \quad (3.4)$$

where $G$ is the total offered channel traffic, i.e., $G = Kp$.

Note that the throughput of the slotted Aloha scheme over a single channel is $Ge^{-G}$ [76] and that of the slotted Aloha scheme over $N$ multiple channels is $Ge^{-G}$ [5]. Thus, we can see that the channel outage reduces the offered traffic from $G$ to $(1 - q)G$. This indicates that a higher offered traffic is required to maximize the throughput when the channel outage exists.

### 3.3.2 Throughput of the persistence slotted Aloha scheme

The system throughput of the persistence slotted Aloha scheme is analysed using a stationary transmission probability variable $\tau$. Applying $\tau$ to (3.3), the system throughput is

$$S = K(1 - q)\tau(1 - (1 - q)\frac{\tau}{N})^{K-1}. \quad (3.5)$$

The stationary transmission probability $\tau$ is defined as

$$\tau = E\{T_i\}, \quad (3.6)$$

where $T_i$ is the transmission probability at the state of the $i$th backoff access, and $m$ is the maximum backoff number. It is an average transmission probability of all states.
In the subsections that follow, an analytic expression for the stationary transmission probability will be derived. We first present a Markov model to capture the state transition behaviour of the transmission update procedure. Then we derive the equation for the stationary transmission probability based on the Markov model. The derived equation is finally calculated with the conditional collision probability for obtaining the system throughput.

The Markov chain model

The transmission update procedure can be expressed as a stochastic process, \( \{b(t)\} \), where \( b(t) \in \{0, 1, ..., m\} \) is the backoff stage process of a user at slot time \( t \).

We model the process \( \{b(t)\} \) as a discrete-time Markov chain which is shown in Figure 3.2. We use \( p_c \) to denote the conditional collision probability that a transmitted packet experiences a collision. \( p_c \) is taken as a constant and independent value.

Let \( P_{i,j} = \Pr(b(t + 1) = j|b(t) = i) \) denote the one-step transition probability from the state \( i \) at slot time \( t \) to the state \( j \) at the next slot time. The transition probabilities in the Markov chain are

\[
\begin{align*}
P_{i,i+1} &= p_c, \quad i \in (0, m); \\
P_{i,0} &= 1 - p_c, \quad i \in (0, m); \\
P_{i,j} &= 0, \quad i \in (0, m), j \in (1, m), j \neq i + 1.
\end{align*}
\]

The first equation in (3.7) accounts for the fact that the backoff stage increases by 1 when an unsuccessful transmission occurs at stage \( i \). The second equation represents that a new transmission starts at stage 0 when a successful transmission occurs at any stage. The
third equation means that the backoff stage cannot increases by more than 1.

The stationary transmission probability

We now derive a closed-form solution of the stationary transmission probability by using a stationary probability of staying at each state and a transmission probability at each backoff stage.

Let $\pi_i$ the stationary probability of staying at state $i$ in the chain. When applying $\pi_i = \sum_{k=0}^{m} \pi_k P_{k,i}$ and (3.7) based on the chain rule, the stationary state probability $\pi_i$ is given by

$$
\pi_i = \begin{cases} 
(1 - p_c)\pi_k, & i = 0; \\
p_c\pi_{i-1}, & 0 < i < m; \\
p_c\pi_{i-1} + p_c\pi_i, & i = m.
\end{cases} \tag{3.8}
$$

Since we have $\sum_{k=0}^{m} \pi_k = 1$, the initial stationary state probability $\pi_0$ is

$$
\pi_0 = p_0,0 \pi_0 + P_{1,0} \pi_1 + \cdots + P_{m,0} \pi_m = (1 - p_c) \sum_{k=0}^{m} \pi_k = 1 - p_c. \tag{3.9}
$$

The stationary state probability is then rewritten as

$$
\pi_i = \begin{cases} 
(1 - p_c), & i = 0; \\
p^i_c \pi_0, & 0 < i < m; \\
p^m_c \frac{1}{1 - p_c} \pi_0, & i = m.
\end{cases} \tag{3.10}
$$

The stationary transmission probability can be expressed as

$$
\tau = \sum_{k=0}^{m} T_i \pi_i, \tag{3.11}
$$

where the transmission probability $T_i$ in backoff stage $k \in (0, 1, \ldots, m)$ takes the form $p^{max} \beta^k$. Taking into account $T_i$ and $\pi_i$, the stationary transmission probability becomes

$$
\tau = \sum_{k=0}^{m-1} p^{max} \beta^k p_c \pi_0 + p^{max} \beta^m \frac{p^m_c}{1 - p_c} \pi_0 = p^{max} \pi_0 \left( \frac{1 - (\beta p_c)^m}{1 - \beta p_c} + \frac{(\beta p_c)^m}{1 - p_c} \right). \tag{3.12}
$$
The conditional collision probability

Since $\tau$ depends on the conditional collision probability $p_c$ which is still unknown, we need an additional equation to find the value of $\tau$.

In a single channel communication, the conditional collision probability of $K$ users is

$$p_c = 1 - (1 - \tau)^{K-1}. \quad (3.13)$$

In the multichannel communication, the collisions may occur differently, depending on the number of transmitting users over multiple channels. Let $Pr(x|y)$ denote the probability that only $x$ users transmit among $y$ users. With the stationary transmission probability $\tau$,

$$Pr(x|y) = \binom{y}{x} \tau^x (1 - \tau)^{y-x}. \quad (3.14)$$

Assuming that a user transmits over a channel among $N$ channels, the probability of selecting a different channel to the transmitting user is $\frac{N-1}{N} = 1 - \frac{1}{N}$. When $x$ users access a channel among $N - 1$ channels, excluding the channel used by the transmitting user, there is no collision with the probability

$$\left(1 - \frac{1}{N}\right)^x Pr(x|K - 1). \quad (3.15)$$

Taking different numbers of transmitting users into account, the conditional collision probability becomes

$$p_c = 1 - \sum_{x=0}^{K-1} \left(1 - \frac{1}{N}\right)^x Pr(x|K - 1)$$

$$= 1 - \sum_{x=0}^{K-1} \left(1 - \frac{1}{N}\right)^x \binom{K-1}{x} \tau^x (1 - \tau)^{K-1-x}. \quad (3.16)$$

Since the channel outage probability affects the transmissions of users, the conditional collision probability needs to be changed as below

$$p_c = 1 - \sum_{x=0}^{K-1} \left(1 - \frac{1}{N}\right)^x \binom{K-1}{x} ((1 - q)\tau)^x$$

$$\times (1 - (1 - q)\tau)^{K-1-x}. \quad (3.17)$$

The numerical value of $\tau$ can be obtained by solving (3.12) with (3.17). Note that (3.12) and (3.17) are a monotonically decreasing function and a monotonically increasing function of $p_c$, respectively. Thus, there exists an unique solution of $\tau$ which can be calculated numerically.
3.3.3 Throughput results

Figure 3.3 shows the throughput curves of the slotted Aloha scheme and the persistence slotted Aloha scheme when operating over a single channel without any channel outage. In Figure 3.3(a), the throughput of the slotted Aloha scheme increases rapidly at small user numbers and decreases greatly at large user numbers. The slotted Aloha scheme has such significant throughput changes because it uses a fixed transmission probability. On the other hand, the throughput of the persistence slotted Aloha scheme gradually increases with user numbers as shown in Figure 3.3(b). Since the persistence slotted Aloha scheme uses the update algorithm to adjust the transmission probability, it can effectively manage collisions at large user numbers, improving throughput.

The effects of multiple channels on throughput are shown in Figure 3.4. Both the slotted Aloha scheme and the persistence slotted Aloha scheme improve their throughput proportionally with channel numbers. This is because users are distributed over more channels, causing fewer collisions. The throughput trends of each scheme are also similar with channel numbers as shown in Figure 3.4(a) and Figure 3.4(b).

The effects of channel outage are shown in Figure 3.5 when $N = 2$ and each channel has the same outage probability. The slotted Aloha scheme improves throughput at large user numbers when the outage probability increases as shown in Figure 3.5(a). Since the high outage probability eliminates more transmitted packets, this causes fewer packet collisions and achieve higher throughput. On the other hand, the persistence slotted Aloha scheme has throughput degradation with the outage probability as shown in Figure 3.5(b). This is because the transmission failure due to the channel outage causes a longer delay in the update algorithm.
Figure 3.3: Throughput curves with an ideal single channel.
Figure 3.4: Throughput curves with ideal multiple channels.
Figure 3.5: Throughput curves with various channel outage probabilities when $N = 2$. 
3.4 The Proposed Outage-aware Access Schemes

This section presents multichannel outage-aware access schemes with frequency-domain backoff and with time-domain backoff.

3.4.1 Outage-aware frequency-domain backoff

The outage-aware frequency-domain backoff (OFB) access schemes can use outage-aware refined channel sets for random access. In order to resolve transmission failure due to collisions or outage effect, the OFB access schemes adopt the fast retrial algorithm using the slotted Aloha scheme and the persistence slotted Aloha scheme.

The outage-aware refined channel sets

Conventionally, existing multichannel access schemes use a random channel selection among all channels and take into account various channel outage probabilities for the channel selection [5, 82]. In the proposed channel selection procedure, we consider a refined channel set for random selection to increase throughput.

Let \( C_i \) denote the order statistic set of channels of user \( i \in \{1, ..., K\} \) in terms of the increasing channel outage probability. Let \( o_{i(t)} \) denote the channel number of user \( i \) of the \( t \)th increasing order of a channel outage probability. Thus, \( C_i \) can be written as \( C_i = \{o_{i(1)}, o_{i(2)}, ..., o_{i(N)}\} \). Instead of random channel selection among all \( N \) channels, a base station selects a refined set with \( h \) lowest-outage channels for user \( i \), denoted by \( C_i(h) \). Then, user \( i \) selects a random channel in \( C_i(h) = \{o_{i(1)}, o_{i(2)}, ..., o_{i(h)}\} \).

Figure 3.6 shows the refined channel sets of \( K \) users for random channel selection when \( N = 10 \) and \( h = 3 \). For example, user 1 has \( C_1 = \{9, 4, 6, 1, 5, 10, 2, 7, 3, 8\} \) and \( |C_1| = N = 10 \). When \( h = 3 \), the refined channel set becomes \( C_1(h = 3) = \{9, 4, 6\} \).

OFB with the slotted Aloha scheme

An outage-aware frequency-domain backoff scheme is proposed by incorporating the outage-aware refined channel sets and the fast retrial algorithm.

The proposed scheme resolves contentions by randomly selecting a channel in the outage-aware refined channel set. It repeats the random channel selection until the number
Figure 3.6: The refined channel sets of $K$ users for random channel selection when $N = 10$ and $h = 3$.

of retrials reaches its maximum number. After then, it uses the slotted Aloha scheme in the 1-persistent manner to resolve contentions for a normal load condition. This access procedure is similar to the fast retrial algorithm.

**OFB with the persistent slotted Aloha scheme**

Another outage-aware frequency-domain backoff scheme follows the same procedure of the fast retrial with the outage-aware refined channel sets for random access, but it uses the persistent slotted Aloha scheme. Since the persistent slotted Aloha scheme dynamically adjust users’ transmission probabilities, it may have better performance for a heavy load condition.

The procedures of the proposed access schemes are illustrated in Figure 3.7 and Figure 3.8, respectively.

### 3.4.2 Outage-aware time-domain backoff

The outage-aware time-domain backoff (OTB) access schemes use a centralised channel allocation scheme with the slotted Aloha scheme and the persistence slotted Aloha scheme.
The outage-aware fast retrial with the slotted Aloha scheme.

![Figure 3.7: The outage-aware fast retrial with the slotted Aloha scheme.](image)

**The outage-aware heuristic algorithm for channel allocation**

The conventional time-domain backoff access schemes consider that users are allocated to specific channels for packet transmission. Unlike the frequency-domain backoff access schemes, users keep the allocated channels. Thus, aggregated channel throughput strongly depends on sets of users who access distinct channels. Under multichannel outage environments, the sets of users also affect the channel throughput because users may experience different channel outage probabilities. Hence, to find optimal sets of users for channel access becomes crucial in order to maximise the channel throughput. This problem can be interpreted as a multiple knapsack problem [84] or a binpacking problem with a fixed size of bins [85]. Since these problems are NP-hard (i.e., the optimal channel allocation needs exhaustive searching to maximise all the channel throughput), efficient heuristic algorithms are needed. Appendix A summarises the knapsack problems.

We propose a lowest-outage increasing heuristic algorithm to find optimal sets of users with respect to the increasing order of the outage probabilities. This heuristic algorithm cannot guarantee the maximum aggregated channel throughput, but it can be implemented...
Figure 3.8: The outage-aware fast retrial with the persistence slotted Aloha scheme.

with reduced complexity obtaining the acceptable aggregated channel throughput. The proposed heuristic algorithm solves the following channel allocation problem,

\[
\begin{align*}
\text{maximise} & \quad \sum_{n \in \{1, \ldots, N\}} \sum_{i \in \{1, \ldots, K\}} (1 - q_{n,i}) x_{n,i} \\
\text{subject to} & \quad \sum_{i} x_{n,i} \leq \left\lceil \frac{K}{N} \right\rceil, \quad n \in \{1, \ldots, N\}, \\
& \quad \sum_{n} x_{n,i} = 1, \quad i \in \{1, \ldots, K\}, \\
& \quad x_{n,i} = 0 \text{ or } 1, \quad i \in \{1, \ldots, K\}, n \in \{1, \ldots, N\}
\end{align*}
\]

(3.18)

where \( \left\lceil a \right\rceil \) denotes the smallest integer number greater than \( a \), and \( x_{n,i} \) denotes the indication function as

\[
x_{n,i} = \begin{cases} 
1 & \text{if user } i \text{ is assigned to channel } n; \\
0 & \text{otherwise.}
\end{cases}
\]

(3.19)

We now describe the lowest-outage increasing heuristic algorithm as follows: First, by using the channel outage matrix, \( Q \), the sequence of users for the channel allocation procedure is obtained in a increasing order of the channel outage probabilities. Then, following the sequence, each user is allocated to the channel which gives the lowest channel
outage probability. If the number of users in a channel is greater than the ideal number of users (i.e., \(|M_n| > \lceil K N \rceil\)), the users who have lower sequential priorities in the channel are redistributed to a channel that gives the next lowest channel outage probability. The procedures are repeated until all users are allocated and the ideal number of users in each channel is satisfied. The time complexity of the heuristic is \(O(KN) + O(K)\).

As an illustration of the proposed heuristic, consider a network with 5 users and 3 channels, i.e., \(K = 5\) and \(N = 3\). The channel outage probabilities of each user over the channels are taken as

\[
Q = \begin{pmatrix}
0.3 & 0.7 & 0.2 \\
0.4 & 0.1 & 0.2 \\
0.7 & 0.4 & 0.3 \\
0.3 & 0.1 & 0.2 \\
0.5 & 0.2 & 0.4
\end{pmatrix}.
\]  

(3.20)

The sequence of users for channel allocation is \(\{2, 4, 1, 5, 3\}\) in an increasing order of channel outage probabilities from \(Q\). The lowest-outage increasing heuristic first allocates all the users based on the sequence as \(M_1 = \{0\}\), \(M_2 = \{2, 4, 5\}\), and \(M_3 = \{1, 3\}\). Then, by using the redistribution of user 5 because the number of users in channel 2 is greater than the ideal number of users, \(\lceil K N \rceil = 2\), the channel allocation is obtained as \(M_1 = \{5\}\), \(M_2 = \{2, 4\}\), and \(M_3 = \{1, 3\}\).

**OTB with the Aloha-type access**

We apply the outage-aware heuristic algorithm for channel allocation to the Aloha-type access schemes: The slotted Aloha scheme and the persistence slotted Aloha scheme. Unlike the OFB schemes, the OTB schemes have no frequency hopping in order to resolve collision. When users are allocated to specific channels, each user keeps its own channel to access a base station. Thus, these schemes needs sophisticated channel allocation operated by a based station as a centralised procedure.
3.5 Persistence Slotted Aloha Game

In this section, we study game-theoretic access approaches that take the channel outage into account. From the perspective of reverse-engineering, a utility function is formulated and three game-theoretic update strategies are discussed [86].

3.5.1 A noncooperative game model

The persistence slotted Aloha scheme is modeled as a noncooperative game when users access a channel. The Aloha game is

$$G_n = [M_n, A_n, \{U_i\}_{i \in M_n}], \quad (3.21)$$

where $M_n$ is the set of players who access channel $n \in \{1, \ldots, N\}$; $A_n$ is the set of all action profiles in channel $n$; $U_i$ is the utility function of user $i \in M_n$.

Note that the transmission probability $p_{n,i}$ of user $i \in M_n$ is an action in $A_{n,i} = \{p_{n,i} | p_{n,i}^{\min} \leq p_{n,i} \leq p_{n,i}^{\max}\}$. The action profile set is $A_n = \times_{i \in M_n} A_{n,i}$.

The utility function

We incorporate the channel outage into the utility function obtained in [86]. The outage-aware utility function is

$$U_i(p_n) = \frac{1}{2}p_{n,i}^{\max}(1 - q_{n,i})p_{n,i}^2 \prod_{j \in M_n \setminus \{i\}} \left(1 - (1 - q_{n,j})p_{n,j}\right)$$
$$+ \frac{1}{3} \beta_{n,i}(1 - q_{n,i})p_{n,i}^3 \left(1 - \prod_{j \in M_n \setminus \{i\}} \left(1 - (1 - q_{n,j})p_{n,j}\right)\right)$$
$$- \frac{1}{3}(1 - q_{n,i})p_{n,i}^3, \quad (3.22)$$

where $q_{n,i}$ is the channel outage probability of user $i \in M_n$, $p_{n,i}$ is the transmission probability, and $p_n$ is the transmission probability set of all users.

