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Simple relation between the $\gamma N \rightarrow N(1535)$ helicity amplitudes

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It is shown that the helicity amplitudes $A_{1/2}$ and $S_{1/2}$ in the $\gamma N \rightarrow N(1535)$ reaction, can be well related by $S_{1/2} = -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_S^2 - M^2}{2M_S Q} A_{1/2}$ in the region $Q^2 > 2 \text{ GeV}^2$, where M and M_S are the nucleon and $N(1535)$ masses, $q^2 = -Q^2$ the four-momentum transfer squared, and $\tau = \frac{Q^2}{(M_S + M)^2}$. This follows from the fact that the Pauli-type transition form factor F_2^* extracted from the experimental data, turns up to show $F_2^* \approx 0$ for $Q^2 > 1.5 \text{ GeV}^2$. The observed relation is tested by the experimentally extracted helicity amplitudes and the MAID parametrization (Mainz group). A direct consequence of the relation is that the assumption $|A_{1/2}| \gg |S_{1/2}|$ is not valid for high Q^2 . Instead, both amplitudes $A_{1/2}$ and $S_{1/2}$ have the same Q^2 dependence in the high Q^2 region; aside from that $S_{1/2}$ has an extra factor, $-\frac{1}{\sqrt{2}} \frac{M_S - M}{2M_S}$. The origin of this relation is interpreted in a perspective of a quark model.

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The electroproduction of a spin 1/2 baryon resonance N^* on a nucleon ($\gamma^* N \rightarrow N^*$) is described by the two independent helicity amplitudes, $A_{1/2}$ and $S_{1/2}$, which depend on the initial and final state polarizations. While the helicity is conserved in the transverse amplitude $A_{1/2}$, it is changed by one unit in the longitudinal amplitude $S_{1/2}$. These amplitudes are frame dependent. For the transition between the nucleon state $|N, S_z = \pm \frac{1}{2}\rangle$ (mass M) and the spin-1/2 nucleon resonance state $|N^*, S'_z = \pm \frac{1}{2}\rangle$ (mass M_R), one can define the helicity amplitudes in the N^* rest frame in terms of the transition current J^μ and the photon polarization vector $\varepsilon_\mu^{(\lambda)}$ with $\lambda = 0$ (longitudinal) or $\lambda = \pm 1$ (transverse) [1]:

$$\begin{aligned} A_{1/2} &= \sqrt{\frac{2\pi\alpha}{K}} \left\langle N^*, S'_z = +\frac{1}{2} \left| \varepsilon^{(+)} \cdot J \right| N, S_z = -\frac{1}{2} \right\rangle, \\ S_{1/2} &= \sqrt{\frac{2\pi\alpha}{K}} \left\langle N^*, S'_z = +\frac{1}{2} \left| \varepsilon^{(0)} \cdot J \right| N, S_z = +\frac{1}{2} \right\rangle \frac{|\mathbf{q}|}{Q}, \end{aligned} \quad (1)$$

where α is the electromagnetic fine structure constant, and

$$K = \frac{M_R^2 - M^2}{2M_R}. \quad (2)$$

In the above $q^2 = -Q^2$ is the four-momentum transfer squared and \mathbf{q} the photon three-momentum in the N^* rest frame. For simplicity we write the current J^μ in units of $e = \sqrt{4\pi\alpha}$.

The helicity amplitudes in the N^* rest frame (1) are defined by the transition current J^μ , which can be defined in terms of the two independent Dirac-type (F_1^*) and Pauli-type (F_2^*) form factors, which are frame independent (covariant) and exclusive functions of Q^2 [the superscript (*) is introduced to indicate the final state is a nucleon excited state N^*].

Although the analysis of the nucleon system is usually made in terms of the covariant electromagnetic form factors [2,3], the data related with nucleon resonances are mostly represented in terms of the helicity amplitudes [4–6] with the exception for the reaction $\gamma N \rightarrow N(1440)$ [1,7].

