Chiral Effective Field Theory
Beyond the Power-Counting Regime

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Abstract

Chiral effective field theory complements numerical simulations of quantum chromodynamics on a spacetime lattice. It provides a model-independent formalism for connecting lattice simulation results at finite volume, and at a variety of quark masses, to the physical region. Knowledge of the power-counting regime of chiral effective field theory, where higher-order terms of the expansion may be regarded as negligible, is as important as knowledge of the expansion. Through the consideration of a variety of renormalization schemes, techniques are established to identify the power-counting regime. Within the power-counting regime, the results of extrapolation are independent of the renormalization scheme.

The nucleon mass is considered as a benchmark for illustrating this approach. Because the power-counting regime is small, the numerical simulation results are also examined to search for the possible presence of an optimal regularization scale, which may be used to describe lattice simulation results outside of the power-counting regime. Such an optimal regularization scale is found for the nucleon mass. The identification of an optimal scale, with its associated systematic uncertainty, measures the degree to which the lattice QCD simulation results extend beyond the power-counting regime, thus quantifying the scheme-dependence of an extrapolation.

The techniques developed for the nucleon mass renormalization are applied to the quenched $\rho$ meson mass, which offers a unique test case for extrapolation schemes. In the absence of a known experimental value, it serves to demonstrate the ability of the extrapolation scheme to make predictions without prior phenomenological bias. The robustness of the procedure for obtaining an optimal regularization scale and performing a reliable chiral extrapolation is confirmed.

The procedure developed is then applied to the magnetic moment and the electric charge radius of the isovector nucleon, to obtain a consistent optimal regularization scale. The consistency of the results for the value of the optimal regularization scale provides strong evidence for the existence of an intrinsic energy scale for the nucleon-pion interaction.
Statement of Originality

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It is with hope and faith that we endeavour to extend our learning to reach new insights just beyond our present reach.
## Contents

1 Introduction  
1.1 Prologue ................................................. 1  
1.2 Overview and Aims ...................................... 5  

2 Lattice QCD  
2.1 Functional Methods .................................. 9  
2.1.1 Wilson Fermions .................................. 11  
2.1.2 Correlation Functions and the Effective Mass .... 12  
2.1.3 Quenching and Computational Alternatives ....... 14  
2.2 Lattice QCD Applicability and Issues ................. 15  

3 Chiral Effective Field Theory  
3.1 Chiral Symmetry ....................................... 19  
3.1.1 Spontaneous Symmetry Breaking .................. 21  
3.1.2 Partial Conservation of the Axial Current ....... 23  
3.1.3 The Sigma Model .................................. 24  
3.2 Chiral Perturbation Theory ............................ 27  
3.2.1 Meson Sector ..................................... 28  
3.2.2 Baryon Sector ..................................... 32  
3.2.3 Electromagnetic Contributions .................... 36  
3.3 Regularization and Renormalization .................... 38  
3.3.1 Historical Overview ............................... 38  
3.3.2 The Power-Counting Regime ........................ 39  
3.3.3 Dimensional Regularization ........................ 41  
3.3.4 Finite-Range Regularization ....................... 42  

4 The Intrinsic Scale of the Nucleon  
4.1 Renormalization Issues for the Nucleon Mass ........ 47  
4.1.1 Chiral Expansion of the Nucleon Mass ............ 48  
4.1.2 Chiral Loop Integrals ............................. 50  
4.1.3 The Sigma Term .................................. 52  
4.1.4 Scheme-Independent Coefficients ................ 54  
4.1.5 Finite-Volume Effects ............................. 55  
4.2 The Intrinsic Scale: An Example by Construction ...... 57
### 5 Results for the Mass of the Nucleon

5.1 Evidence for an Intrinsic Scale in the Nucleon Mass
- 5.1.1 Renormalization Flow Analysis
- 5.1.2 Analysis of Systematic Uncertainties
- 5.1.3 Effects at Higher Chiral Order

5.2 Summary and Specific Issues for the Nucleon Mass

### 6 Results for the Mass of the Quenched $\rho$ Meson

6.1 Renormalization of the Quenched $\rho$ Meson Mass
- 6.1.1 Chiral Expansion of the Quenched $\rho$ Meson Mass
- 6.1.2 Chiral Loop Integrals
- 6.1.3 Scheme-Independent Coefficients

6.2 Extrapolating the Quenched $\rho$ Meson Mass
- 6.2.1 Renormalization Flow Analysis
- 6.2.2 Intrinsic Scale and Systematic Uncertainties
- 6.2.3 Optimal Pion Mass Region and Systematic Uncertainties

6.3 Summary and Specific Issues for the Quenched $\rho$ Meson

### 7 Electromagnetic Properties of the Nucleon

7.1 Renormalization of the Magnetic Moment
- 7.1.1 Chiral Expansion of the Magnetic Moment
- 7.1.2 Chiral Loop Integrals

7.2 Evidence for an Intrinsic Scale in the Magnetic Moment
- 7.2.1 Renormalization Flow Analysis
- 7.2.2 Analysis of Systematic Uncertainties
- 7.2.3 Chiral Extrapolation Results

7.3 Finite-Volume Considerations for the Electric Charge Radius
- 7.3.1 Chiral Loop Integrals
- 7.3.2 Chiral Expansion of the Electric Charge Radius

7.4 Evidence for an Intrinsic Scale in the Electric Charge Radius
- 7.4.1 Renormalization Flow Analysis
- 7.4.2 Analysis of Systematic Uncertainties
- 7.4.3 Chiral Extrapolation Results

7.5 Summary and Specific Issues for the Electromagnetic Properties of the Nucleon

### 8 Conclusion

8.1 Evaluation and Summary Analysis
8.2 Future Studies and Further Developments
8.3 Codetta