Nash equilibrium

The existence of Nash equilibrium was proved by showing the utility function $U_i$ of user $i$ is continuous and quasi-concave on the user $i$’s action set which is a nonempty compact
convex subset of Euclidean space \[86\]. Taking into consideration the channel outage, the
Nash equilibrium is given by
\[
p_{n,i}^* = \frac{p_{n,i}^{max} \prod_{j \in M_n \setminus \{i\}} \left(1 - (1 - q_{n,j})p_{n,j}^*(t)\right)}{1 - \beta_{n,i} \left(1 - \prod_{j \in M_n \setminus \{i\}} \left(1 - (1 - q_{n,j})p_{n,j}^*(t)\right)\right)}.
\] (3.23)

### 3.5.2 Game-theoretic update strategies

Based on the reverse-engineering results in \[86\], we consider the following three update

In the gradient update, each user updates its transmission probability to the direction of
maximising its utility using the gradient. The expected transmission probability becomes
\[
\hat{p}_{n,i}(t) = p_{n,i}(t) + \left. \frac{\partial U_i(p_n)}{\partial p_{n,i}} \right|_{p_n=p_n(t)}.
\]

The gradient update algorithm is
\[
p_{n,i}(t+1) = \min\{p_{n,i}^{max}, \hat{p}_{n,i}(t)\}.
\] (3.24)

The stochastic subgradient update approximates the next transmission probability
based on the current access results to maximise its utility, while the gradient update uses
the exact gradient of its utility function. Note that, from Theorem 2 in \[86\], the stochastic
subgradient update is equivalent to the update of the persistence slotted Aloha scheme.

By slightly abusing the notation, the stochastic subgradient update is rewritten as
\[
p_{n,i}(t + 1) = \min\{p_{n,i}^{max}, p_{n,i}(t)I_{z=0} + p_{n,i}^{max}I_{z=1} + p_{n,i}(t)\beta I_{z=2}\}.
\] (3.25)

For the best response update algorithm, let \(p_{n,-i}\) denote the fixed transmission prob-
babilities of the other users except user \(i\) in channel \(n\), and \(B_{n,i}(p_{n,-i})\) denote the best
response function of user \(i\). The best response function is defined as \(B_{n,i}(p_{n,-i}) = \arg\max U_i(p_{n,i}, p_{n,-i})\) in which each user \(i\) maximise his utility function against the
other users’ persistence probabilities in channel \(n\). From the Nash equilibrium, the best
response update is given by
\[
p_{n,i}(t + 1) = B_{n,i}(p_{n,-i}(t))
= \frac{p_{n,i}^{max} \prod_{j \in M_n \setminus \{i\}} \left(1 - (1 - q_{n,j})p_{n,j}(t)\right)}{1 - \beta_{n,i} \left(1 - \prod_{j \in M_n \setminus \{i\}} \left(1 - (1 - q_{n,j})p_{n,j}(t)\right)\right)}.
\] (3.26)
Table 3.1: System parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network model</td>
<td></td>
</tr>
<tr>
<td>Channel number ($N$)</td>
<td>2 to 10</td>
</tr>
<tr>
<td>User number ($K$)</td>
<td>10 to 200</td>
</tr>
<tr>
<td>Access model</td>
<td></td>
</tr>
<tr>
<td>Maximum transmission probability ($p^{max}$)</td>
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</tr>
<tr>
<td>Transmission reducing factor ($\beta$)</td>
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</tr>
<tr>
<td>Maximum backoff number ($m$)</td>
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</tr>
<tr>
<td>Channel model</td>
<td></td>
</tr>
<tr>
<td>Outage probability ($q$)</td>
<td>0.0 to 0.6</td>
</tr>
</tbody>
</table>

3.5.3 The convergence of the update strategies

Even though a Nash equilibrium exists, it does not guarantee uniqueness of the Nash equilibrium. From Theorem 4 in [86], the uniqueness and convergence of the Nash equilibrium was proved for the best response update. The proof uses a contraction mapping theorem in which if there exists a contraction mapping function, it has a unique fixed point and its sequence converges to the unique fixed point. In other words, if $\|J\|_{\infty} < 1$, where $J$ is the Jocobian matrix of the best response function, the best response function is a contraction mapping and both uniqueness and convergence are proved.

With the channel outage, the uniqueness and convergence condition of the Nash equilibrium (3.26) can be obtained as follows

$$\|J\|_{\infty} \leq \frac{p^{max}(1-q)K}{4\beta(1-(1-q)p^{max})N} < 1.$$  \hspace{1cm} (3.27)

3.6 Simulation Results

This section presents throughput performance of the multichannel outage-aware access schemes and the game-theoretic update algorithms, comparing with the conventional Aloha access schemes.
3.6.1 Parameters

Table 3.1 presents the system parameters for the network, access, and channel models discussed in Section 3.2.

The slotted Aloha scheme uses the transmission probability $p^{max}$. As a backoff procedure, the persistence slotted Aloha scheme uses the maximum transmission probability $p^{max}$, the reducing factor $\beta$, and the maximum backoff number $m$.

Each simulation result is an average of 50 runs in which each run has 5000 time slots.

3.6.2 Throughput comparisons of the outage-aware schemes

Figure 3.9 shows throughput curves of the proposed outage-aware access schemes when $N = 10$ and $q = 0.4$.

In Figure 3.9(a), the throughput performance of the time-domain backoff schemes is presented. The dashed lines indicate the results of the conventional access schemes, i.e., the slotted Aloha scheme (SA) and the persistence slotted Aloha scheme (PSA). These access schemes use a random channel allocation. Since the slotted Aloha scheme uses the fixed transmission probability, it outperforms the persistence slotted Aloha scheme at small user numbers. The outage-aware time-domain backoff access schemes (i.e., OTB SA and OTB PSA) improve throughput significantly. The OTB with PSA obtains better performance than the conventional PSA at all user numbers. The OTB with SA achieves the highest throughput until the user number becomes around 70. However, the throughput of the OTB with SA dramatically decreases when the user number becomes large because of the fixed transmission probability, although the outage-aware channel allocation is used.

Figure 3.9(b) shows the throughput curves of the frequency-domain backoff access schemes. The outage-aware frequency-domain backoff schemes (i.e., OFB SA and OFB PSA) achieve higher throughput than the fast retrial schemes (i.e., FR SA and FR PSA) at small user numbers. This is because the outage-aware refined sets provide more successful transmissions, reducing the channel outage effect. However, when the number of users increase, the OFB SA degrades throughput because more collisions occur with the outage-aware refined channel sets.
Figure 3.9: Throughput curves of the proposed outage-aware access schemes compared with the conventional access schemes when $N = 10$ and $q = 0.4$. 

(a) Time-domain backoff access schemes.

(b) Frequency-domain backoff access schemes.
Figure 3.10 shows the throughput curves of the proposed outage-aware access schemes with different channel numbers. When the number of channels is small (e.g., $N = 4$) as shown in Figure 3.10(a), the OTB SA outperforms the others at small user numbers because the centralised channel allocation improves throughput, but it has significant throughput degradation when the user number increases in which the outage-aware channel allocation causes more collision. The OFB PSA dominates the others at large user numbers with the outage-aware frequency hopping. When the number of channel becomes large (e.g., $N = 10$) as shown in Figure 3.10(b), the OFB PSA achieves the best throughput at small user numbers. This is because the frequency-domain backoff scheme has more advantages with more channels and less users. On the other hand, at large user numbers, the OFB PSA becomes less efficient than the OTB PSA in which hopping channels causes more transmission failure with more users.

3.6.3 Throughput comparisons of the game-theoretic updates

The gradient update and the best response update are compared in Figure 3.11 for their convergence and throughput effects. The stochastic subgradient update is not discussed because it is the basic update of the persistence slotted Aloha scheme.

Figure 3.11(a) shows the converge results of the gradient update and the best response update with respect to their persistence probabilities. The gradient update always converges to a certain value because its transmission probability is equivalent to an expected update probability. On the other hand, the best response update depends on the convergence condition in (3.27), and may not converge if there are more users, fewer channels, or lower outage probability.

Figure 3.11(b) shows the throughput curves of both the updates with respect to different user numbers. When the number of users exceeds 80, the best response update does not converge because its transmission probability fluctuates significantly. Thus, the throughput of the best response update significantly decreases. On the other hand, the gradient update has gradual throughput degradation because of its convergence. From this result, we observe that the convergence of the updates significantly affects throughput performance.

Figure 3.12 shows throughput comparisons of the game-theoretic updates with the
Figure 3.10: Throughput curves of the proposed outage-aware access schemes when $N \in \{4, 10\}$ and $q = 0.4$. 
Figure 3.11: The persistence probabilities and throughput curves of the gradient update and the best response update when $p^{\text{max}} = 0.25$, $\beta = 0.5$, $q = 0.2$, and $N = 6$ (the solid line and the dotted line indicate the results of the gradient update and the best response update, respectively).
Figure 3.12: Throughput curves of the slotted Aloha scheme, the persistence(P)-slotted Aloha scheme, the gradient update, and the best response update according to different user numbers with two channel outage probabilities when $p_{\text{max}} = 0.25$, $\beta = 0.5$, and $N = 6$. 
slotted Aloha scheme and the persistence slotted Aloha scheme with two channel outage probabilities when $p_{\text{max}} = 0.25$, $\beta = 0.5$, and $N = 6$. Note that the persistence slotted Aloha scheme is the stochastic subgradient update. The gradient update improves the throughput in which the conventional Aloha schemes cannot obtain it at certain user numbers. Similarly, the best response has the same effect if it converges. This is because it can update the transmission probability to the direction of maximising its utility. The slotted Aloha scheme still achieves higher throughput than the others at small user numbers, while the persistence slotted Aloha scheme outperforms the others at large user numbers. The trends of throughput curves are similar with different outage probabilities as shown in Figure 3.12(a) and Figure 3.12(b). The gradient update significantly improves the optimal range of user numbers in which it outperforms other update schemes when the channel outage probability increases. Table 3.2 presents optimal ranges of user numbers for the schemes with various channel numbers and channel outage probabilities when $p_{\text{max}} = 0.25$ and $\beta = 0.5$. These table results can be used to select an optimal scheme depending on the network environments.

### 3.7 Conclusion

The multichannel outage-aware access schemes have been proposed with the frequency-domain backoff access and the time-domain backoff access for wireless networks. The frequency-domain backoff schemes mitigate the channel outage effect by using the refined outage-aware channel sets for random access. The time-domain backoff access schemes also reduce the channel outage effect with the lowest-outage increasing heuristic algorithm for channel allocation. These outage-aware schemes achieve higher system throughput than the conventional Aloha access schemes because they provide more successful transmissions under the channel outage environments. In addition, the game-theoretic update algorithms have been incorporated with the channel outage. The use of the game-theoretic algorithms enables the transmission probability update to become more efficient because they maximise the outage-aware utility function. The numerical and simulation results showed that the outage-aware access schemes and the game-theoretic algorithms outperform the conventional Aloha schemes, and improve system throughput.
Table 3.2: Optimal schemes according to the user numbers when $p = p^{\text{max}} = 0.25$ and $\beta = 0.5$

<table>
<thead>
<tr>
<th>Channels ($N$)</th>
<th>Outage ($q$)</th>
<th>Slotted Aloha</th>
<th>Gradient Update</th>
<th>P-slotted Aloha</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>$-\leq 0$</td>
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Chapter 4

Multichannel Sensing and Access for Wireless Networks

A new backoff access policy is proposed to operate with a myopic sensing policy. It takes into account imperfect information of channel conditions and user transmission behaviours for multichannel wireless networks. The sensing and access problem is formulated as a Partially Observable Markov Decision Process (POMDP) and is solved with simple and efficient heuristic approaches. Simulation results show that system throughput is improved when the proposed sensing and access policies are used jointly.
4.1 Introduction

Recent wireless communications have been developed to use multiple channels simultaneously to improve system throughput by applying advanced access techniques such as OFDMA.

In OFDMA wireless networks, efficient sensing and access policies are required to improve system throughput by selecting a channel or a subset of channels to sense, and by adjusting transmissions to access. However, developing the sensing and access policies that maximise the system throughput is not trivial because users may not sense all channel conditions to transmit and cannot observe all other users’ transmission behaviours fully due to hardware and energy constraints.

Existing sensing and access policies mainly focus on their own objectives individually: For the sensing policies, myopic sensing approaches have been developed with a simple access policy for spectrum sensing [87–89]. Some access policies have been used without applying sensing policies for packet transmissions [5]. Although some existing schemes that take into account both sensing and access policies have been proposed [72, 73, 87], they are not designed to fully improve system throughput under the multichannel environments with partial information of networks.

In this chapter, the sensing and access problem is posed as a Partially Observable Markov Decision Process (POMDP) problem because of the imperfect network information [87, 90]. Since finding optimal policies for the POMDP problem is often intractable, existing policies mainly focus on greedy, heuristic, or approximation approaches [88, 91]. In order to provide suboptimal solutions that improve system throughput, we follow a simple greedy approach for sensing and develop an efficient heuristic solution for access. The impacts of the joint sensing and access policies on system throughput are investigated under imperfect information of multichannel conditions and user transmission behaviours.
4.2 Problem Formulation

4.2.1 System model

A wireless network is used by $K$ users to perform up-link communication to the base station over $N$ wireless channels. The wireless users experience different channel conditions at each slot time of transmission. Before using a channel, the user will conduct the channel sensing procedure to assess the channel condition. Transmission will take place only if the channel is good. The access results such as successful transmission, idle, and collision, are assumed to be known to the user immediately after accessing the channel.

4.2.2 Network state, user action and reward function

The state of a network is expressed as

$$S(t) = \{C(t), X(t)\}, \quad (4.1)$$

where $C(t)$ is the channel condition state in slot $t$, and $X(t)$ is the access state in slot $t$. The channel condition state $C(t)$ is defined as $C(t) = c_{ij}(t))_{i \in \mathcal{K}, j \in \mathcal{N}}$, where $c_{ij}(t) \in \{0(\text{bad}), 1(\text{good})\}$ denotes the channel condition of user $i$ to channel $j$ in slot $t$, $\mathcal{K}$ is the set of users, and $\mathcal{N}$ is the set of channels. The access state $X(t)$ is defined as $X(t) = (x_{ij}(t))_{i \in \mathcal{K}, j \in \mathcal{N}}$, where $x_{ij}(t) \in \{-1(\text{collision}), 0(\text{idle}), 1(\text{success})\}$ denotes the access result of user $i$ to channel $j$ in slot $t$.

The user action is a set of channel sensing and access actions. The user action is given by

$$A(t) = \{H(t), P(t)\}, \quad (4.2)$$

where $H(t)$ is the sensing action and $P(t)$ is the access action. The sensing action is defined as $H(t) = (h_i(t))_{i \in \mathcal{K}}$, where user $i$ selects a channel $h_i(t) \in \{1, \ldots, N\}$. The

---

1The simple representation of the channel condition state has the following advantages: (i) easy implementation of sensing policies, and (ii) derivation of an optimal solution under a certain condition. Although the channel condition state can be represented by a sophisticated probability distribution function, it may be difficult to implement and address a solution of the user collision problem.
access action is defined as $P(t) = (p_{ij}(t))_{i \in K, j \in N}$, where user $i$ accesses to channel $j$ with a probability $p_{ij}(t) \in \{1, \frac{1}{2}, \ldots, \frac{1}{N}\}$.

Given the network state and the user action, the reward function can be defined as the total number of bits delivered from all users, i.e., the total throughput of the system. The reward function is generally expressed as

$$r(S(t), A(t)) = \sum_{i \in K} \sum_{j \in N} c_{ij}(t)p_{ij}(t) \prod_{k \in K, k \neq i} (1 - c_{kj}(t)p_{kj}(t))B_j,$$

(4.3)

where $B_j$ is a bandwidth of channel $j$.

The objective is to find an optimal channel selection policy for sensing and transmission control. The optimal policy is derived by solving

$$\max_{\pi} \mathbb{E}\left\{ \sum_{t=0}^{T} r(S(t), A(t)) | S(0) \right\},$$

(4.4)

where $\pi$ is a policy that determines the user action $A(t)$ given the network state $S(t)$, with $S(0)$ being the initial network state.

### 4.2.3 POMDP formulation

Because the network state cannot be fully observed in a multichannel environment, further formulation to solve (4.4) is required. The Partially Observable Markov Decision Process (POMDP) is of interest because of its easy integration of the sensing and access problem under a decision-theoretic framework [87].

In POMDP, the internal state of the underlying Markov process is taken as unknown. In an attempt to summarise the knowledge of the internal state, a belief vector $\Lambda(t)$ is introduced. The belief state is defined as

$$\Lambda(t) = (\lambda_s(t))_{s \in S} = [\lambda_1(t), \ldots, \lambda_{|S|}(t)],$$

(4.5)

where $\lambda_s$ is a discrete probability of a network state $s \in S$, and $|S|$ denotes the size of all network states.

A policy $\pi$ for the POMDP is then given by a sequence of functions, each maps from the current belief vector $\Lambda(t)$ to the user action $A(t)$ to be taken in slot $t$. The optimisation problem in (4.4) is reformulated as

$$\max_{\pi} \mathbb{E}\left\{ \sum_{t=0}^{T} r(\Lambda(t), A(t)) | \Lambda(0) \right\},$$

(4.6)
where $\Lambda(0)$ is the initial belief vector. Solving (4.6) requires the belief vector to be updated recursively. This will be explained in the section to follow.

### 4.3 Multichannel Sensing and Access Policies

#### 4.3.1 Optimal sensing and access policies

The optimisation problem in (4.6) can be solved by Bellman’s optimality equation, which is written as

$$V_t(\Lambda(t)) = \max_{A(t) \in A} \left( r(\Lambda(t), A(t)) + \mathbb{E}\{V_{t+1}(\Lambda(t+1)|\Lambda(t))\} \right),$$

(4.7)

where $V_t(\Lambda(t))$ is the maximum expected remaining reward, given in the current belief vector $\Lambda(t)$. There are two parts in (4.7): (i) the immediate reward at slot $t$, that is, $r(\Lambda(t), A(t))$, and (ii) the maximum expected remaining reward starting from slot $t + 1$ with the updated belief vector $\Lambda(t+1)$, that is, $\mathbb{E}\{V_{t+1}(\Lambda(t+1)|\Lambda(t))\}$.

From Bayes’ rule, the updated belief state is

$$\lambda_s(t+1) = \frac{\Pr(s|\Lambda(t), A(t), \Theta(t))}{\Pr(\Theta(t)|\Lambda(t), A(t))} = \frac{\Pr(s|\Lambda(t), A(t)) \sum_{s' \in S} \Pr(s'|\Lambda(t), A(t)) \lambda_{s'}(t+1)}{\Pr(\Theta(t)|\Lambda(t), A(t))},$$

(4.8)

where $\Pr(\Theta(t)|\Lambda(t), A(t)) = \sum_{s \in S} \Pr(s|\Lambda(t), A(t)) \sum_{s' \in S} \Pr(s'|\Lambda(t), A(t)) \lambda_{s'}(t+1)$,

(4.9)

and $\Theta(t)$ is the observation state.