In this work we study the $\gamma N \rightarrow N(1535)$ reaction based on the covariant form factor representation. The empirically extracted results for the transition form factors will lead to a new, simple, and important relation between the helicity amplitudes. In the literature the amplitude $S_{1/2}$ is generally neglected compared to $A_{1/2}$. It was only recently that both the $S_{1/2}$ and $A_{1/2}$ amplitudes were extracted simultaneously in the analysis of the cross section data. Presently, we have results for both amplitudes from a MAID analysis of old data [4], and recent results from CLAS (Jefferson Lab) [5].

The $N(1535)$ resonance is an S_{11} state with negative parity. The transition current for $\gamma N \rightarrow S_{11}$ can be represented [8] as

$$J^\mu = \bar{u}_S(P_+) \left[F_1^* \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2^* \frac{i\sigma^{\mu\nu} q_\nu}{M_S + M} \right] \gamma_5 u(P_-), \quad (3)$$

where the subscript S stands for the quantities associated with S_{11} . We note that there are several equivalent definitions for the transition current [1,9,10], but we choose the present one, due to the similarity with the nucleon and $N(1440)$ cases.

The transition form factors, F_1^* and F_2^* , extracted from the data for the $\gamma N \rightarrow N(1535)$ reaction, are shown in Fig. 1. A remarkable observation in the figure is that $F_2^* \approx 0$ for $Q^2 > 1.5 \text{ GeV}^2$. Although this is due to the definition of F_2^* in the current Eq. (3), it has an important consequence when expressed in terms of the helicity amplitudes $A_{1/2}$ and $S_{1/2}$ as we will show next.

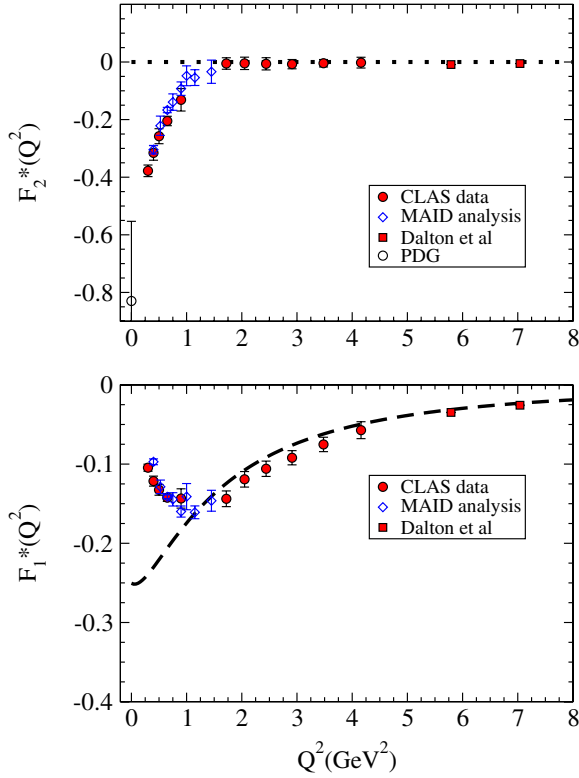


FIG. 1 (color online). $\gamma N \rightarrow N(1535)$ transition form factors. The dashed line for F_1^* represents the spectator quark model result from Ref. [8]. Data are from Refs. [4–6].

The relations for the form factors and the helicity amplitudes are given by

$$F_1^* = -\frac{Q^2}{Q_-^2} \frac{1}{2b} \left[A_{1/2} - \sqrt{2} \frac{M_S - M}{|q|} S_{1/2} \right], \quad (4)$$

$$F_2^* = -\frac{M_S^2 - M^2}{Q_-^2} \frac{1}{2b} \left[A_{1/2} + \sqrt{2} \frac{Q^2}{(M_S - M)|q|} S_{1/2} \right], \quad (5)$$

where $Q_\pm^2 = (M_S \pm M)^2 + Q^2$, and

$$b = e \sqrt{\frac{Q_+^2}{8M(M_S^2 - M^2)}}. \quad (6)$$

In the above,

$$|q| = \frac{\sqrt{Q_+^2 Q_-^2}}{2M_S} \quad (7)$$

is the absolute value of the photon three-momentum in the S_{11} rest frame.