Since $V_t(\Lambda(t))$ is convex and piecewise linear [90], the optimisation problem can be solved using a linear programming algorithm when the state size is small. Unfortunately, most real world applications are high dimension in states and actions, computing the expectation and updating the value function are often intractable [91]. Finding an exact solution for a general POMDP is thus computationally prohibitive. In this chapter, the focus is on exploiting specific structure of the problem and developing suboptimal strategies that are computationally less complex.
4.3.2 Sensing policies

In this section, we consider three sensing policies: myopic sensing, ideal sensing, and random sensing.

Myopic sensing policy

Myopic sensing is a greedy approach. It selects a channel that provides the maximum expected reward, based on the channel condition estimations [87]. We will use myopic sensing to reduce the computational complexity in a sensing policy. The effects of the myopic sensing on the system throughput will be investigated with ideal and random sensing policies.

In order to find an optimal policy, the myopic sensing policy uses a sufficient statistic \( \Omega_i = (\omega_{ij})_{i \in N} \), which is the probability distribution of available channels of user \( i \) conditioned on the sensing and decision history. As shown in [87], the dimension of sufficient statistic grows linearly with \( N \).

Assuming that channels evolve independently, a simple Markov channel model as shown in Figure 4.1 is used to estimate channel conditions with known state transition probabilities. The probability that channel \( j \) will be available for user \( i \) is \( w_{ij}(t) \beta_{ij} + (1 - w_{ij}(t)) \alpha_{ij} \), where \( \alpha_{ij} \) is the probability for transiting from state 0 to state 1 at channel \( j \) for user \( i \), and \( \beta_{ij} \) is the probability for staying in state 1 from state 1 at channel \( j \) for user \( i \).

For the greedy approach, the optimal sensing action of user \( i \) is to maximise the ex-
pected channel condition, i.e.,

\[ h_{is}(t) = \arg \max_{j \in N} (w_{ij}(t) \beta_{ij} + (1 - w_{ij}(t)) \alpha_{ij}), \quad (4.10) \]

where \( h_{is}(t) \) is the optimal sensing action of user \( i \) and \( h_{is}(t) \in \{1, ..., N\} \).

Belief vector update based on the sensing action \( h_{is}(t) \) and the observation \( \Theta_{h_{is}}(t) \) is given by

\[ w_{ij}(t+1) = \begin{cases} 
1, & \text{if } h_{is}(t) = j, \Theta_{h_{is}}(t) = 1, \\
0, & \text{if } h_{is}(t) = j, \Theta_{h_{is}}(t) = 0, \\
w_{ij}(t) \beta_{ij} + (1 - w_{ij}(t)) \alpha_{ij}, & \text{if } h_{is}(t) \neq j. 
\end{cases} \quad (4.11) \]

**Ideal sensing policy**

If the exact channel conditions are known, the ideal sensing policy is to select a channel among channels that have good conditions. The optimal sensing policy of user \( i \) at slot \( t \) is \( h_{is}(t) \in \hat{N}_i \), where \( \hat{N}_i \) is the set of channels that satisfy \( c_{ij}(t) = 1, j \in \mathcal{N} \). Since we cannot obtain the channel conditions exactly, this approach is impractical.

**Random sensing policy**

The simplest sensing policy is the random sensing - it selects a channel randomly with no requirement to know the channel conditions. The random sensing policy of user \( i \) at \( t \) is \( h_i(t) \in \mathcal{N} \).

### 4.3.3 Access policies

We now consider three access policies: ideal access, random access and backoff access. When using an efficient access policy associated with a sensing policy, the system throughput will be improved by adjusting the transmission probability.

**Ideal access policy**

Let \( \mathcal{N}_j \) denote the set of users who access channel \( j \) with a good channel condition and \( N_j \) denote the size of the set \( \mathcal{N}_j \). Assuming that the base station knows the user number
of channel \( j \) and each user has the same transmission probability \( p_j \), the throughput of channel \( j \) is given by

\[
S_j = p_j(1 - p_j)^{N_j-1}.
\]  
(4.12)

The optimal transmission probability is simply obtained as \( p_j = \frac{1}{N_j} \) by applying the derivative of \( S_j \) as

\[
\frac{d}{dp_j} S_j = (1 - p_j)^{N_j-1} - p_j(N_j - 1)(1 - p_j)^{N_j-2} = 0.
\]

In a random access environment where users change their access channels frequently, this assumption is impractical. However, if the channel access becomes less frequent, then it would be feasible to estimate the number of users who access each channel.

**Random access policy**

A simple approach is a random selection among possible probabilities. Let \( \hat{p}_{ij}(t) \) denote the selected random transmission probability of user \( i \) to access channel \( j \) in slot \( t \). Since the selected channel may have different numbers of users ranging from 1 to \( N_j \), the optimal transmission probability can range from \( \frac{1}{N} \) to \( \frac{1}{N} \), i.e., the random transmission probability \( \hat{p}_{ij}(t) \) is selected within \( \{1, \frac{1}{2}, ..., \frac{1}{N}\} \).

As user \( i \) select a channel \( h_{ix}(t) \) from the myopic sensing strategy, the user sets \( p_{ij}(t) = 0 \) if \( h_{ix}(t) \neq j \). It is summarised as

\[
p_{ij}(t) = \begin{cases} 
0, & \text{if } h_{ix}(t) \neq j, \\
\hat{p}_{ij}(t), & \text{if } h_{ix}(t) = j.
\end{cases}
\]  
(4.13)

With the random transmission probability selected after the sensing policies, the system throughput in slot \( t \) can be rewritten as

\[
\sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{N}} c_{ij}(t)p_{ij}(t) \prod_{k \in \mathbb{K}, k \neq i} (1 - c_{kj}(t)p_{kj}(t))B_j 
= \sum_{j \in \mathbb{N}} \sum_{i \in \mathbb{N}_j(t)} \hat{p}_{ij}(t) \prod_{k \in \mathbb{N}_j(t), k \neq i} (1 - \hat{p}_{kj})B_j.
\]  
(4.14)

**Proposed backoff access policy**

Existing backoff policies usually operate in a network where the number of users is not frequently changed over a channel. The backoff policies use a default transmission probability and reduce its probability when collision occurs with a certain reducing factor \( 0 \leq \gamma \leq 1 \).
In the multichannel environment, each user experiences different channel conditions and the number of users who access a specific channel is frequently changed. Hence, the existing backoff policies are no longer suitable because users’ transmission behaviours are hard to predict when adjusting their transmission probabilities.

In this chapter, we extend the random access policy by applying the backoff concept. Each user selects a random transmission probability for accessing a channel. When the user experiences a collision, the probability is reduced with the factor. If the user transmits successfully or becomes idle, the probability increases by dividing it with the factor. The backoff access policy is

$$p_{ij}(t + 1) = \begin{cases} \hat{p}_{ij}(t)/\gamma, & \text{if } h_{is}(t) = j \text{ and success or idle,} \\ \hat{p}_{ij}(t)\gamma, & \text{if } h_{is}(t) = j \text{ and collision,} \\ 0, & \text{if } h_{is}(t) \neq j. \end{cases}$$

This extended backoff policy is useful for accessing multiple channels when the users have imperfect information of channel conditions and other users’ transmission behaviours. Note that the reducing factor can be adjusted effectively by using advanced backoff schemes. Chapter 5 will discuss a joint backoff control in the time and frequency domains to mitigate packet collisions.

### 4.4 Simulation Results

#### 4.4.1 Parameters for networks and policies

Table 4.1 presents the system parameters for the network, access, and channel models discussed in Section 4.2. These parameters we used follow the parameters in [87].

We consider three independent wireless channels. Their channel conditions depend on the Markov model with the transition probabilities, i.e., $\alpha$ and $\beta$.

The default access policy uses the fixed transmission probability $\hat{p}$. The backoff access policy uses the reducing factor $\gamma$ with $\hat{p}$.

Each simulation result is an average of 50 runs in which each run has 500 time slots.
Table 4.1: System parameters.

<table>
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<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
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<td></td>
</tr>
<tr>
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<tr>
<td>User number ((K))</td>
<td>1 to 10</td>
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<tr>
<td><strong>Access model</strong></td>
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<tr>
<td>Maximum transmission probability ((p^{max}))</td>
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</tr>
<tr>
<td>Transmission reducing factor ((\gamma))</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Channel model</strong></td>
<td></td>
</tr>
<tr>
<td>Transition probabilities ((\alpha, \beta))</td>
<td>(\alpha = 0.8, \beta = 0.2)</td>
</tr>
</tbody>
</table>

### 4.4.2 Simulation study

We present the effects of different sensing policies on the system throughput when the user number is small. Figure 4.2 shows the throughput curves of ideal, myopic, and random sensing policies when the number of channels is \(N = 3\) and the number of user is \(K = 2\). The three sensing policies operate with the default access policy that each user directly transmit a packet after selecting a channel (i.e., the fixed transmission probability \(p = 1\)). This result shows that the sensing policy with more accurate channel estimation achieves higher throughput because the selected channels have better conditions for transmissions.

When the number of users is large as shown in Figure 4.3, the sensing policy with better channel estimation achieves lower throughput. This unexpected result occurs because the advanced sensing policy may allow more users to access specific channels of good conditions resulting in more collisions. If the sensing policy is not accurate, some users may select bad channels occurring less transmissions resulting in less collisions.

In Figure 4.4, the throughput performance of different sensing policies are presented when the number of users vary and each user has the fixed transmission probability \(p = 1\). As mentioned in the previous result, with more users, the ideal sensing achieve the worst performance, while the random sensing policy achieves the best performance. This result implies that instead of using the fixed access, the advanced sensing policy needs a more efficient access policy in order to take advantage of using channels of good conditions. Note that other parameter setting such as in channel number or transmission probability...
Figure 4.2: Throughput performance of three sensing policies when $K = 2$, $N = 3$, and $p = 1$.

Figure 4.3: Throughput performance of different sensing policies when $K = 10$, $N = 3$, and $p = 1$. 
will lead to the similar trend of the throughput performance of the sensing policies with the fixed access policy.

The throughput curves of different access policies are shown in Figure 4.5 in which the myopic sensing policy is used. If the base station knows the number of users who access each channel at each slot time, the optimal access shows the best throughput performance. However, this assumption is impractical for real applications. As a simple and efficient policy, the backoff access achieves better throughput than the random access and the fixed access because it can avoid collisions by adjusting transmission probabilities. This result shows that in order to achieve better throughput, an efficient sensing policy must cooperate with the advanced access policy. The proposed backoff access is useful in real applications because it is implementable with a simple access structure, and it offers relatively good throughput performance.

### 4.5 Conclusion

We presented the myopic sensing and backoff access policies that perform efficiently with imperfect network information in an multichannel wireless network. The myopic
sensing policy provides an optimal sensing action by selecting a channel based on the expected channel conditions. The backoff access policy mitigates packet collisions by adjusting its transmission probability with a reducing factor. These sensing and access policies are jointly performed in order to improve the system throughput. Simulation results showed that when using only the advanced sensing policy the system throughput cannot be improved; when jointly applying the efficient access policy a better throughput can be achieved.
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Chapter 5

Joint Backoff in Time and Frequency for Multichannel Wireless Networks and its Markov Model for Analysis

A new joint random access scheme is proposed to enable effective uplink access when users are partially aware of channel conditions in multichannel wireless systems. The proposed scheme mitigates packet collisions with joint backoff control in the time and frequency domains, cooperating with a sensing method that exploits the channel conditions. The performance analysis of the joint access scheme is facilitated by a Markov model that provides a closed-form throughput expression. Simulation results show that this channel access scheme, working together with a simple sensing method, offers significant improvements to system throughput.
5.1 Introduction

Multichannel systems have received increasing attention for implementing next generation wireless communication systems such as B3G (beyond third generation) and 4G (fourth generation). Simultaneous usage of multiple channels is made possible by using advanced multiplexing techniques that can eliminate interference from adjacent signals and limit channel bandwidth. This multichannel usage improves system performance and offers better access opportunities for users than single channel usage [4].

In multichannel wireless systems, mitigating packet collisions is important. When users access multiple channels, collision mitigation can be effectively achieved through a joint access approach that resolves packet collisions in the time domain and the frequency domain. More specifically, a joint access scheme can mitigate the packet collisions by delaying access time as in a conventional backoff access scheme (i.e., a time-domain backoff scheme), and by accessing different channels in the manner of a hopping access scheme (i.e., a frequency-domain backoff scheme).

One existing joint access scheme is the truncated binary backoff scheme for the IEEE 802.16e WiMAX system [6,32,74]. This backoff scheme resolves a collision by adjusting the user’s transmission time in the process of random channel selection. The time and frequency domain backoff approach is also used in a joint access scheme based on carrier sense multiple access with collision avoidance (CSMA/CA) [48], and a fast retrial access scheme that reduces an excessive amount of access delay [5,80].

These existing joint access schemes have shown to provide better opportunities to resolve packet collisions over multiple channels. However, when wireless channels are time-varying and fading as in all multichannel systems, the existing schemes become inefficient [72,73,92]. These schemes may access channels of bad conditions without realizing changes in channel conditions, leading to performance degradation. Incorporating packet collision resolution with channel condition exploitation is thus a sensible strategy for achieving high throughput performance.

In this chapter, we propose a new joint access scheme that enables effective uplink access when users are only partially aware of channel conditions in multichannel wireless systems. The proposed scheme is supported by a new analytical model that can derive system performance for adjusting design parameters of the joint access procedure. In
order to exploit the channel conditions, we use a fast channel sensing method that requires only partial channel information [87, 88]. This sensing method, called myopic sensing, is suitable for real-time wireless communications because it has low computational overhead and a short sensing time. By integrating the joint access procedure with myopic sensing, our proposed scheme offers better control flexibility in reducing packet collisions and a better response to channel condition changes than the existing joint access schemes.

5.2 System Description

We consider an uplink wireless system. A number of users transmit their packets to a base station, and can access multiple time-varying wireless channels simultaneously. Figure 5.1 shows the relation between the users, the channels, and the base station.

The network operating environment is one of random access for the users. Aloha-type access [32] is used because of its efficiency in channel usage. CSMA/CA-type access [29] is not used as it is less efficient due to the additional time needed for carrier sensing.

The communication is multi-channel. The channel conditions can vary over time independently among users. The user messages transmitted over the network are real-time messages. During the network access, users provide only their local and limited channel state information. The availability of the channel information differs from that of conventional opportunistic access schemes which assume that full channel state information is available [72, 73].
Figure 5.2 shows the slot structure used in this study for transmitting an uplink message. Time is slotted for each transmission, and the channel conditions of users remain constant for the duration of one slot. At the beginning of each slot, each user senses some channels to obtain partial state information. After sensing, the user selects a suitable channel and transmits. When no collision occurs (i.e., no more than one packet is transmitted simultaneously over a channel at a slot) and the channel conditions are good, the transmitted packet is successfully received at the base station. The user receives an acknowledgement packet from the base station at the end of the slot. We will discuss how to handle collisions and poor channel conditions in the sections that follow.

5.3 Joint Random Access and Myopic Sensing

Two main issues for a multi-channel uplink system are access control for packet transmission and sensing for channel selection. The joint random access scheme for transmission control will first be introduced. The myopic sensing for channel selection will then be considered.

5.3.1 Joint random access for transmission control

The joint random access scheme uses the time- and frequency-domain backoff methods selectively to resolve collisions. We use the time-domain exponential backoff scheme [6, 32, 74] and the frequency-domain hopping access scheme [5, 80].

We observe that in general, the time-domain scheme is efficient when contention is high, and the frequency-domain scheme achieves better performance with low contention. Below is the access procedure for the proposed joint random access scheme.
**Joint Random Access Procedure:**

Step 1. Each user selects a channel using a sensing method at each slot.

Step 2. If a previous transmission channel is selected, then the exponential backoff access method is used:

2a. Each user transmits a packet with a transmission probability.

2b. When a collision occurs, the number of retransmissions increases.

2c. At each collision, the user’s transmission probability is adjusted based on an update function for retransmission.

2d. If the number of retransmission does not exceed its maximum number, repeat from step 2b to step 2c.

Step 3. Otherwise, the hopping access method is applied:

3a. Each user transmits with the same transmission probability used in the previous transmission attempt.

3b. When a collision occurs, the number of retransmissions increases.

3c. If the number of retransmissions exceeds the maximum retransmission number, the hopping access method replaces the exponential backoff access method.

Remark 1: The channel selection depends on a certain sensing method which will be discussed in Subsection 3.2.

Remark 2: The update function in the joint random access procedure is important to resolve packet collisions because it adjusts the transmission probability. Since the update function operates as the exponential backoff access, the transmission probability is updated by selecting the minimum value between the maximum transmission probability and the adjusted transmission probability, i.e.,

\[
\min\{p^{max}, p_{ij}(t)I_{z=0} + p^{max}I_{z=1} + p_{ij}(t)\gamma I_{z=2}\}, \quad (5.1)
\]
where $p^{max}$ is the maximum transmission probability, $p_{ij}(t)$ is the transmission probability that user $i \in \{1, ..., K\}$ accesses channel $j \in \{1, ..., N\}$ at slot $t$, and $\gamma$ is the reducing factor between 0 and 1, i.e., $0 < \gamma < 1$. The indication function $I_z$ is 1 when the event $z$ occurs, where $z$ is 0 when no transmission is taken place, $z$ is 1 for a successful transmission, and $z$ is 2 when there is a collision.

Remark 3: The number of retransmissions is limited to the maximum retransmission number of each scheme. When the exponential backoff access scheme reaches its maximum retransmission number, it does not update the transmission probability and uses the current transmission probability. On the other hand, when the hopping access scheme exceeds its maximum retransmission number, it then replaces the exponential backoff access scheme.

Example. The effectiveness of the joint random access procedure in resolving collision is illustrated with a scenario when two packets collide. Figure 5.3 shows the sequence of events when a collision occurs. Each user defers access time as in the exponential backoff access method if the selected channel is the same as that in the previous slot. Otherwise, the user immediately accesses a different channel as in the hopping access method. The channel selection depends on a channel sensing method having a certain selection probability $p_0$. Since the joint access scheme mitigates the collision in both the time and frequency domains, it has more opportunities for resolution.
Note that we will compare the proposed access scheme with the following joint access schemes: the truncated binary exponential backoff and the fast retrial. The truncated binary exponential backoff method uses the same update function as in a conventional binary exponential backoff approach, but accesses a channel randomly with its updated transmission probability. On the other hand, as a hopping access method, the fast retrial access scheme resolves the contention by immediately accessing a different channel at the next time slot [5]. This algorithm repeats the random channel selection and transmits with a certain transmission probability like the slotted Aloha until the retrial number reaches the maximum number of retrials.

5.3.2 Myopic sensing for channel selection

A suitable good channel for transmission can be identified by optimally sensing or measuring full conditions of channels [96, 97]. However, sensing all channels is impractical in resource-limited wireless systems. Since sensing more channels consumes more resources, fewer resources remain for effective data transmissions.

When it is impractical to fully sense all channels, myopic sensing is a useful technique. This myopic sensing technique has been successfully employed in opportunistic spectrum access methods such as those in [87, 88].

In myopic sensing, only some of the channels are selected to be sensed. When estimating channel conditions, myopic sensing uses a sufficient statistic \( \Omega_i(t) = (\omega_{ij}(t))_{j \in \{1, \ldots, N\}} \), which is the probability distribution of \( N \) available channels to user \( i \) at slot \( t \). The channel condition probability in \( \Omega_i(t) \) depends on the sensing and decision history updated from the initial statistics \( \Omega_i(0) \).

When the channels evolve independently, a Markov channel model can be used to estimate channel conditions with known state transition probabilities [87, 88]. Figure 5.4 shows the Markov channel model with two states: good (1) or bad (0), and with \( \alpha_{ij} \) the transition probability from state 0 to state 1 at channel \( j \) for user \( i \), and \( \beta_{ij} \) the probability of staying in state 1 for user \( i \) at channel \( j \).

Using the sufficient statistics \( \Omega_i(t) \) and the two-state Markov channel model, the myopic sensing method chooses a channel \( x^*_i(t) \) that has the maximum probability of channel
Figure 5.4: The Markov channel model with two channel states: bad (0) and good (1).

\[ x_i^*(t) = \arg \max_{j \in \{1, \ldots, N\}} (w_{ij}(t)\beta_{ij} + (1 - w_{ij}(t))\alpha_{ij}). \] (5.2)

The statistic update for the next slot, i.e., \( \Omega_i(t+1) \), is derived based on the current sensing action \( x_i^*(t) \in \{1, \ldots, N\} \) and its observation \( \theta_{ij}(t) | j = x_i^*(t) \in \{0, 1\} \) from

\[
\begin{aligned}
    w_{ij}(t+1) = & \begin{cases} 
        1, & \text{if } j = x_i^*(t), \theta_{ij}(t) | j = x_i^*(t) = 1, \\
        0, & \text{if } j = x_i^*(t), \theta_{ij}(t) | j = x_i^*(t) = 0, \\
        w_{ij}(t)\beta_{ij} + (1 - w_{ij}(t))\alpha_{ij}, & \text{if } j \neq x_i^*(t).
    \end{cases}
\end{aligned} \] (5.3)

Equation (5.3) shows that when the selected channel to sense is identified as having good conditions (i.e., \( \theta_{ij}(t) | j = x_i^*(t) = 1 \)), the probability of \( w_{ij}(t+1) \) is set to 1. This good channel is likely to be used for the next access. If the channel condition is deemed bad, (i.e., \( \theta_{ij}(t) | j = x_i^*(t) = 0 \)), the updated probability is set to 0. The other channels are updated based on \( w_{ij}(t)\beta_{ij} + (1 - w_{ij}(t))\alpha_{ij} \).

It is noteworthy that a belief vector, a probability distribution of all channel states, has been used to summarise the knowledge of unknown internal states of an underlying Markov process [90]. Although stochastic optimal control with a finite state dimension is possible [98, 99], the direct use of the belief vector is often intractable because the size of the total states grows exponentially with the number of channels. Fortunately, it has been shown that when channels evolve independently, the sufficient statistic \( \Omega_i(t) \) can replace the belief vector with reduced complexity [87].
5.4 Performance Analysis

5.4.1 Throughput of the joint random access scheme

System throughput is defined as the average aggregated number of packets delivered from all users through multiple channels at each slot, and is analysed using a stationary transmission probability variable.

We assume that the system is saturated as in [100–102], and that all \( K \) users are identical and access a channel fairly over \( N \) channels. With the stationary transmission probability in each channel being \( \frac{\tau}{N} \), the system throughput is

\[
S = N \binom{K}{1} \frac{\tau}{N} (1 - \frac{\tau}{N})^{K-1}
= K \tau (1 - \frac{\tau}{N})^{K-1},
\]

(5.4)

where \( \binom{K}{1} \) is a binomial coefficient. Note that the system throughput for a single channel is \( S = K \tau (1 - \tau)^{K-1} \) as in [101].

The stationary transmission probability \( \tau \) is defined as

\[
\tau = \mathbb{E}\{T_{ij}\}, \forall i \in (0, M), \forall j \in (0, H),
\]

(5.5)

where \( T_{ij} \) is the transmission probability at the state of the \( i \)th backoff access and \( j \)th hopping access, \( M \) is the maximum backoff number, and \( H \) is the maximum hopping number.

The stationary probability \( \tau \) is an average transmission probability of all states, because the transmission probability \( T_{ij} \) of each user depends on the state of access in the joint random access scheme.

In the sections that follow, an analytic expression for the stationary transmission probability will be derived. We first present a Markov model to capture the state transition behaviour of the joint access procedure. We then derive the equation for the stationary transmission probability based on the Markov model. The derived equation is calculated with the conditional collision probability for obtaining the system throughput.

The Markov model for the joint access state

The joint access procedure of the backoff and hopping stages can be expressed as a two-dimensional process, \( \{b(t), g(t)\} \), where \( b(t) \in \{0, 1, ..., M\} \) is the backoff stage process
Figure 5.5: The joint access Markov model showing the state transition diagram: $M$ backoff stages and $H$ hopping stages.

of a user at slot time $t$, and $g(t) \in \{0, 1, \ldots, H\}$ is the hopping stage process.

We model the process $\{b(t), g(t)\}$ as a discrete-time Markov chain which is shown in Figure 5.5. We use $p_c$ to denote the conditional collision probability that a transmitted packet experiences a collision, $p_0$ to denote the probability of selecting the backoff access, and $p_c$ to denote the probability of selecting the hopping access. Both $p_c$ and $p_0$ are taken as constant and independent values. The value of $p_1$ is obtained from $p_1 = 1 - p_0$.

Let $P(i, j|m, h) = \Pr(b(t + 1) = i, g(t + 1) = j | b(t) = m, g(t) = h)$ denote the transition probability from the state $\{m, h\}$ at slot time $t$ to the state $\{i, j\}$ at the next slot.
time. The transition probabilities in the Markov chain are

\[
P(i, j + 1 | i, j) = p_c p_1, \quad i \in (0, M), j \in (0, H);
\]

\[
P(i + 1, j | i, j) = p_c p_0, \quad i \in (0, M), j \in (0, H - 1);
\]

\[
P(i + 1, j | i, j) = p_c, \quad i \in (0, M), j = H;
\]

\[
P(0, 0 | i, j) = 1 - p_c, \quad i \in (0, M), j \in (0, H);
\]

\[
P(m, h | i, j) = 0, \quad m \in (0, M), m \neq i, m \neq i + 1, \quad h \in (0, H), h \neq j, h \neq j + 1.
\]

We now derive a closed-form solution of \( \pi_{i,j} \), the stationary probability of state \( \{i, j\} \) in the chain, where \( \pi_{i,j} = \lim_{t \to \infty} \Pr(b(t) = i, g(t) = j) \). The derivation will use the chain rules and the following two steady state equations:

\[
\pi_{i,j} = \sum_{m=0}^{M} \sum_{h=0}^{H} \pi_{m,h} P(i, j | m, h). \tag{5.7}
\]

and

\[
\sum_{i=0}^{M} \sum_{j=0}^{H} \pi_{i,j} = 1. \tag{5.8}
\]

When applying (5.7) to (5.6) based on the chain rules, we obtain the stationary probability of state \( \{i, j\} \) as

\[
\pi_{i,j} = \begin{cases} 
\sum_{m=0}^{M} \sum_{h=0}^{H} (1 - p_c) \pi_{m,h}, & i = 0, j = 0; \\
p_c p_0 \pi_{i-1,j}, & 0 < i < M, j = 0; \\
p_c p_0 \pi_{i-1,j} + p_c p_0 \pi_{i,j}, & i = M, j = 0; \\
p_c p_1 \pi_{i,j-1}, & i = 0, 0 < j < H; \\
p_c p_0 \pi_{i-1,j} + p_c p_1 \pi_{i,j-1}, & 0 < i < M, 0 < j < H; \tag{9a} \\
p_c \pi_{i-1,j} + p_c p_1 \pi_{i,j-1}, & 0 < i < M, j = H; \tag{9b} \\
p_c p_1 \pi_{i,j-1} + p_c p_0 \pi_{i-1,j} + p_c p_0 \pi_{i,j}, & i = M, 0 < j < H; \tag{9c} \\
p_c \pi_{i-1,j} + p_c p_1 \pi_{i,j-1} + p_c \pi_{i,j}, & i = M, j = H.
\end{cases}
\]

Since we have \( \pi_{0,0} = \sum_{m=0}^{M} \sum_{h=0}^{H} \pi_{m,h} (1 - p_c) \) from (5.9), the stationary probability of the initial state becomes \( \pi_{0,0} = (1 - p_c) \) by applying (5.8). With \( \pi_{0,0} \), (5.7), and (5.8),
the stationary state probability is

$$\pi_{i,j} = \begin{cases} 
1 - p_c, & i = 0, j = 0; \\
(p_c p_0)^i \pi_{0,0}, & 0 < i < M, j = 0; \\
\frac{(p_c p_0)^i (1 - p_c p_0)}{p_0}, & i = M, j = 0; \\
(p_c p_1)^j \pi_{0,0}, & i = 0, 0 < j \leq H; \\
\pi_{A,i,j}, & 0 < i < M, 0 < j < H; \\
\pi_{B,i,j}, & 0 < i < M, j = H; \\
\pi_{C,i,j}, & i = M, 0 < j < H; \\
\pi_{D,i,j}, & i = M, j = H,
\end{cases} \tag{5.10}$$

where

$$\begin{align*}
\pi_{A,i,j} &= \binom{i + j}{j} p_c^i p_0^j \pi_{0,0}, \\
\pi_{B,i,j} &= p_c^{i+H} p_1^H \pi_{0,0} + \sum_{x=1}^i \binom{x + H - 1}{H - 1} p_c^{i+x} p_0^x p_1^H \pi_{0,0}, \\
\pi_{C,i,j} &= \sum_{x=0}^j \frac{1}{(1 - p_c p_0)^x+1} \binom{M - 1 + j - x}{j - x} p_c^{M+j} p_0^x p_1^H \pi_{0,0}, \\
\pi_{D,i,j} &= \frac{1}{1 - p_c} \left( p_c^{M+H} p_1^H \pi_{0,0} + \sum_{x=1}^{M-1} \binom{x + H - 1}{H - 1} p_c^{M+x} p_0^x p_1^H \pi_{0,0} \right) \\
&\quad + \sum_{x=0}^{H-1} \frac{1}{(1 - p_c p_0)^x+1} \binom{M + H - 2 - x}{H - 1 - x} p_c^{M+x} p_0^x p_1^H \pi_{0,0}\right). 
\end{align*}$$

Remarks: The closed-form solution of the stationary state probability has been derived by applying the chain rules to the steady state equations. Since the chain rules are based on the Markov model as shown in Figure 5.5, the stationary state probability strongly depends on the state transition diagram of the proposed joint access procedure.

The verification of (5.10) is done with the following three propositions on different backoff and hopping stages:

**Proposition 5.4.1** The stationary state probability of $\pi_{i,j}$ for $0 < i < M$ and $0 < j < H$ in (10a) is obtained with the following binomial coefficients:

$$\pi_{i,j}^A = \binom{i + j}{j} p_c^i p_0^j \pi_{0,0}, \quad 0 < i < M, 0 < j < H. \tag{5.11}$$
Proof: From the chain equation in (9a), we have \( \pi_{i,j} = p_c p_0 \pi_{i-1,j} + p_c p_1 \pi_{i,j-1} \) for \( i \)th backoff and \( j \)th hopping. By applying the chain rule, we have the following results:

\[
\begin{align*}
\pi_{0,0} &= (p_c p_0)^1 \pi_{0,0} \quad (p_c p_0)^2 \pi_{0,0} \quad (p_c p_0)^3 \pi_{0,0} \quad \cdots \quad (p_c p_0)^{M-1} \pi_{0,0} \\
(p_c p_1)^1 \pi_{0,0} &= 2p_c^2 p_0 p_1 \pi_{0,0} \quad 3p_c^2 p_0^2 p_1 \pi_{0,0} \quad 4p_c^2 p_0^3 p_1 \pi_{0,0} \quad \cdots \\
(p_c p_1)^2 \pi_{0,0} &= 3p_c^3 p_0^2 p_1^2 \pi_{0,0} \quad 6p_c^3 p_0^3 p_1^2 \pi_{0,0} \quad \cdots \\
(p_c p_1)^3 \pi_{0,0} &= 4p_c^4 p_0^3 p_1^3 \pi_{0,0} \quad \cdots \\
& \vdots \quad \vdots \quad \vdots \\
(p_c p_1)^{H-1} \pi_{0,0} \\
\end{align*}
\]

Since the hopping and backoff processes occur with collisions, \( \pi_{i,j} \) is the stationary probability of total \( i + j \) collisions. This implies that the total number of cases of selecting \( j \) hopping among the total collisions \( i + j \) becomes \( \binom{i+j}{j} = \binom{i+j}{i} \). From this equation, we can see that the coefficient of \( \pi_{i,j} \) becomes a binomial coefficient from Pascal’s Triangle because it has the following relation as

\[
\binom{i+j-1}{j} + \binom{i+j-1}{j-1} = \binom{i+j}{j}, \tag{5.13}
\]

where the results in (5.12) are satisfied with \( p_c p_0 \pi_{i-1,j} + p_c p_1 \pi_{i,j-1} = \pi_{i,j} \). Therefore, the proposition is proved. \( \square \)

Proposition 5.4.2 The stationary state probability of \( \pi_{i,j} \) for \( 0 < i < M \) and \( j = H \) in (10b) is:

\[
\pi_{i,j}^B = p_c^{i+H} p_1^H \pi_{0,0} + \sum_{x=1}^{i} \left( \frac{x + H - 1}{H - 1} \right) p_c^x p_0^i p_1^H \pi_{0,0}. \tag{5.14}
\]

Proof: We use the following chain equation in (9b): \( \pi_{i,H} = p_c \pi_{i-1,H} + p_c p_1 \pi_{i,H-1} \) for the maximum hopping stage (i.e., \( j = H \)) and \( 0 < i < M \). From this chain rule, we have the following relation:

\[
\begin{align*}
\pi_{1,H} &= p_c \pi_{0,H} + p_c p_1 \pi_{1,H-1} \\
\pi_{2,H} &= p_c \pi_{1,H} + p_c p_1 \pi_{2,H-1} = p_c^2 \pi_{0,H} + p_c^2 p_1 \pi_{1,H-1} + p_c p_1 \pi_{2,H-1} \\
& \vdots \\
\pi_{i,H} &= p_c \pi_{i-1,H} + p_c p_1 \pi_{i,H-1} = p_c^i \pi_{0,H} + p_c^i p_1 \pi_{1,H-1} + p_c^{i-1} p_1 \pi_{2,H-1} + \cdots + p_c p_1 \pi_{i,H-1}. \tag{5.15}
\end{align*}
\]
The probability $\pi_{i,H}$ becomes

$$\pi_{i,H} = p_c^i \pi_{i,0,H} + \sum_{x=1}^{i} p_c^{i-x+1} p_1 \pi_{x,H-1}. \quad (5.16)$$

Using $\pi_{0,H} = (p_c p_1)^H \pi_{0,0}$ and $\pi_{x,H-1} = \left(\frac{x+H-1}{H-1}\right) p_c^{x+H-1} p_0^x p_1^{H-1} \pi_{0,0}$ from (5.10), we can obtain the probability $\pi_{i,H}$ as

$$\begin{align*}
\pi_{i,H} &= p_c^i \pi_{i,0,H} + \sum_{x=1}^{i} p_c^{i-x+1} p_1 \left(\frac{x+H-1}{H-1}\right) p_c^{x+H-1} p_0^x p_1^{H-1} \pi_{0,0} \\
&= p_c^i \pi_{i,0,H} + \sum_{x=1}^{i} \left(\frac{x+H-1}{H-1}\right) p_c^{i-x+1} p_1 \pi_{i,H-1} \pi_{0,0}. \quad (5.17)
\end{align*}$$

Hence, the proposition is proved. \qed

**Proposition 5.4.3** The stationary state probability of $\pi_{i,j}$ for $i = M$ and $0 < j < H$ in (10c) is:

$$\pi_{i,j}^C = \sum_{x=0}^{j} \frac{1}{(1-p_c p_0)^x+1} \left(\frac{M-1+j-x}{j-x}\right) p_c^{M+j} p_0^M p_1^j \pi_{0,0}. \quad (5.18)$$

**Proof:** When the backoff stage reaches its maximum (i.e., $i = M$) and $0 < j < H$, the chain equation in (9c) is used as $\pi_{M,j} = p_c p_0^M \pi_{M-1,j} + p_c p_1 \pi_{M,j-1} + p_c p_0 \pi_{M,j}$. The probability is rewritten as

$$\pi_{M,j} = \frac{1}{1-p_c p_0} \left( p_c p_0^M \pi_{M-1,j} + p_c p_1 \pi_{M,j-1} \right). \quad (5.19)$$

By applying the recursive approach with the probability $\pi_{M,0} = (p_c p_0)^M \pi_{0,0}$, we have the following relation:

$$\begin{align*}
\pi_{M,1} &= \frac{1}{1-p_c p_0} \left(\frac{M-1+1}{1}\right) p_c^{M+1} p_0^M p_1 \pi_{0,0} + \frac{1}{(1-p_c p_0)^2} \left(\frac{M-1+j}{j}\right) p_c^{M+j} p_0^M p_1^j \pi_{0,0} \\
&\vdots \\
\pi_{M,j} &= \frac{1}{1-p_c p_0} \left(\frac{M-1+j}{j}\right) p_c^{M+j} p_0^M p_1^j \pi_{0,0} + \frac{1}{(1-p_c p_0)^j} \left(\frac{M-1+j-1}{j-1}\right) p_c^{M+j} p_0^M p_1^j \pi_{0,0} \\
&\quad + \cdots + \frac{1}{(1-p_c p_0)^{j+1}} \left(\frac{M-1+j-1}{j-1}\right) p_c^{M+j} p_0^M p_1^j \pi_{0,0}. \quad (5.20)
\end{align*}$$

From the relation, we can generalise the probability $\pi_{M,j}$ for $0 < j < H$ as

$$\sum_{x=0}^{j} \frac{1}{(1-p_c p_0)^{x+1}} \left(\frac{M-1+j-x}{j-x}\right) p_c^{M+j} p_0^M p_1^j \pi_{0,0}. \quad (5.21)$$

\qed
The stationary transmission probability

With the stationary state probability \( \pi_{i,j} \) from (5.10), the average of transmission probabilities in (5.5) can be further expressed as

\[
\tau = \sum_{i=0}^{M} \sum_{j=0}^{H} T_{ij} \pi_{i,j}. \tag{5.22}
\]

The transmission probability \( T_{ij} \) at the state of \( i \)th backoff and \( j \)th hopping access takes the form

\[
T_{ij} = \begin{cases} 
  p_{\text{max}}, & i = 0, 0 \leq j \leq H; \\
  p_{\text{max}} \gamma^i, & 0 < i \leq M, 0 < j \leq H,
\end{cases} \tag{5.23}
\]

where, as introduced in Section 3.1, \( p_{\text{max}} \) is the maximum transmission probability and \( \gamma \) is the reducing factor of the exponential backoff access scheme.