From Eq. (5), the condition $F_2^* \approx 0$ is equivalent to

$$S_{1/2} \approx -\frac{1}{\sqrt{2}} \frac{(M_S - M)|q|}{Q^2} A_{1/2}. \quad (8)$$

We can simplify Eq. (7) for $Q^2 \gg (M_S - M)^2 \approx 0.355 \text{ GeV}^2$ as

$$|q| \approx \sqrt{1 + \tau} \frac{(M_S + M)}{2M_S} Q, \quad (9)$$

where $\tau = \frac{Q^2}{(M_S + M)^2}$. This approximation has a precision better than 10% for $Q^2 > 1.8 \text{ GeV}^2$. Combining Eqs. (8) and (9), we obtain a simple relation,

$$S_{1/2} \approx -\frac{\sqrt{1 + \tau}}{\sqrt{2}} \frac{M_S^2 - M^2}{2M_S Q} A_{1/2}, \quad (10)$$

for the region $Q^2 > 1.8 \text{ GeV}^2$. The relation of Eq. (10) is the main result of this work. We call this relation *scaling* (between $S_{1/2}$ and $A_{1/2}$).

Another interesting point concerning the $\gamma N \rightarrow N(1535)$ reaction is the pQCD estimate of $Q^3 A_{1/2}$ in the high Q^2 region [11], where it gives a magnitude much larger than the other resonance cases, e.g., than for the $\gamma N \rightarrow \Delta$ reaction [11]. However, we will not discuss this point here. (See Refs. [6,8] for more details.)

Scaling using the MAID fit: To test the scaling relation Eq. (10), we use the MAID parametrization for the amplitudes $A_{1/2}$ and $S_{1/2}$. The MAID parametrization is a fit to the MAID analysis data that can be extended for the high Q^2 region [4]. In Fig. 2 we compare the CLAS data for $\gamma N \rightarrow N(1535)$ with the MAID fit (solid line). In addition to $S_{1/2}$ we calculate the result estimated by Eq. (10) (dashed-dotted line). From the figure we conclude that the relation (10) is indeed a good approximation for MAID parametrization for $S_{1/2}$ in the region $Q^2 > 1.5 \text{ GeV}^2$. Note that in the figure both the $S_{1/2}$ parametrization and the results of $S_{1/2}$ derived from the scaling, are within the error bars. One can also see that the MAID parametrization and the approximation are indistinguishable for $Q^2 > 5.5 \text{ GeV}^2$. However, one must be careful in the extension of the MAID parametrization for the very high Q^2 region, since the parametrization is based on the analytical expression regulated by exponential functions such as $e^{-\beta Q^2}$. This is successful in the intermediate Q^2 region data, but differs asymptotically from the expected power law behavior predicted by pQCD, and also from the partial scaling suggested by the data from other reactions. Finally note the differences in scales for $A_{1/2}$ and $S_{1/2}$ in Fig. 2.

Spectator quark model: We consider now the $\gamma N \rightarrow N(1535)$ reaction based on a constituent quark model. By this, we intend to demonstrate the usefulness of the *scaling relation*, and shed some light on the underlying physics. The use of a quark model instead of a phenomenological parametrization has an advantage to relate the obtained results with the microscopic mechanisms. In the present case we can decompose the contributions for the form factors from the valence quark structure and those from the quark-antiquark excitations,