Taking into account the different stationary state probabilities (5.10) and the associated transmission probabilities (5.23), the stationary transmission probability in (5.22) becomes

\[
\tau = p_{\text{max}} \pi_{0,0} + \sum_{i=1}^{M-1} p_{\text{max}} \gamma^i \pi_{i,0} + p_{\text{max}} \gamma^M \pi_{M,0} + \sum_{j=1}^{H} p_{\text{max}} \pi_{0,j} \\
+ \sum_{i=1}^{M-1} \sum_{j=1}^{H-1} p_{\text{max}} \gamma^i \pi_{i,j} + \sum_{i=1}^{M-1} p_{\text{max}} \gamma^i \pi_{i,H} + \sum_{j=1}^{H-1} p_{\text{max}} \gamma^M \pi_{M,j} + p_{\text{max}} \gamma^M \pi_{M,H}. \tag{5.24}
\]

Substituting (5.10) into (5.24) yields

\[
\tau = p_{\text{max}} (1 - p_c) \left( 1 + \frac{\gamma p_c p_0 (1 - (\gamma p_c p_0)^{M-1})}{1 - \gamma p_c p_0} + \frac{(\gamma p_c p_0)^M}{1 - \gamma p_c p_0} + \frac{p_c p_1 (1 - (p_c p_1)^H)}{1 - p_c p_1} \\
+ \frac{1 - p_c}{1 - p_c} \sum_{i=1}^{M-1} \gamma^i \left( \sum_{j=1}^{H-1} \pi_{i,j} + \pi_{i,H} \right) + \gamma^M \frac{1 - p_c}{1 - p_c} \left( \sum_{j=1}^{H-1} \pi_{i,j} + \pi_{i,H} \right) \right). \tag{5.25}
\]

Remarks: If the probability \( p_0 \) is set to 1, the stationary probability of the joint random access becomes that of the exponential backoff access. When the maximum number of hops \( H \) is 0, the joint random access becomes the exponential backoff access.

Since the transmission probability in (5.25) is a function of \( p_c \), which is unknown, we therefore cannot obtain \( \tau \) directly. We will derive an additional equation for the conditional collision probability, which is a function of \( \tau \), so that we can calculate the stationary transmission probability.
The conditional collision probability

In a single channel communication, the conditional collision probability of $K$ users is

$$p_c = 1 - (1 - \tau)^{K-1}. \quad (5.26)$$

In the multichannel communication, the collisions may occur differently, depending on the number of transmitting users over multiple channels. Let $\Pr(x|y)$ denote the probability that only $x$ users transmit among $y$ users. With the stationary transmission probability $\tau$,

$$\Pr(x|y) = \binom{y}{x} \tau^x (1 - \tau)^{y-x}. \quad (5.27)$$

Assuming that a user transmits over a channel among $N$ channels, the probability of selecting a different channel to the transmitting user is $\frac{N-1}{N} = 1 - \frac{1}{N}$. When $x$ users access a channel among $N - 1$ channels, excluding the channel used by the transmitting user, there is no collision with the probability

$$\left(1 - \frac{1}{N}\right)^x \Pr(x|K-1). \quad (5.28)$$

Taking different numbers of transmitting users into account, the conditional collision probability becomes

$$p_c = 1 - \sum_{x=0}^{K-1} \left(1 - \frac{1}{N}\right)^x \Pr(x|K-1)$$
$$= 1 - \sum_{x=0}^{K-1} \left(1 - \frac{1}{N}\right)^x \binom{K-1}{x} \tau^x (1 - \tau)^{K-1-x}. \quad (5.29)$$

Equations (5.25) and (5.29) are two nonlinear equations with two unknown variables, $p_c$ and $\tau$. No closed-form expression for the solution exists. We use numerical techniques to find the solution. The solution is unique, as shown below.

**Proposition 5.4.4** A unique solution for the stationary transmission probability $\tau$ exists, and can be found by solving (5.25) and (5.29).

**Proof:** Unfortunately, the set of (5.25) and (5.29) represents a nonlinear system with the two unknown variables, $\tau$ and $p_c$, and, in general, closed form expressions of these variables are not available. Thus, we need to use numerical techniques to find them.
The stationary transmission probability \( \tau \) in (5.25) is a function of the conditional collision probability \( p_c \), denoted by \( f_1(p_c) \). Similarly, in (5.29), we consider \( p_c \) as a function of \( \tau \), i.e., \( p_c = f_2(\tau) \). It is shown that \( f_1(p_c) \) is a continuous and monotonically decreasing function in the range \( p_c \in (0, 1) \), that starts from \( f_1(0) = p^{max} \) and decreases to \( f_1(1) = 0 \). On the other hand, \( f_2(\tau) \) is a continuous and monotonically increasing function in the range of \( \tau \in (0, 1) \) that starts from \( f_2(0) = 0 \) and increases up to \( f_2(1) = 1 \).

Following [101], we can prove that the solution of the nonlinear system is unique since the first order conditions of \( f_1 \) and \( f_2 \) are all positive or 0 as \( \frac{\partial f_1(p_c)}{\partial p_c} \geq 0 \) and \( \frac{\partial f_2(\tau)}{\partial \tau} \geq 0 \).

Summary: The system throughput in (6.12) is found by solving numerically the unique solution of the stationary transmission probability in (5.25) with the conditional collision probability in (5.29). The stationary state probability in (5.10), which characterises the joint random access procedure and is used in (5.25), is derived from the joint access Markov model.

### 5.4.2 Impact of the myopic sensing on the system throughput

In order to find the system throughput with myopic sensing, we first analyse stationary probabilities of channel states based on the Markov channel model and then define a channel condition probability obtained from the myopic sensing method. We assume that each channel condition follows the Markov model with two channel states, i.e., bad (0) and good (1), as shown in Figure 5.4. The state transition probabilities, \( \alpha \) and \( \beta \), are assumed to be known.

#### The stationary probability of a channel state

Let \( q_0 \) and \( q_1 \) denote the stationary probabilities of a bad state and a good state. From the Markov channel model, we have the following two equations: \( q_0 = q_0(1 - \alpha) + q_1(1 - \beta) \) and \( q_1 = q_0\alpha + q_1\beta \). With \( q_0 + q_1 = 1 \), the stationary probabilities are

\[
q_0 = \frac{1 - \beta}{1 + \alpha - \beta} \quad \text{and} \quad q_1 = \frac{\alpha}{1 + \alpha - \beta}.
\]  

(5.30)

When randomly accessing a channel, the selected channel follows the good state condition as given in (5.30). When applying myopic sensing, the selected channel may have
a different condition than \( q_1 \). Hence, finding a channel condition probability is crucial for analysing the impact of the myopic sensing method on the system throughput.

**The channel condition probability**

Let \( \Pr(\text{good} | \theta) \) denote the probability of selecting a good channel from observing the channel condition \( \theta \). Having observed a bad condition (i.e., \( \theta = 0 \)), the probability of selecting a good channel among \( N \) channels is

\[
\Pr(\text{good} | \theta = 0) = \alpha \frac{1}{N} + q_1 \left( 1 - \frac{1}{N} \right). \tag{5.31}
\]

In the good channel observation (i.e., \( \theta = 1 \)), we have \( \Pr(\text{good} | \theta = 1) = 1 \) because the myopic sensing method selects the observed channel with probability 1.

The good channel condition probability of myopic sensing, \( q \), is given by

\[
q = q_0 \Pr(\text{good} | \theta = 0) + q_1 \Pr(\text{good} | \theta = 1)
= q_0 \left( \alpha \frac{1}{N} + q_1 \left( 1 - \frac{1}{N} \right) \right) + q_1. \tag{5.32}
\]

Since myopic sensing selects the best channel among good channels, the good condition probability of the selected channel is a probability that at least one channel has a good condition.

**The system throughput with myopic sensing**

With the stationary probability of the channel condition being good, the system throughput is obtained as

\[
S = K q \tau \left( 1 - q \frac{\tau}{N} \right)^{K-1}. \tag{5.33}
\]

In order to calculate the system throughput, the conditional collision probability in (5.29) also needs to be changed with the channel condition probability \( q \) as below

\[
p_c = 1 - \sum_{x=0}^{K-1} \left( 1 - \frac{1}{N} \right)^x \binom{K-1}{x} (q \tau)^x (1 - q \tau)^{K-1-x}. \tag{5.34}
\]

This is because packet transmissions are affected by conditions of selected channels.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network model</td>
<td></td>
</tr>
<tr>
<td>Channel number ((N))</td>
<td>2 to 5</td>
</tr>
<tr>
<td>User number ((K))</td>
<td>1 to 50</td>
</tr>
<tr>
<td>Access model</td>
<td></td>
</tr>
<tr>
<td>Maximum transmission probability ((p_{\text{max}}))</td>
<td>0.5</td>
</tr>
<tr>
<td>Transmission reducing factor ((\gamma))</td>
<td>0.5</td>
</tr>
<tr>
<td>Maximum backoff number ((M))</td>
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<td>Maximum hopping number ((H))</td>
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<tr>
<td>Selection probability ((p_0))</td>
<td>(1/N)</td>
</tr>
<tr>
<td>Channel model</td>
<td></td>
</tr>
<tr>
<td>Transition probabilities ((\alpha, \beta))</td>
<td>(\alpha = 0.8, \beta = 0.2)</td>
</tr>
</tbody>
</table>

The channel condition probability can be summarised as

\[
q = \begin{cases} 
q_1, & \text{if random sensing;} \\
q_1 + q_0 \left( \alpha \frac{1}{N} + q_1 \frac{N-1}{N} \right), & \text{if myopic sensing.}
\end{cases}
\] (5.35)

For random sensing, the channel condition only depends on the channel model since each user selects a channel fairly over \(N\) channels. When using myopic sensing, the probability of selecting a good channel increases because each user estimates a better channel based on the channel observation in a greedy manner.

### 5.5 Numerical and Simulation Results

The proposed scheme is compared with two well-known existing joint access schemes: the truncated binary backoff scheme that operates as a backoff access with a random channel selection [6] and the fast retriial scheme that works as a hopping access without a time-domain backoff [5]. Numerical and simulation studies are conducted to evaluate throughput performance of the joint access schemes and throughput impacts of the sensing methods.
5.5.1 Parameters

Table 5.1 presents the system parameters for the network, access, and channel models discussed in Section 5.3.

The truncated binary backoff scheme uses the transmission probability $p^{\text{max}}$, the transmission reduction factor $\gamma$, and the maximum backoff number $M$ with a random channel selection. The fast retrial scheme uses the same $p^{\text{max}}$ with $\gamma = 0$, so it becomes a hopping access scheme with a fixed transmission probability. The proposed scheme uses all parameters including the maximum hopping number $H$ and the probability of selecting the backoff access scheme $p_0$.

Each simulation result is an average of 50 runs, where each run has 5000 time slots.

5.5.2 Throughput comparison under perfect channel conditions

Figure 5.6 shows the throughput comparison of the proposed scheme, the fast retrial scheme, and the truncated backoff scheme. Perfect channel conditions are taken into account in which the success of packet transmissions only depends on packet collisions.

In Figure 5.6(a) and Figure 5.6(b), the proposed scheme is compared with the fast retrial scheme and the truncated backoff scheme. Intuitively, as the channel number increases, the access schemes improve their throughput proportionally. The fast retrial scheme is the most efficient with a small number of users, but as the user number increases, its throughput is greatly reduced because more collisions occur with its fixed transmission probability. The truncated backoff scheme uses the update function to avoid collisions, gradually increases the throughput and obtains the highest throughput with a large number of users. By jointly using the hopping and backoff methods, the proposed scheme provides a better opportunity to mitigate collisions, and outperforms the fast retrial scheme at large user numbers and the truncated backoff scheme at small user numbers. The simulation results verify the analytical results.

5.5.3 Throughput performance with access parameters

Figure 5.7 presents the throughput performance of the proposed scheme with respect to transmission probabilities and hopping numbers under perfect channel conditions when
Figure 5.6: Throughput comparison of the proposed scheme with the fast retrial scheme and the truncated backoff scheme under perfect channel conditions. The simulation results are represented by symbols, while the analytical results are represented by solid, dashed, and dash-dotted lines for the proposed scheme, the fast retrial scheme, and the truncated backoff scheme.
Figure 5.7: Throughput performance of the proposed scheme with respect to transmission probabilities and hopping numbers under perfect channel conditions.
Shown in Figure 5.7(a) is the impact of the transmission probabilities. The proposed scheme increases the system throughput by adjusting a transmission probability through the fast access inherited from the frequency-domain backoff and the update procedure of the time-domain backoff access. Figure 5.7(b) shows the impact of the hopping numbers. When the hopping number is large, the frequency-domain backoff procedure in the proposed scheme becomes prominent, and it increases the system throughput at small user numbers. These results imply that the proposed scheme has more control flexibility to improve the system throughput by adjusting the access parameters.

5.5.4 The impact of sensing methods on throughput performance

Under imperfect channel conditions, the success of packet transmissions depends on the packet collisions as well as the channel conditions. Hence, we can see from Figure 5.8 that the joint access schemes have different impacts on the system throughput with the sensing methods. For clear presentation, only the simulation results are included. Note that the simulation results still verify the analytical results.

Figure 5.8(a) shows the throughput comparison of the joint access schemes with myopic sensing and random sensing when channel conditions are relatively good (i.e., $\alpha = 0.8$). Since myopic sensing can select a good channel with a higher possibility, it allows more packets to be transmitted over selected good channels than random sensing. However, more collisions may occur if there is no efficient access scheme. The fast re-trial scheme achieves better throughput with myopic sensing when the number of users is small because its fast access procedure uses the selected good channels. The truncated backoff scheme always has better throughput with myopic sensing because it can reduce collisions at all user numbers. By effectively using the selected good channels at small user numbers and mitigating the collisions at large user numbers, the proposed scheme cooperates with the myopic sensing well.

When the channel conditions are relatively bad (i.e., $\alpha = 0.2$) as shown in Figure 5.8(b), the proposed scheme with the myopic sensing is the most efficient in throughput performance because the selected good channels are fully used for transmissions. The conventional backoff access scheme is less efficient than both the proposed scheme and
Figure 5.8: Throughput comparison of the three access schemes when using the myopic sensing and the random sensing with imperfect channel conditions.
the fast retrial scheme because the time-domain backoff procedure causes significant delay when transmission failures occur due to bad channel conditions.

5.6 Conclusion

We have proposed a joint random access scheme that selectively uses the hopping and backoff access methods for multichannel wireless networks. We have shown that the proposed scheme offers better control flexibility to reduce packet collisions than the existing joint schemes. It improves the system throughput that cannot be obtained by the hopping access or the backoff access methods alone. The use of the myopic sensing method enables the proposed scheme to become more efficient because it fully uses the selected good channels for transmissions through the joint backoff control. The analysis of the system throughput has been done using the Markov models that capture the access and channel states, providing the closed-form throughput expression. The numerical and simulation results showed that the proposed scheme outperforms the existing joint schemes and improves the system throughput.
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Chapter 6

A Refined MAC Protocol with Multipacket Reception

Medium access control (MAC) protocols making use of multipacket reception (MPR) capability achieve better throughput than conventional MAC protocols. When a wireless network operates with MPR capable nodes and non-MPR nodes, the MAC protocols must not only utilise the MPR capability to maximise throughput, but must also enable the co-existence with these two types of nodes. A new MPR MAC protocol is proposed to achieve the co-existence requirement by adopting a request-to-send/clear-to-send mechanism in IEEE 802.11 MAC standards. This MPR MAC protocol also improves throughput by allowing additional data transmissions to use the MPR capability fully. We analyse the system throughput of the co-existence of different link characteristics of nodes, and optimise its throughput by adjusting contention window sizes with respect to certain throughput requirements of the nodes.
6.1 Introduction

Multipacket reception (MPR) has been introduced to improve the performance of random access in wireless networks. The MPR capability, which can detect multiple packets simultaneously, is obtained by using advanced signal processing or antenna array techniques [8]. For instance, a receiver can have the MPR capability by employing code division multiple access (CDMA) where different waveforms (i.e., codes) are assigned to each user and using multiuser detection techniques [7]. It is also available by using spatial division multiple access (SDMA) with an antenna array [3].

In order to use the MPR capability, MPR-enabled medium access control (MAC) protocols are required. Conventional MAC protocols [28, 29] cannot support the MPR capability because of the limitation of single packet reception. A number of MPR MAC protocols have been proposed to improve throughput based on the concept of cross-layer design, which requires signal processing at the physical layer and protocol design at the MAC layer [21, 22, 64, 65, 103]. These MPR MAC protocols work well with a central coordinator for packet transmission, but they are not suitable for distributed wireless networks such as ad hoc networks. By adopting the request-to-send (RTS)/clear-to-send (CTS) mechanism [28, 29], some MPR MAC protocols recently have been developed to support multiple access in the distributed networks [66, 67].

When a wireless network operates with MPR-capable nodes and non-MPR nodes, the MPR MAC protocols must allow these two types of nodes to co-exist. If the protocols do not take into account the co-existence, the MPR-capable nodes will have an unfair advantage in achieving better throughput in the network. To our knowledge, none of the existing MPR MAC protocols support such co-existence.

In this chapter, we propose a new MPR MAC protocol that improves overall system throughput and ensures throughput fairness when the MPR and non-MPR nodes co-exist in a wireless network. The throughput improvement is achieved by allowing additional data transmissions to use the MPR capability fully. We use the RTS/CTS mechanism since it can operate in both central and distributed manners in a co-existence environment. The selection of the contention window size is the key factor in the trade-off between throughput and fairness. To find the optimal contention window size, we first derive a closed-form throughput expression that takes into account the co-existence of different
Figure 6.1: Co-existence of direct-link (non-MPR) nodes and up-link (MPR) nodes in a wireless network when the base station is equipped with $\alpha$ antennas.

link characteristics with and without the MPR capability. We then use a constrained optimisation procedure to find the solution. Numerical and simulation results show that optimal throughput can be achieved by adjusting the contention window sizes with respect to the specified throughput requirements of wireless nodes.

6.2 Network Model and Medium Access Mechanisms

6.2.1 Network model

We consider a wireless network that consists of a base station and wireless nodes. The base station is equipped with $\alpha$ antennas to facilitate the MPR capability using antenna array techniques. The number of antennas affects the maximum number of packets that can be simultaneously received at the base station (i.e., the MPR capability) [3]. Among the wireless nodes, there can be two types of nodes: (i) up-link nodes that attempt to transmit packets to the base station, and (ii) direct-link nodes that attempt to transmit packets to another nodes. Since the base station has the MPR capability, the up-link nodes are referred to as MPR nodes, while the direct-link nodes are referred to as non-MPR nodes in this chapter. Figure 6.1 shows the co-existence of the direct-link nodes and the up-link nodes with the base station of $\alpha$ MPR capability. It is assumed that the degree of the MPR capability depends on the number of antennas.
The focus of this work is the throughput improvement of the network when the up-link nodes and the direct-link nodes co-exist. Because of the MPR capability, the up-link nodes require an MPR MAC protocol to facilitate throughput improvement of up-link transmissions, while the direct-link nodes use a conventional non-MPR protocol to communicate with other nodes without rerouting their transmissions through the base station. For the direct-link nodes, the RTS/CTS mechanism is used as a conventional distributed MAC protocol since the direct-link nodes have a distributed nature on their transmissions. In order to simultaneously operate with the direct-link nodes, the extended RTS/CTS mechanism that uses the MPR capability is used for the up-link nodes.  

### 6.2.2 The conventional RTS/CTS mechanism for the direct-links

The RTS/CTS access mechanism is provided in the IEEE 802.11 distributed coordination function [28, 29], and uses a four-way handshake for data transmission with two control packets, called RTS and CTS packets.

Figure 6.2 shows the four-way handshake in the RTS/CTS mechanism. When a source node attempts to transmit a data packet, the RTS packet is sent to a destination node. After a short interframe space (SIFS) time, the destination node responds by sending back the CTS packet. Each transmission waits for the SIFS time before transmitting packets. After receiving the CTS packet, the source node transmits its data packet and the destination

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1Although a MAC protocol for down-link transmission is important, it is not considered in the chapter as we focus on the impact of the MPR capability. The throughput of down-link can be improved using beamforming, directional antenna, or spatial multiplexing techniques [104] in terms of multipacket transmission. This will be considered in our future work.
confirms the successful data transmission by sending an acknowledge (ACK) packet to the
source node. In order to avoid packet transmissions of other nodes within the transmission
range of the source and destination nodes, a network allocation vector (NAV) is used. The
NAV informs all other nodes of the total transmission period that is indicated in the RTS
and CTS packets. After a distributed interframe space (DIFS) time, new transmission
starts with a backoff rule.

The backoff rule uses a binary exponential rule for packet transmission described as
follows. Nodes set a random interval chosen within an initial contention window size.
As long as the nodes sense that a channel is idle, the backoff interval is decreased by
one at a slot time. If the channel is busy, the backoff interval is frozen until the channel
becomes idle. When the backoff interval reaches 0, the nodes transmit the RTS packets to
their destination. If only one packet is transmitted, the four-way handshake is performed.
Otherwise, collisions occur and the contention window size is doubled for packet retrans-
mission. The contention window increases to the maximum contention window size.

The RTS/CTS mechanism is effective under the condition of high contention to ac-
access a radio channel with a large packet size because its RTS and CTS frames reduce the
retransmission period when collisions occur [101, 105]. This conventional access mech-
anism is suitable for nodes on direct-link transmissions. For up-link transmissions, the
conventional mechanism is not suitable because it does not exploit the MPR capability
and it only considers multiple packet transmissions as collisions.

6.2.3 The extended RTS/CTS mechanism for the up-links

We propose an extension of the RTS/CTS mechanism to use the MPR capability fully by
allowing multiple simultaneous data transmissions for the up-link nodes.

In the extended mechanism, multiple RTS packets are used to establish data trans-
missions of the up-link nodes, similar to the four-way handshake of the conventional
RTS/CTS mechanism. If the number of the transmitted RTS packets does not exceed the
MPR capability of the base station, the proposed mechanism allows the requested data
transmissions by sending back the CTS packet to the up-link nodes. Furthermore, if the
number of the requested data transmissions is less than the maximum MPR capability, the
proposed mechanism selects additional data transmissions up to the maximum MPR ca-
Figure 6.3: The extended RTS/CTS access mechanism for up-links with the MPR capability, $\alpha$.

pability. The additional data transmission information is also included in the CTS packet. After receiving the multiple data transmissions, the ACK packet is transmitted. On the other hand, if the number of the transmitted RTS packets exceeds the maximum MPR capability, the proposed mechanism cannot establish the data transmissions since the base station cannot detect the RTS packets because of the collisions.

Note that for the extended RTS/CTS mechanism, the formats of the CTS and ACK packets need to be modified, while the formats of the RTS and data packets need not [66]. However, for simplicity, we assume that the current CTS and ACK packets can include additional information that other nodes understand.

The extended RTS/CTS mechanism for the up-links is illustrated in Figure 6.3. A base station of $\alpha$ MPR capability receives $k$ RTS packets from up-link nodes (i.e., the base station is a destination and the up-link nodes are sources). Since $k < \alpha$, the base station can detect the RTS packets. The base station then sends back the CTS packet to the

---

2In this chapter, we assume that the additional data transmissions are chosen randomly among the up-link MPR nodes to simplify the throughput analysis. However, there could be a certain rule to select the additional data transmissions to improve delay and this issue will be investigated in the near future.
(a) Two simultaneous RTS transmissions from Node \( D_1 \) to Node \( D_2 \) and from Node \( U_1 \) to BS (the RTS of Node \( U_1 \) is only received by BS).

(b) A CTS transmission from BS with the up-links information for simultaneous data transmissions (Node \( U_4 \) is randomly selected as an additional up-link).

(c) Two simultaneous data transmissions from Nodes \( U_1 \) and \( U_4 \) and successful MPR of BS by noticing an ACK transmission.

Figure 6.4: An example of the extended RTS/CTS mechanism when coexisting up-link nodes and direct-link nodes in a wireless network. The base station (BS) has the MPR capability of receiving two simultaneous packets while each node does not.
up-link nodes to establish the \( k \) requested data transmissions. Moreover, the base station randomly selects additional \( \alpha - k \) data transmissions to use the MPR capability fully. The CTS packet includes the additional transmission information. Hence, the number of the established data transmissions becomes \( \alpha \), the MPR capability.

### 6.2.4 An example of the extended RTS/CTS mechanism for the up-link MPR nodes.

Figure 6.4 illustrates an example of the extended RTS/CTS mechanism for MPR nodes when non-MPR nodes that use the conventional RTS/CTS mechanism coexist in a wireless network. We assume that the base station can receive two packets transmitted simultaneously (i.e., \( \alpha = 2 \)). As shown in Figure 6.4(a), when there are two simultaneous RTS transmissions, the base station successfully detects a RTS packet from a up-link node because of the MPR capability, while the direct-link node who receives the other RTS packet does not as it has no MPR capability. After receiving the RTS packet, the base station broadcast a CTS packet that includes the up-links information as shown in Figure 6.4(b). Based on the extended RTS/CTS mechanism with \( \alpha = 2 \), the base station allows an up-link transmission requested by the RTS packet and the other up-link transmission by an additional selection rule. Then, two up-link transmissions are established followed by an ACK packet that confirms the successful transmissions as shown in Figure 6.4(c).

### 6.3 Throughput Analysis

In this section, we analyze the system throughput when the up-link transmissions and the direct-link transmissions co-exist.

#### 6.3.1 Throughput

The system throughput, denoted by \( S \), is defined as

\[
S = \frac{\text{Average payload transmitted in a slot time}}{\text{Average length of a slot time}}.
\]  
(6.1)
If there are no collisions in the system, the maximum throughput of the up-link transmissions with $\alpha$ MPR capability is $\frac{\alpha L}{T_s}$, where $L$ is the amount of payload and $T_s$ is the successful transmission time. For the direct-link transmissions using no MPR capability, the maximum throughput is $\frac{L}{T_s}$.

If there are collisions, we must take into account the idle, collision, and successful transmission periods when expressing the throughput. Thus, the throughput is

$$S = \frac{E[\text{pload}]}{E[\text{idle}] + E[\text{coll}] + E[\text{succ}]}$$

where $E[\text{pload}]$ is the average amount of payload successfully transmitted, $E[\text{idle}]$, $E[\text{coll}]$, and $E[\text{succ}]$ are the average idle, collision, and successful transmission periods, respectively.

### 6.3.2 Throughput of up-link and direct-link transmissions

In this subsection, we derive $E[\text{idle}]$, $E[\text{coll}]$, $E[\text{succ}]$ and $E[\text{pload}]$ for the system throughput.

Consider a network with $M$ number of MPR nodes and $N - M$ number of non-MPR nodes. The base station is equipped with $\alpha$ antennas. The network is under (i) the saturated-traffic condition in which each node always has a packet available for transmission and (ii) the ideal channel condition where a transmission failure only occurs at a packet collision.

Let $p^d_t$ and $p^u_t$ denote the packet transmission probabilities in a randomly chosen slot time for the direct-link (or non-MPR) nodes and the up-link (or MPR) nodes, respectively. The probability that there is at least one packet transmission among $N$ nodes in a randomly selected slot time, denoted by $P_{tr}$, is

$$P_{tr} = 1 - (1 - p^u_t)^M (1 - p^d_t)^{(N-M)}.$$  

The probability that a single direct-link can be established successfully, $P_{tr}^{d[1]|N}$, is

$$P_{tr}^{d[1]|N} = (N-M) (1-p^u_t)^M (p^d_t)^{(N-M-1)}.$$  

The conditional probability that a direct-link is successfully established under the assumption that there is at least one transmission, denoted by $P^d_s$, is

$$P^d_s = \frac{P_{tr}^{d[1]|N}}{P_{tr}}.$$
Let $P_{tr}^{u|k|N}$ denote the probability that up-links can be established successfully when $k$ packets are simultaneously transmitted by $M$ up-link nodes and $N - M$ direct-link nodes. It can be shown that

$$P_{tr}^{u|k|N} = \sum_{i=1}^{k} \binom{M}{i} \binom{N - M - i}{k - i} (p^u_i)^i (1 - p^u_i)^{M-i} \times (p^d_i)^{k-i} (1 - p^d_i)^{N-M+k-i}.$$  \quad (6.6)

Since the MPR capability can detect $\alpha$ simultaneous packets, the multiple up-links can be successfully established if the number of total transmitted packets (i.e., the $k$ packets) does not exceed the MPR capability of the $\alpha$ packets. By taking into account the MPR capability with $P_{tr}^{u|k|N}$, the conditional probability that multiple up-links are successfully established with at least one transmission, denoted by $P^u$, is given by

$$P^u = P_{tr}^{u|1|N} + P_{tr}^{u|2|N} + \cdots + P_{tr}^{u|\alpha|N} \div P_{tr} = \sum_{k=1}^{\alpha} \frac{P_{tr}^{u|k|N}}{P_{tr}}.$$  \quad (6.7)

With $P_{tr}$, $P^d$ and $P^u$, we can obtain the probabilities of idle, collision and successful transmission as

$$P_{idle} = (1 - P_{tr}),$$
$$P_{coll} = P_{tr}(1 - P^d - P^u),$$
$$P_{succ} = P_{tr}(P^d + P^u).$$  \quad (6.8)

Hence, the average idle, collision, and successful transmission periods are given by

$$E[idle] = P_{idle}\sigma = (1 - P_{tr})\sigma,$$
$$E[coll] = P_{coll}T_c = P_{tr}(1 - P^d - P^u)T_c,$$
$$E[succ] = P_{succ}T_s = P_{tr}(P^d + P^u)T_s,$$  \quad (6.9)

where $\sigma$ is the empty slot time and $T_c$ is the collision time. The successful transmission time and the collision time of the RTS/CTS mechanism in IEEE 802.11 standard are obtained from

$$T_s = \text{RTS} + \text{SIFS} + \delta + \text{CTS} + \text{SIFS} + \delta + H + T_p + \text{SIFS} + \delta + \text{ACK} + \text{DIFS} + \delta.$$
\[ T_c = \text{RTS} + \text{DIFS} + \delta, \quad (6.10) \]

where \( \delta \) is the propagation delay, \( H \) is the total overhead time to transmit the packet headers of physical and MAC layers, and \( T_p \) is the transmission time of the payload.

The average amount of payload successfully transmitted in a slot time is

\[
E[p\text{load}] = P_{tr}P^d_s L + P_{tr}P^u_s \alpha L \\
= P_{tr}(P^d_s + \alpha P^u_s) L.
\quad (6.11)
\]

The payload of each transmission of a direct-link is \( L \). For the up-links, the payload is \( \alpha L \).

Since the extended RTS/CTS mechanism can allocate additional data transmissions if the number of RTS packets does not exceed the MPR capability, the mechanism provides a fixed number of data transmissions, \( \alpha \). Thus, the average amount of payload for the direct-links is \( P_{tr}P^d_s L \) and that for the up-links is \( P_{tr}P^u_s \alpha L \).

Applying (6.9) and (6.11) into (6.2), the system throughput of up-link and direct-link transmissions is

\[
S = \frac{P_{tr}(P^d_s + \alpha P^u_s) L}{(1 - P_{tr}) \sigma + P_{tr}(P^d_s + P^u_s) T_s + P_{tr}(1 - P^d_s - P^u_s) T_c}.
\quad (6.12)
\]

The throughput in (6.12) can be calculated from the transmission probabilities, \( p^d \) and \( p^u \).

### 6.3.3 Throughput calculation with contention window sizes.

The contention window size strongly affects the transmission probability when the back-off rule of the RTS/CTS mechanism is used for the packet transmission. The backoff rule can be represented using a Markov chain model for the contention window size [101]. The Markov model allows the transmission probability to be expressed in terms of its contention window size and conditional collision probability. Appendix B shows the throughput analysis based on the Markov model of the backoff rule in IEEE 802.11.

\footnote{If the number of up-links is not fixed such as in the existing MPR mechanism [66, 67], the payload of up-links is \( kL \) when \( k \) RTS packets are successfully transmitted, \( 1 \leq k \leq \alpha \). This is because the number of up-links only depends on the number of successful RTS packets. Hence, the average amount of payload for the up-links is \( P_{tr} \sum_{k=1}^{\alpha} kP^u_s L \), then we obtain the result of (16) in [66].}
Let \( W^d \) denote the contention window size of direct-links and \( W^u \) denote that of up-links. Since the extended RTS/CTS mechanism also adopts the backoff rule, the transmission probabilities, \( p_{t}^d \) and \( p_{t}^u \), are expressed as follows

\[
p_{t}^d = \frac{2(1 - 2p_{c}^d)}{(1 - 2p_{c}^d)(W^d + 1) + p_{c}^d W^d (1 - (2p_{c}^d)^m)}
\]

and

\[
p_{t}^u = \frac{2(1 - 2p_{c}^u)}{(1 - 2p_{c}^u)(W^u + 1) + p_{c}^u W^u (1 - (2p_{c}^u)^m)}
\]

where \( m \) is the maximum backoff stage, \( p_{c}^d \) and \( p_{c}^u \) are the conditional collision probabilities of direct-links and up-links, respectively.

To find the values of \( p_{t}^d \) and \( p_{t}^u \), we need additional equations which are related with the unknown variables, \( p_{c}^d \) and \( p_{c}^u \). The following equations of the conditional collision probabilities provide a sufficient condition for finding the solution of \( p_{t}^d \) and \( p_{t}^u \).

The conditional collision probability, \( p_{c}^d \), is the probability that at least one of the \( N - 1 \) remaining nodes transmits as follows

\[
p_{c}^d = 1 - P_{tr}^{d[0](N-1)} = 1 - (1 - p_{t}^u)^M (1 - p_{t}^d)^{(N-M-1)},
\]

where \( P_{tr}^{d[0](N-1)} \) is the probability that no packet is transmitted among the \( N - 1 \) remaining nodes.

The conditional collision probability, \( p_{c}^u \), is the probability where the transmitted packet collides if there are more than \( \alpha \) packets transmitted. It follows that

\[
p_{c}^u = 1 - P_{tr}^{u[0](N-1)} - P_{tr}^{u[1](N-1)} - \ldots - P_{tr}^{u[\alpha-1](N-1)}
\]

\[
= 1 - \sum_{k=0}^{\alpha-1} P_{tr}^{u[k](N-1)} = 1 - \sum_{k=0}^{\alpha-1} \sum_{i=0}^{k} \binom{M - 1}{i} \binom{N - M}{k - i} (p_{t}^u)^{i} \times (1 - p_{t}^u)^{(M-1-i)(p_{t}^d)^{k+i}(1 - p_{t}^d)^{N-M-k+i}},
\]

where \( P_{tr}^{u[k](N-1)} \) is the probability that the up-links can be established successfully while the \( N - 1 \) remaining nodes simultaneously transmit \( k \) (\( \leq \alpha - 1 \)) packets.

Since this set of four equations represents a nonlinear system with four unknown variables, no closed-form expression for the solution exists. Thus, numerical techniques
are used to find the solution. In Appendix C.1, we show that the solution of $p_{tr}^d$, $p_{tr}^u$, $p_{c}^d$, and $p_{c}^u$ is unique.