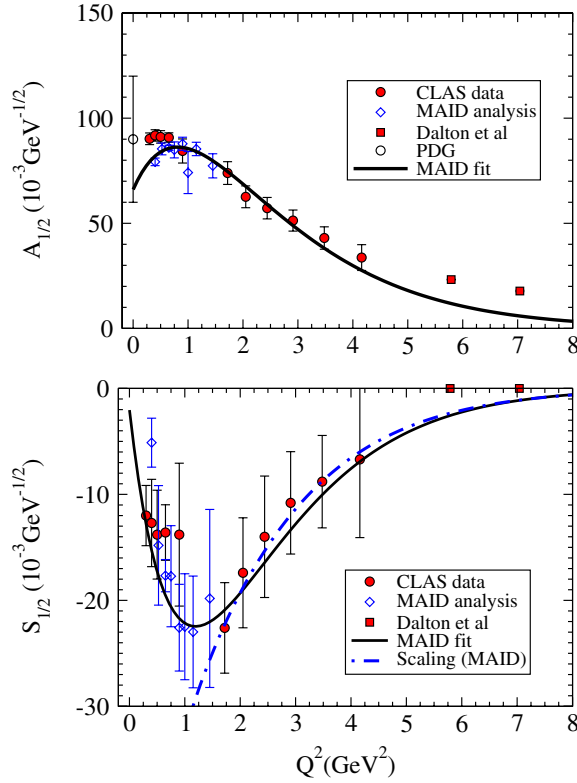


FIG. 2 (color online). $\gamma N \rightarrow N(1535)$ helicity amplitudes compared with the MAID parametrization. Data are from Refs. [4–6]. The filled squares corresponding to the data from Dalton [6] for $S_{1/2}$ are included to emphasize that $S_{1/2} = 0$ is assumed in the determination of $A_{1/2}$.

which are interpreted as meson cloud excitations in the low Q^2 region. In Ref. [8] it was shown that the spectator quark model predictions for F_2^* are consistent with the estimates of the EBAC group for the contributions from the bare core near $Q^2 = 2 \text{ GeV}^2$ [12], when the meson cloud is turned off. Also in Ref. [13], where $N(1535)$ was described as a dynamically generated resonance and therefore only meson cloud was taken in consideration, the contributions for F_2^* are negative but with a magnitude close to the results of the spectator quark model [14]. These results suggest that $F_2^* \simeq 0$ in the region $Q^2 > 1.5 \text{ GeV}^2$, can be interpreted as the cancellation between the valence quark effects and those of the meson cloud. Although the meson cloud contributions are expected to fall faster than those of the valence quarks, they can be significant in some nonleading order form factors in the intermediate Q^2 region. An example of the dominance of the meson cloud effects over the valence quark effects are the electric and Coulomb transition quadrupole form factors in the $\gamma N \rightarrow \Delta$ reaction for $Q^2 = 0\text{--}6 \text{ GeV}^2$ [15,16].

Based on the discussions made above, below we apply the covariant spectator quark model developed in Ref. [8] to the $\gamma N \rightarrow N(1535)$ reaction in the region $Q^2 \gg (\frac{M_S^2 - M^2}{2M_S})^2 = 0.23 \text{ GeV}^2$, without having any

adjustable parameters. As shown in Fig. 1, the model (dashed line) gives an excellent description for the F_1^* data in the region $Q^2 > 2.3 \text{ GeV}^2$. This is not surprising, since the valence quark contributions are in general small in the high Q^2 region. Combining the result for F_1^* and the assumption that $F_2^* = 0$ for $Q^2 > 1.5 \text{ GeV}^2$ (see Fig. 1), we get

$$A_{1/2}(Q^2) = -2bF_1^*(Q^2), \quad (11)$$

$$S_{1/2}(Q^2) = \sqrt{2}b \frac{(M_S - M)|\mathbf{q}|}{Q^2} F_1^*(Q^2).$$

This set of equations is consistent with the scaling relation Eq. (10) for $Q^2 > 1.8 \text{ GeV}^2$, but provides also a method to calculate $A_{1/2}$ through F_1^* . The results obtained in the spectator quark model (dashed line) are presented in Fig. 3. One can see the excellent agreement with the helicity amplitude data in the region $Q^2 > 2.3 \text{ GeV}^2$. Assuming that the scaling relation, or relations in Eqs. (11), hold for very high Q^2 , we expect that the ratio, $S_{1/2}/A_{1/2}$, converges to $-\frac{1}{\sqrt{2}} \frac{M_S - M}{2M_S} \simeq -0.13$ in the limit $Q^2 \rightarrow \infty$. However, note that the approximation works only in the region $Q^2 \gg (M_S + M)^2 = 6.1 \text{ GeV}^2$, meaning that the convergence is very slow.