Using the transmission probabilities, we can obtain the probabilities, $P_{tr}$, $P_{sd}$, and $P_{su}$, by substituting $p_{tr}^d$ and $p_{tr}^u$ into (6.3), (6.5), and (6.7). The throughput of (6.12) then yields with $E[\text{idle}]$, $E[\text{coll}]$, and $E[\text{succ}]$ in (6.9), and $E[\text{pload}]$ in (6.11) using $P_{tr}$, $P_{sd}$, and $P_{su}$.

### 6.3.4 Throughput optimisation

The extended RTC/CTS MAC protocol provides an additional mechanism that fully exploits the MPR capability to achieve better system throughput. In order to optimise the system throughput, we consider the optimisation problem of finding the contention window sizes to

$$\text{maximise} \quad f(W^d, W^u)$$

$$\text{subject to} \quad W^d, W^u \in \mathbf{W},$$

(6.17)

where $f$ is the objective function and $\mathbf{W}$ is the set of contention window sizes which can be $\{2, 4, 8, \ldots, 1024\}$ depending on the binary backoff rule.

We use

$$f = S(W^d, W^u) - D(W^d, W^u),$$

(6.18)

where $S(W^d, W^u)$ is the system throughput in terms of the contention window sizes, and $D(W^d, W^u)$ is the throughput difference between the obtained throughput and the throughput requirement. The $S(W^d, W^u)$ term is used to achieve the maximum system throughput and the $D(W^d, W^u)$ is used to achieve certain throughput specified by the application needs. In this study, we are interested in achieving fairness in throughput for both the MPR nodes and the non-MPR nodes, thus, we define the throughput difference as

$$D(W^d, W^u) = |S^u(W^d, W^u) - \lambda S(W^d, W^u)|,$$

(6.19)

where $S^u(W^d, W^u)$ is the throughput of up-links and $\lambda$ is a ratio of the throughput requirement for up-links (i.e., $0 \leq \lambda \leq 1$). From (6.12), the throughput of up-links is obtained as

$$S^u = \frac{P_{tr}P_{su}^u\alpha L}{(1 - P_{tr})\sigma + P_{tr}(P_{sd} + P_{su}^u)T_s + P_{tr}(1 - P_{sd} - P_{su}^u)T_c}.$$
By adjusting $\lambda$, the objective function can be used for specific applications which have different system requirements. For example, if some applications need 70% of the system throughput for up-links (or 30% of system throughput for direct-links), then set $\lambda$ to 0.7.

We propose the following algorithm to solve the optimisation problem in (6.17) with the associated cost functions (6.18) and (6.19) to obtain the required contention window sizes. Let $(W^{ds}, W^{*u})$ be the optimal solution, and $f^*$ be the associated objective function value.

**Algorithm 1.**

1) Set $(W^{ds}, W^{*u}) = (2, 2)$ as a default solution from $W$ and $f^* = 0$ as a default value.

2) Given $W^d$ and $W^u$, find $p_{dt}^d$ and $p_{ut}^u$ by solving (6.13), (6.14), (6.15), and (6.16). Calculate $S$ from (6.12) and $S^u$ from (6.20).

3) Calculate $f$ from (6.18) and (6.19). Set $f^* = f$ and $(W^{ds}, W^{*u}) = (W^d, W^u)$, if $f > f^*$; otherwise, make no change to $f^*$ and $(W^{ds}, W^{*u})$.

4) Repeat from 2) to 4) until all the set of $W^d$ and $W^u$ from $W$ are compared.

Note that with the analytical model of (6.12), the maximum achievable throughput can also be obtained by taking the derivatives of $S(p_{dt}^d, p_{ut}^u)$ with respect to two variables, $p_{dt}^d$ and $p_{ut}^u$. The derivatives give two equations with unknown variables $(p_{dt}^d, p_{ut}^u)$. The solution of the optimal transmission probabilities, $(p_{dt}^{ds}, p_{ut}^{*u})$, can be found using numerical techniques. Appendix C.2 shows the throughput maximisation of (6.12).

### 6.4 Numerical and Simulation Results

This section presents analytical and simulation results of the throughput performance of the proposed MPR mechanism with different MPR capabilities, network sizes, and contention window sizes.

The results have been obtained under the saturated-traffic and ideal channel conditions. The parameters we used follow the IEEE 802.11g MAC specification as shown in Table 6.1 assuming that all the packets have constant payload length. The conventional
Table 6.1: System parameters in IEEE 802.11g MAC specification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Packet payload, ( L )</td>
<td>8184 bits</td>
</tr>
<tr>
<td>MAC header</td>
<td>272 bits</td>
</tr>
<tr>
<td>PHY overhead</td>
<td>26 ( \mu s )</td>
</tr>
<tr>
<td>RTS packet</td>
<td>160 bits + PHY overhead</td>
</tr>
<tr>
<td>CTS packet</td>
<td>112 bits + PHY overhead</td>
</tr>
<tr>
<td>ACK packet</td>
<td>112 bits + PHY overhead</td>
</tr>
<tr>
<td>DIFS</td>
<td>28( \mu s )</td>
</tr>
<tr>
<td>SIFS</td>
<td>10( \mu s )</td>
</tr>
<tr>
<td>Slot time, ( \sigma )</td>
<td>9 ( \mu s )</td>
</tr>
<tr>
<td>Propagation delay, ( \delta )</td>
<td>1( \mu s )</td>
</tr>
<tr>
<td>Data rate</td>
<td>54 Mbps</td>
</tr>
<tr>
<td>Basic rate</td>
<td>6 Mbps</td>
</tr>
<tr>
<td>Contention window, ( W )</td>
<td>2, 4, 8, ..., 1024</td>
</tr>
</tbody>
</table>

and extended RTS/CTS mechanisms discussed in Sections 6.2.2 and 6.2.3, respectively, have been implemented as an event-driven simulation using the Java programming language. Each simulation result is an average of 50 runs in which each run has 100,000 time slots.

### 6.4.1 Impacts of different MPR capabilities.

We present the impacts of different MPR capabilities on the transmission probability and the throughput performance when the direct-links and up-links have the same size of contention window. The network has 20 nodes and the number of up-links is 5 (i.e., \( N = 20 \) and \( M = 5 \)).

The transmission probabilities of direct-links and up-links, \( p^d_t \) and \( p^u_t \), are important to evaluate the throughput based on the throughput analysis in Section 6.3.3. We show analytical results of \( p^d_t \) and \( p^u_t \) with different MPR capabilities, \( \alpha \), in Figure 6.5. As shown in (6.13) and (6.14), \( p^d_t \) and \( p^u_t \) decrease with the size of the contention window. When the MPR capability increases, the up-links achieve the higher transmission probability and the direct-links have the slightly lower transmission probability. From the results of trans-
mission probabilities, we can expect that the system throughput will be improved because more up-links can be transmitted according to the MPR capability. Meanwhile, the MPR capability of up-links may reduce the transmissions of direct-links since the up-links and the direct-links co-exist. Thus, the impact of the MPR capability on the transmission probabilities may cause different throughput performances of the direct-links and the up-links.

The system throughput can be calculated with the obtained transmission probabilities as discussed in Section 6.3.2. Figure 6.6 presents analytical and simulated throughput curves with different $\alpha$. The throughput results show the throughput improvement with the MPR capability because of the higher transmission probability of up-links as we expected in Figure 6.5. The results also show that the maximum throughput can be obtained with relatively smaller contention window size when the MPR capability increases. Since the transmission probability significantly increases with the small contention window size, the throughput can be maximised. For example, when $\alpha = 1$ (i.e., no MPR capability), the maximum throughput can be achieved around the contention window size $W = 32$, which is also the initial size of contention window according to IEEE 802.11
standard. On the other hand, with the higher MPR capability, the maximum throughput can be obtained with relatively smaller contention window sizes such as 4 or 8. Note that the simulation results verify the analytical results.  

In Fig 6.7, we show analytical throughput comparison of the proposed and the existing MPR mechanisms [66, 67] with different $\alpha$. Since the proposed MPR mechanism always allows the fixed number of up-links up to the maximum MPR capability, it is clear that as the MPR capability increases, the proposed MPR mechanism achieves better throughput than the existing MPR mechanism which has various up-links. Meanwhile, since both the MPR mechanisms adopt the RTS/CTS mechanism, their throughput results have the similar trends in terms of the contention window size with various $\alpha$. We thus only consider the proposed MPR mechanism which improves throughput when coexisting with MPR nodes and non-MPR nodes.

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4Since the analytical model assumes the steady state of transmissions and uses the approximated transmission probabilities that may be not the same as in simulation, this may cause minor differences between the analytical results and simulation results.
6.4.2 Impacts of different network sizes.

The impacts of different network sizes on the system throughput are presented when the direct-links and up-links have the same contention window size, \( W \) with \( \alpha = 2 \).

In Figure 6.8, we show analytical and simulated throughput curves with different \( N \). It is clear that the system throughput decreases with the number of nodes as there are more collisions for a larger number of nodes. In addition, because of the MPR capability, the maximum throughput is achieved when \( W = 8 \), which is smaller than the conventional size \( W = 32 \) given in the IEEE 802.11 standard. Again, the simulation results verify the analytical results in Figure 6.8.

Analytical and simulated throughput curves with different \( M \) are presented in Figure 6.9. When the number of the up-links increases, the system throughput is improved because given a fixed number of the nodes (\( N = 20 \)), more up-links rather than more direct-links can increase successful transmissions. It is also shown that the optimal size of the contention window increases with \( M \) for the maximum throughput. More up-links cause frequent collisions when the contention window size becomes smaller. This is why the maximum throughput is not obtained with smaller contention window size. For exam-
ple, when $M = 0$, the throughput becomes the maximum with $W = 32$ because the MPR capability is not used with no up-links. As $M$ increases, the system throughput becomes higher and the maximum throughput is obtained with larger contention window sizes such as $W = 4$ for $M = 2$, $4$ and $W = 8$ for $M = 6$.

### 6.4.3 Impacts of different contention window sizes.

We present the impacts of the different contention window sizes, $W^d$ and $W^u$, on the system throughput when $\alpha = 2$, $N = 20$, and $M = 5$.

In Figure 6.10, we show analytical throughput curves in terms of $W^u$ with different $W^d$. When $W^u$ increases, the system throughput is mainly improved up to a certain size of $W^u$ because the successful transmissions increases with relatively low idle time. Then, the system throughput gradually decreases with larger $W^u$ that causes longer idle time. The maximum throughput is obtained when $W^u = 8$ and $W^d = 1024$ since the up-links maximally exploit the MPR capability with the smaller $W^u$. Intuitively, we can see that the direct-links have very lower throughput performance than the up-links with the largest $W^d$. 
Figure 6.9: Throughput with different $M$ when $N = 20$.

Figure 6.10: Throughput in terms of $W^u$. 
From the view point of $W^d$, the system throughput is plotted in Figure 6.11. The throughput is improved when $W^d$ increases. This implies that with larger $W^d$, the direct-links have longer idle time and less transmissions. Thus, the up-links have more chances of being successfully transmitted. The successful transmissions of up-links improve the throughput by fully taking advantage of the MPR capability. However, unfair transmissions of the direct-links become critical although the throughput is significantly improved. Analytical and simulation results of the throughput are also presented with respect to various $W^d$ and $W^u$ in Figure 6.12. In general, it is shown that the analytical results in Figure 6.10 and Figure 6.11 agree with the simulation results in Figure 6.12.

### 6.4.4 Optimal throughput performance

In order to find an optimal contention window size that maximises the system throughput with the minimum throughput difference, the throughput performances of direct-links and up-links are presented in Figure 6.13 based on the previous analytical results in Figure 6.12. For the throughput optimisation, we assume that the throughput requirement of up-links is 50% of the system throughput (i.e., $\lambda = 0.5$).
Figure 6.12: Throughput with various $W^d$ and $W^u$ (solid lines represent analytical results, while bars with dots represent simulation results).

Figure 6.13(a) shows the throughput performances of direct-links and up-links in terms of the same contention window size (i.e., $W^d = W^u$). It is shown that the maximum throughput is achieved with the contention window size of 8. As the up-links have the advantages of the MPR capabilities for transmissions, they have much higher throughput than the direct-links. The throughput difference between the direct-links and the up-links becomes significant. This is not desirable for the direct-links in terms of the fair throughput performance that satisfies the throughput requirement even though the system throughput is maximised.

The contention window sizes that minimise the throughput difference in terms of $W^d$ are shown in Figure 6.13(b). The optimal solution obtained by using Algorithm 1 is $(W^d^*, W^u^*) = (32, 64)$ with the maximum throughput (24.7 Mbps) and the throughput difference (0.44). Unlike the solutions of only maximising the system throughput, the optimal solution achieves the throughput fairness which minimises the throughput difference between up-links and direct-links by increasing $W^u$ given a fixed $W^d$ (i.e., $W^u \geq W^d$) because the larger $W^u$ reduces the throughput of up-links improving the throughput of direct-links. Since the system throughput decreases as the throughput difference is min-
(a) Throughput results when the direct-links and up-links use the same contention window sizes.

(b) Throughput results that have the smallest throughput differences.

Figure 6.13: Throughput performances of direct-links and up-links and their throughput fairness.
Table 6.2: The values of $W^d$ and $W^u$ for the maximum throughput ($S_{max}$) and the optimal throughput ($S_{opt}$).

<table>
<thead>
<tr>
<th>System throughput</th>
<th>Throughput difference</th>
<th>Contention window size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{max}$ = 38.26</td>
<td>$S^d = 0.04$</td>
<td>$W^d = 1024$</td>
</tr>
<tr>
<td></td>
<td>$S^u = 38.22$</td>
<td>$W^u = 8$</td>
</tr>
<tr>
<td>$S_{opt}$ = 24.74</td>
<td>$S^d = 12.15$</td>
<td>$W^d = 32$</td>
</tr>
<tr>
<td></td>
<td>$S^u = 12.59$</td>
<td>$W^u = 64$</td>
</tr>
</tbody>
</table>

imised, the relationship of maximising the system throughput and improving the throughput fairness is a tradeoff.

Table 6.2 summaries the values of $W^d$ and $W^u$ for the maximum throughput and the optimal throughput. The maximum system throughput is obtained at $(W^d, W^u) = (1024, 8)$, but we can see unfair throughput performances between the direct-links and the up-links. On the other hand, the optimal throughput occurs at $(W^d, W^u) = (32, 64)$ with the minimum throughput difference. Although the optimal throughput decreases to 35% of the maximum throughput, it reduces the throughput difference to 98.8% of the maximum throughput.

### 6.5 Conclusion

We have extended the RTS/CTS mechanism to improve the system throughput by making use of the MPR capability, and to allow the co-existence of MPR and non-MPR nodes in a wireless network. In order to optimise the throughput and its fairness, we have derived the throughput analytically by taking into account the co-existence of up-links and direct-links of different MPR capability. For the throughput analysis, the new collision model was considered in which simultaneous multiple packet transmissions are allowed in a given slot time. Numerical and simulation results showed that when the number of antennas and the number of up-links increase, the proposed mechanism can improve the throughput because the MPR capability is fully utilised for up-link transmissions. In addition, it was shown that by optimally adjusting the contention window sizes of direct-
links and up-links, the maximum throughput is obtained with the minimum throughput difference between direct-links and up-links for the throughput fairness.
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Chapter 7

Conclusion and Future Work

7.1 Summary

This thesis presents a study of the performance of various random access schemes with advanced multichannel usage and multipacket reception capabilities in wireless networks. We have addressed several random access problems associated with imperfect channel conditions over multiple channels and packet collisions with multipacket reception capability. As described in Chapter 1, we have focussed our work on the divisions of multichannel random access that exploits time-varying channel conditions and MPR random access that support coexistence of MPR-enable and non-MPR nodes. We now summarise our key contributions:

In Chapter 3, we proposed the outage-aware access protocols to improve system throughput under multichannel outage environments. In the frequency-domain backoff protocols, the refined outage-aware channel set was used for random access, while the lowest-outage increasing heuristic was applied in the time-domain backoff access protocols for channel allocation. We also studied throughput performance with the game-theoretic update algorithms which provides the transmission update strategies based on the basic update of the persistence slotted Aloha. Our findings showed that the outage-aware access protocols outperform the conventional Aloha protocols with the refined channel set, the outage-aware heuristic, and the game-theoretic update algorithms.

Chapter 4 presented the random backoff access scheme that performed efficiently with imperfect network information for multichannel wireless networks. By recognising the
sensing and access as the partially observable Markov decision process problem, the proposed backoff access jointly performed with the myopic sensing as the suboptimal solutions for the POMDP. Our findings showed that the proposed backoff access scheme achieves better throughput than other access schemes that simply apply the channel sensing policy.

In Chapter 5, a joint random access scheme was proposed in which the hopping and backoff access methods were selectively used for multichannel wireless networks. Under time-varying channel conditions, by cooperating with the myopic sensing method, the proposed joint access scheme achieved a system throughput that cannot be obtained by the hopping access or the backoff access methods alone. In order to obtain the closed-form throughput expression, we analysed the throughput performance with the Markov models of the access states and the channel states. The performance of the proposed joint access scheme was verified through numerical and simulation results, showing its superiority in throughput performance compared with the single usage of the hopping access or the backoff access methods.

In Chapter 6, by taking into account the multipacket reception capability, we extended the RTS/CTS mechanism to improve the system throughput. The extended MAC protocol is suitable when MPR and non-MPR nodes co-exist in a wireless network. The throughput expression was derived analytically with the new collision model in which simultaneous multiple packet transmissions are allowed in a given slot time. Our outcomes showed that the proposed protocol can improve the throughput because the MPR capability is fully utilised for up-link transmissions. In addition, it was shown that by optimally adjusting the contention window sizes of direct-links and up-links, the maximum throughput is obtained with the minimum throughput difference between direct-links and up-links for the throughput fairness.

7.2 Future Research

Our studies in this thesis will lay the foundation for future work on optimising access control of wireless networks. The future research that extends the proposed schemes of the previous chapters will take the following directions: (i) cross-layer probabilistic
modelling and (ii) stochastic network utility maximisation.

Given the random access schemes developed in this thesis, optimising interactions of control variables at different communication layers will be prominent to support ubiquitous access and high data transfer rates. However, such cross-layer optimisation that can achieve different performance objectives is nontrivial in wireless systems.

In the future research, the cross-layer optimisation will be studied by applying efficient probabilistic models into an optimisation framework. First, we will develop probabilistic models that can characterise the interactions of cross-layer control variables and the time-varying environments. We will then incorporate the probabilistic models to advance optimisation methods in which stochastic methods are integrated under a network utility maximisation framework. This integrated approach will provide optimal control strategies with low computational complexity.