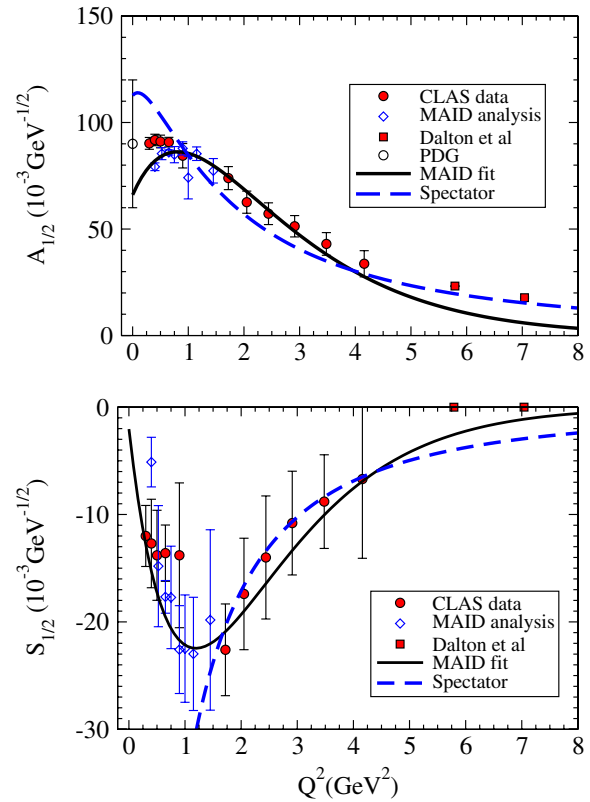


FIG. 3 (color online). $\gamma N \rightarrow N(1535)$ helicity amplitudes calculated in the spectator quark model [8] (dashed line). They are calculated using the results for F_1^* and Eqs. (11). Data are from Refs. [4–6].

Summary: In this work we have found a novel scaling relation between the $A_{1/2}$ and $S_{1/2}$ helicity amplitudes for the $\gamma N \rightarrow N(1535)$ reaction, given by Eq. (10), for $Q^2 > 1.8 \text{ GeV}^2$. The scaling relation is a consequence of the experimental result for the Pauli-type form factor: $F_2^* \approx 0$ for $Q^2 > 1.5 \text{ GeV}^2$. This is very surprising, and has never been observed in similar reactions like $\gamma N \rightarrow N$ or $\gamma N \rightarrow N(1440)$. The scaling relation between the helicity amplitudes, found in this work, is also supported by the MAID parametrization for $Q^2 > 1.5 \text{ GeV}^2$. In a quark model formalism the result can be interpreted as the cancellation between the valence quark and meson cloud effects for F_2^* . As a consequence, the helicity amplitudes $A_{1/2}$ and $S_{1/2}$ can be simultaneously predicted using a valence quark model with the results of F_1^* for $Q^2 > 1.5 \text{ GeV}^2$. We have demonstrated this using the covariant spectator quark model of Ref. [8], which is valid for $Q^2 > 2.3 \text{ GeV}^2$. We conclude that although $N(1535)$ may possibly be described as a dynamically generated

resonance [12,13,17], the transition form factors for $\gamma N \rightarrow N(1535)$ can be very well described in a constituent quark model for high Q^2 . We also note that although the scaling relation is consistent with the spectator quark model and the MAID parametrization, they give different predictions for $Q^2 > 5 \text{ GeV}^2$ (see Fig. 3). Then, a precise experimental determination of the helicity amplitudes, particularly for $S_{1/2}$ in the high Q^2 region, will be essential to clarify the role of the helicity amplitudes and to test the scaling relation given by Eq. (10).

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