### 7.2.1 Cross-layer probabilistic models

Efficient cross-layer models need to capture the interaction of different layer parameters collectively and the time-varying nature of communication environments. Understanding the interactive behaviours and the dynamic environments is critical to accomplishing various objectives of communication systems. These objectives include overall throughput maximisation, power optimisation, delay minimisation, and network stability.

A number of cross-layer modelling approaches have been proposed and studied in \([8, 93, 106–108]\). However, their parameter relationships are still complex and not well understood. In the future research, new cross-layer models that interplay between control parameters at different layers will be developed and tested by using state estimation techniques that only need partial information. The state estimation techniques can be applied for estimating channel conditions, access probability, network connectivity, and traffic patterns. Through modelling and experimentation, we will further derive optimal control strategies under a decision-making framework that incorporates the cross-layer interactions and the environmental uncertainty.
7.2.2 Stochastic network utility maximisation

For a cross-layer control, a network utility maximisation framework is a promising control framework. It has received great research attention because of its innovative idea on network resource allocation since 1997 [93]. The innovative idea is that an optimisation problem of resource allocation is formulated globally, and it is solved locally by decomposing the global problem into smaller subproblems. The subproblems can be solved with local variables based on local state observations. This approach has a substantial set of theory, algorithms and applications, and it is suitable for solving our cross-layer optimisation problems that deal with diverse network requirements. Appendix D summaries the network utility maximisation.

Since the optimisation problems are related with the time-varying environments, a stochastic approach can be applicable to the network utility maximisation framework with feedback system models as in control theory, and pricing-based supply-demand models as in economics theory [109]. Although there have been developments in stochastic network utility maximisation, there are still many open problems and unexplored topics with respect to cross-layer interactions and time-varying network natures. In future research, the probabilistic models will be effectively incorporated with this stochastic optimisation framework. The work on decision feedback and adaptive control [110–112] will further contribute to the decision-theoretic concept of this work.

The integration of the network utility maximisation and the stochastic optimisation will lead us to further study on how to adopt relaxation techniques to allow hybrid optimisation techniques, such as statistical learning, iterative dynamic programming, and optimisation heuristics, on different parts of the objective and constraints that suit the conditions. The work is challenging, and we will make use of extensive optimisation techniques in stochastic optimisation, min-max, multi-objectives, and imprecise computation [113–116].
7.2.3 Remark

Future research will advance the knowledge base through providing new results in feedback control from the physical to the upper layer algorithms in time-varying network environments. The research will explore the strengths of cross-layer designs to provide flexible solutions with optimal control strategies, minimal computation effort, and efficient verifications for a huge diversity of applications. Intelligent interplay between control parameters will provide the reliability and efficiency of network systems.

Future research will benefit many wireless applications through adaptive implementation. These applications range from sensor networks to local area networks, cellular and satellite networks: sensor monitoring and surveillance that need to support swift data information under harsh environments and uncertain transmission conditions; reliable data transmission during user mobility that possibly cause heterogeneous link configuration between users; and uplink access for requesting quality of service among cellular users under time-varying multichannel environments in OFDMA networks.
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Appendix A

Knapsack Problems

A knapsack problem, also called a bin-packing problem, is to fill up a knapsack by selecting various possible objects that will give maximum comfort [84]. It can be mathematically formulated as a constrained optimisation problem as follows:

\[
\text{maximise } \sum_{j=1}^{n} p_j x_j \\
\text{subject to } \sum_{j=1}^{n} w_j x_j \leq c, \tag{A.0.1}
\]

where \(x_j\) is a vector of binary variables having the following meaning: \(x_j = 1\) if object \(j\) is selected; Otherwise, \(x_j = 0\). \(p_j\) is a measure of the comfort given by object \(j\), \(w_j\) is the size of the object \(j\), \(c\) is the size of the knapsack.

This knapsack problem can be considered as an investment problem in which a capital of \(c\) dollars is distributed over \(n\) possible investments, with the profit from investment \(j\) \((p_j\) ) and the amount of dollars \((w_j)\). The optimal solution of this problem indicates the best possible choice of investments.

This type of problems including bounded knapsack, multiple knapsack, generalised assignment, and bin-packing is known to be NP-hard [117]. For such problems, no pseudo-polynomial algorithm can exist, unless \(P = NP\), since the problems can be proved to be NP-hard in the strong sense.

A solution approach is to examine all possible binary vector \(x\), selecting the best of those which satisfy the constraint. Unfortunately, the number of such vector is \(2^n\), so even a hypothetical computer, capable of examining one billion vectors per second, would
require more than 30 years for \( n = 60 \), more than 60 years for \( n = 61 \), ten centuries for \( n = 65 \), and so on. However, specialised algorithms can, in most case, solve a problem with \( n = 100,000 \) in a few seconds on a personal computer.

The following two approaches can be considered:

(i) enumerative algorithms (having, in the worst case, running times which grow exponentially with the input size) to determine optimal solutions;

(ii) approximate algorithms (with running times bounded by a polynomial in the input size) to determine feasible solutions whose value is a lower bound on the optimal solution value;

The average running times of such algorithms are experimentally evaluated through execution of the corresponding computer codes on different classes of randomly-generated test problems. It will be seen that the average behaviour of the enumerative algorithms is in many cases much better than the worst-case bound, allowing optimal solution of large-size problems with acceptable running times.

The performance of an approximate algorithm for a specific instance is measure through the ratio between the solution value found by the algorithm and the optimal solution value. Besides the experimental evaluation, it is useful to provide, when possible, a theoretical measure of performance through worst-case analysis [118].
Appendix B

A Markov Model Analysis for the Backoff Scheme in IEEE 802.11 DCF

We briefly summarise the exponential backoff scheme in IEEE 802.11 DCF and present the throughput analysis using its Markov model [101].

The exponential backoff procedure is described as follows: At each packet transmission, the backoff time is uniformly chosen in the range \((0, w-1)\). The value \(w\) is called contention window, and depends on the number of transmission failures. At the first transmission attempt, \(w\) is set equal to a value \(CW_{\text{min}}\) called minimum contention window. After each unsuccessful transmission, \(w\) is doubled, up to a maximum value \(CW_{\text{max}} = 2^m CW_{\text{min}}\), where \(m\) is the maximum backoff stage. The backoff time is decremented as long as a channel is sensed idle. It is frozen when the channel is used for a transmission, and reactivated when the channel is sensed idle again for more than a period of distributed interframe space (DIFS).

The system throughput is analysed by modelling the exponential backoff procedure. Define \(W = CW_{\text{min}}\). Then, we have \(CW_{\text{max}} = 2^m W\), and let us adopt the notation \(W_i = 2^i W\), where \(i \in (0, m)\) is called the backoff stage.

Let \(p\) denote the conditional collision probability. Once independence is assumed, and \(p\) is supposed to be a constant value. It is then possible to model the backoff procedure as a bidimensional process \(\{s(t), b(t)\}\), where \(s(t)\) represents the backoff stage \((0, ..., m)\), and \(b(t)\) represents the backoff time. A Markov chain model of the bidimensional process is depicted in Figure B.1.
The transmission probability \( \tau \) that a station transmits in a randomly chosen slot time is expressed as

\[
\tau = \frac{2(1 - 2p)}{(1 - 2p)(W + 1) + pW(1 - (2p)^m)}.
\]

The transmission probability is obtained by applying the following approaches: (i) find one-step transition probabilities from the Markov chain, (ii) obtain stationary probabilities of being at state \( \{s(t), b(t)\} \), and (iii) calculate \( \tau \) with an additional equation of the conditional collision probability.

Let \( P(i, k|v, h) = \Pr(s(t + 1) = i, b(t + 1) = k, s(t) = v, b(t) = h) \) denote the one-step transition probability from the state \( \{v, h\} \) at slot time \( t \) to the state \( \{i, j\} \) at the next slot time. From the Markov chain in Figure B.1, we directly have

\[
P(i, k|i, k + 1) = 1, \quad k \in (0, W_i - 2), i \in (0, m);
\]

\[
P(i, k|i - 1, 0) = p/W_i, \quad k \in (0, W_i - 1), i \in (0, m);
\]

\[
P(i, k|0, 0) = (1-p)/W_0, \quad k \in (0, W_0 - 1), i \in (0, m);
\]

\[
P(m, k|m, 0) = p/W_m, \quad k \in (0, W_m - 1).
\]

Let \( \pi_{i,k} \) denote the stationary probability of the state \( \{i, k\} \), i.e., \( \pi_{i,k} = \lim_{t \to \infty} \Pr(s(t) = i, b(t) = k) \). With a steady state equation \( \pi_{i,k} = \sum_{v=0}^{m} \sum_{h=0}^{W_v - 1} \pi_{v,h} P(i, k|v, h) \), we have

Figure B.1: A Markov chain model for the backoff window size.
the following relation:

\[ \pi_{i,0} = p^i \pi_{0,0}, \quad i \in (0, m); \]
\[ \pi_{m,0} = \frac{p^m}{1-p} \pi_{0,0}, \quad i = m; \]
\[ \pi_{i,k} = \frac{W_i-k}{W_i} \pi_{i,0}, \quad i \in (0, m), k \in (0, W_i - 1). \]  

(B.0.3)

From the backoff procedure, the transmission probability \( \tau \) is defined as

\[ \tau = \sum_{i=0}^{m} \pi_{i,0} \]
\[ = \sum_{i=0}^{m-1} p^i \pi_{0,0} + \frac{p^m}{1-p} \pi_{0,0} \]
\[ = \frac{1}{1-p}, \]  

(B.0.4)

where \( \pi_{0,0} \) is the stationary probability of the initial state. It can be calculated with a steady state equation \( \sum_{i}^{m} \sum_{k}^{W_i-1} \pi_{i,k} = 1 \).

The calculation of \( \tau \) requires an additional equation of the conditional collision probability,

\[ p = 1 - (1 - \tau)^{n-1}, \]  

(B.0.5)

where \( n \) is the number of nodes.

The system throughput of the IEEE 802.11 DCF is obtained by using \( \tau \) [101]. It is expressed as

\[ \frac{P_{tr} P_s E[P]}{(1 - P_{tr}) \sigma + P_{tr} P_s T_s + P_{tr} (1 - P_s) T_c}, \]  

(B.0.6)

where \( P_{tr} \) is the probability that there is at least one transmission in a slot time, and \( P_s \) is the probability of successful transmission, conditioned on the fact that at least one node transmits. Both \( P_{tr} \) and \( P_s \) are functions of \( \tau \):

\[ P_{tr} = 1 - (1 - \tau)^n, \]
\[ P_s = \frac{n \tau (1 - \tau)^{n-1}}{P_{tr}}. \]  

(B.0.7)

The other parameters are constant values depending on the IEEE 802.11 standard: \( \sigma \) is the slot time, \( T_s \) is the average time of successful transmission, \( T_c \) is the average time of unsuccessful transmission due to collision, and \( E[P] \) is the average packet payload size.
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Appendix C

Unique Solution and Optimisation

C.1 Existence of the unique solution

Unfortunately, the set of (6.13), (6.14), (6.15), and (6.16) represents a nonlinear system with the four unknown variables, $p_d^d$, $p_u^u$, $p_d^c$, and $p_u^c$, and, in general, closed form expressions of these variables are not available. Thus, we need to use numerical techniques to find them.

Let $g_1(p_c^d)$, $g_2(p_c^u)$, $g_3(p_d^d, p_u^d)$, and $g_4(p_d^d, p_u^d)$ denote $p_d^d$, $p_u^u$, $p_d^c$, and $p_u^c$, respectively. In [101], it is shown that $g_1(p_c^d)$ and $g_2(p_c^u)$ are continuous and monotone decreasing functions in the range $p_c^d, p_c^u \in (0, 1)$, that start from $g_1(0) = g_2(0) = 2/(W + 1)$ and decrease up to $g_1(1) = g_2(1) = 2/(1 + 2^mW)$. On the other hand, $g_3(p_d^d, p_u^d)$ and $g_4(p_d^d, p_u^d)$ are continuous and monotone increasing functions in the range of $p_d^d, p_u^d \in (0, 1)$ that start from $g_3(0, 0) = g_4(0, 0) = 0$ and increase up to $g_3(1, 1) = g_4(1, 1) = 1$. We can prove that the solution of the nonlinear system is unique since the first order conditions of $g_3$ and $g_4$ are all positive or 0 as $\frac{\partial g_3(p_d^d, p_u^d)}{\partial p_d^d}$, $\frac{\partial g_3(p_d^d, p_u^d)}{\partial p_u^d}$, $\frac{\partial g_4(p_d^d, p_u^d)}{\partial p_d^d}$, and $\frac{\partial g_4(p_d^d, p_u^d)}{\partial p_u^d} \geq 0$. 

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C.2 Optimal transmission probabilities for the maximum throughput

Let \( f_1, f_2, f_3, \) and \( f_4 \) denote \( E[\text{pload}], E[\text{idle}], E[\text{coll}], \) and \( E[\text{succ}] \), respectively, as functions of \( p^d_t \) and \( p^u_t \). Then, we have

\[
\begin{align*}
    f_1 &= \left( h_1(p^d_t, p^u_t) + \alpha h_2(p^d_t, p^u_t) \right) L, \\
    f_2 &= \left( 1 - h_3(p^d_t, p^u_t) \right) \sigma, \\
    f_3 &= \left( h_3(p^d_t, p^u_t) + h_2(p^d_t, p^u_t) \right) T_s, \\
    f_4 &= \left( h_3(p^d_t, p^u_t) - h_1(p^d_t, p^u_t) - h_2(p^d_t, p^u_t) \right) T_c, \\
\end{align*}
\]

(C.2.1)

where

\[
\begin{align*}
    h_1(p^d_t, p^u_t) &= P_{tr} P^d_s, \\
    h_2(p^d_t, p^u_t) &= P_{tr} P^u_s, \quad \text{and} \\
    h_3(p^d_t, p^u_t) &= P_{tr}. 
\end{align*}
\]

With (C.2.1), the throughput is rewritten as

\[
S(p^d_t, p^u_t) = \frac{E[\text{pload}]}{f_1} = \frac{E[\text{idle}] + E[\text{coll}] + E[\text{succ}]}{f_2 + f_3 + f_4}. 
\]

(C.2.2)

To obtain \( p^d_{t*} \) and \( p^u_{t*} \), first, take the derivative of \( S(p^d_t, p^u_t) \) with respect to \( p^d_t \) as follows

\[
\frac{\partial S(p^d_t, p^u_t)}{\partial p^d_t} = \frac{A}{B} = 0, 
\]

(C.2.3)

where

\[
\begin{align*}
    A &= \frac{\partial f_1}{\partial p^d_t} \left( f_2 + f_3 + f_4 \right) - \frac{\partial (f_2 + f_3 + f_4)}{\partial p^d_t} f_1, \\
    B &= \left( f_2 + f_3 + f_4 \right)^2. 
\end{align*}
\]

(C.2.4)

Then, we can obtain the following equation with two unknown variables:

\[
\frac{\partial f_1}{\partial p^d_t} \left( f_2 + f_3 + f_4 \right) - \frac{\partial (f_2 + f_3 + f_4)}{\partial p^d_t} f_1 = 0, 
\]

(C.2.5)

where

\[
\begin{align*}
    \frac{\partial f_1}{\partial p^d_t} &= \left( \frac{\partial h_1(p^d_t, p^u_t)}{\partial p^d_t} + \alpha \frac{\partial h_2(p^d_t, p^u_t)}{\partial p^d_t} \right) L, \\
    \frac{\partial f_2}{\partial p^d_t} &= \frac{\partial h_3(p^d_t, p^u_t)}{\partial p^d_t} \sigma. 
\end{align*}
\]
\[
\frac{\partial f_3}{\partial p_t^d} \left( \frac{\partial h_1(p_t^d, p_t^u)}{\partial p_t^d} + \frac{\partial h_2(p_t^d, p_t^u)}{\partial p_t^d} \right) T_s \\
\frac{\partial f_4}{\partial p_t^d} \left( \frac{\partial h_3(p_t^d, p_t^u)}{\partial p_t^d} - \frac{\partial h_1(p_t^d, p_t^u)}{\partial p_t^d} - \frac{\partial h_2(p_t^d, p_t^u)}{\partial p_t^d} \right) T_c.
\]

(C.2.6)

We can have the other derivative of \( S(p_t^d, p_t^u) \) with respect to \( p_t^u \) such as \( \frac{\partial f_1}{\partial p_t^u}(f_2 + f_3 + f_4) - \frac{\partial (f_2 + f_3 + f_4)}{\partial (p_t^u)} f_1 = 0 \). These two derivatives of \( S(p_t^d, p_t^u) \) provide two equations with unknown variables \((p_t^d, p_t^u)\). The solution of the two equations becomes \((p_t^{d*}, p_t^{u*})\). Using numerical techniques, we can find the solution\(^1\).

\(^1\)Although we are unable to show the existence of a unique solution by analysis, the solution seems unique based on our observation with a number of numerical examples.
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Appendix D

Network Utility Maximisation

Communication network protocols have been developed as in layered architectures. Recent efforts on such layered protocols are to analyse holistically and to design systematically for an overall communication network.

A network utility maximisation (NUM) framework provides a general method to model the overall communication network in the face of layered interactions between protocol stacks [93]. In the NUM framework, the network protocols are designed to solve a global optimisation problem by decomposing the global problem into smaller subproblems. Each decomposed subproblem corresponds to a different layering protocol architecture.

Using the NUM framework, many protocols in layers 2-4, i.e., medium access control (MAC), routing, and congestion control protocols, have been extensively studied. Typically, the optimisation framework has found many applications in the transport layer to allocate end-to-end flow rate through congestion control protocols [119–121]. Utility-optimal MAC protocols have been also designed through the framework in which contentions to access a communication link are adjusted [86, 122].

As cross-layer approaches, the optimisation framework has provided joint congestion control and routing in which the congestion control adjust each flow rate to uses all available bandwidth among competing flows, while routing determines which flows pass through which links. [107, 108, 123]. In addition, the MAC protocols have been jointly studied with the congestion control to ensure high system performance and to achieve a fair bandwidth usage among competing nodes and flows [124, 125]. Moreover, the
joint control of all MAC, routing, and congestion control has been considered in [126]. However, the existing joint control of all layers has not been fully developed when the network topology becomes clustered and different constraints may exist for intra-cluster and inter-cluster communications.
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