A stochastic model for the fracture network in the Habanero enhanced geothermal system

Seifollahi, S., Dowd, P-A and Xu, C

University of Adelaide

Sattar.seifollahi@adelaide.edu.au

Fracture Network Modelling (FNM) plays an important role in many areas where the characterization of discontinuities in deep ground is required. Applications of the FNM include, but not limited, hydrocarbon reservoir production, mineral extraction, tunnelling, underground storage or disposal of hazardous wastes and geothermal systems. One important step in FNM is to estimate the density of fractures and geometries and properties of individual fractures such as the size and orientation. Due to the lack of data, the tortuous nature of fractures and the great uncertainty involved in practice, the only feasible approach is via a stochastic modelling. This paper describes a general optimization approach to modelling the fracture network in a geothermal reservoir, conditioned on the seismic events several kilometres beneath the surface detected during the fracture stimulation process. Two key aspects of our method are the construction of an appropriate objective function and the derivation of an efficient updating scheme, which still remain to be the two challenging issues of most global optimization techniques. In our application, the objective function consists of two important components: the minimisation of squared distances of the seismic points to the fracture model and the minimisation of number of fractures or the amount of fracturing, which corresponds to the least consumption of fracturing energy. The model updating process includes several proposals for perturbing the parameters of individual fractures and also to alter the size of the fracture network in order to get a global optimal solution. As a case study, the model is applied to Geodynamics' Habanero reservoir in the Cooper Basin of South Australia.

Keywords: global optimization, simulated annealing, stochastic fracture modelling, enhanced geothermal systems, hot dry rock

Introduction

Earlier mathematical models for fracture networks include continuum models that assume a fractured rock mass can be represented as an "equivalent porous medium" (Long and Witherspoon, 1985; Odling, 1992) and discrete fracture models that rely on a detailed description of discontinuity geometry (Dershowitz and LaPointe, 1994; Einstein 2003; Mardia et al., 2007a). In a discrete fracture model, the characterization of a fracture network is still a very challenging problem as the whole fracture system is not observable on any meaningful scale and the system can only be described via a stochastic model. Stochastic modelling is a general approach in which the fracture characteristics such as location, size and orientation are treated as random variables with inferred probability distributions.

In modelling practice, it is impossible to represent all fractures by tortuous surfaces and some simplification is required. Common approaches for fracture geometry include circular discs, elliptical discs, planer polygons or planes with infinite extents. Here we use an ellipse to represent a fracture, i.e. $H = (x, y, z, \alpha, \beta, \gamma, a, b)$ where x, y, z are the coordinates of the fracture centre point, α, β the dip direction and dip angle of the plane, γ the rotation angle of the major axis against the dip direction of the ellipse, and a, b the major and minor axes of the ellipse. This configuration, even with the 'best' fitted model, can not intersect all seismic points due to fracture tortuosity but the distance of the points to fracture planes can be used as one of the criteria to assess the goodness-of-fit of the fracture model. The seismic events in this context refer to those detected during the stimulation process at the development stage of a geothermal reservoir.

Recently, a Markov chain Monte Carlo (MCMC) approach has been used to condition a fracture model to borehole data in 2D application (Mardia et al, 2007b). The model, then, was extended to 3D applications, where the early results are promising (Mardia et al, 2007a). In both works, the number of fractures varies during the learning process. Another extension of MCMC in FNM is to condition the fracture model on the seismic events recorded during fracture stimulation of the reservoir (Xu et al., 2011). In this model the number of fractures is fixed in advance and it doesn't vary during the optimization process. Therefore, determining an optimal number of fractures to properly fit the seismic event points is one of the remaining challenging issues in this approach. In the work presented in this paper, we address this issue via a general optimization model by defining proper objective functions and using two sets of proposals: one for updating the parameters of individual fractures and the other for adjusting the number of fractures in the model.

Distance-Directional Transform

The Distance-Directional Transform (DD-Transform), originally proposed for line detection by Seifollahi et al. (2012), is extended to 3D applications in our approach to generate parameters of new fractures. In 2D applications, the approach can generate the midpoint and the orientation of fractures. It incorporates the distance and the orientation and represents the best orientations of planes embedded in the points. In the following extended version, the approach is used to detect five parameters (out of eight) of the fracture, which are the centre coordinates, (x, y, z) the dip direction and the dip angle. Other three parameters of the fracture (size of the ellipse and rotation) can be generated by specified distributions. Suppose D denotes the conditioning data points, then the steps of the DD-Transform are as follows:

- Set $A = 0$, where A is an $N \times 3$ matrix, and N is the number of points in D
- Select the point $p_i \in D, i = 1,..., N.$
- Initialize $A^i = 0$ where A^i is an $i_d \times i_a$ matrix; i_d and i_a correspond to the possible indices of the dip direction and dip angle; here $i_d = 360$ and. $i_a = 90$ For $s = 1,2,..., S$ do the following:
  a) Select two points. $p_i, p_{j'} \in D, j, j' \neq i$
  b) Determine a plane passing through the points p_i, p_j and $p_{j'}$.
  c) Calculate the dip direction and dip angle of the plane and update $(k,l)^{th}$ element of the matrix

  $$A_{kl}^i = A_{kl}^i + \min\left(b, d_{ij}^{-\alpha} + d_{ij'}^{-\alpha}\right)$$

  where k and l correspond to the index of dip direction and dip angle, respectively, $d_{ij}^{-\alpha}, d_{ij'}^{-\alpha}$ are the distances of p_i to p_j and $p_{j'}$ α and b are two positive constants (here $b = 10$, $\alpha = 0.5$).
- Sort A^i in terms of the highest values. Store the row corresponding to the highest value in i^{th} row of matrix A . If $i \geq N$ go to the next step; otherwise, repeat from second line.
- Sort the rows of A in a descending order with respect to the values obtained in previous step.
- The i^{th} row of A corresponds to the parameters of the i^{th} best new fracture, which includes the dip direction, dip angle and the index of the centre.

Stochastic optimization model

We propose an optimization approach to construct a fracture map fitted to the seismic point cloud with the best possible goodness-of-fit and a most reasonable size. In the initialization step, a prior map,

$H = \{H^i\}_{i=1}^m$ of fractures is generated, where H^i is an ellipse as a first approximation of the i^{th} fracture and m is the number of fractures. The centres of initial fractures are selected from the seismic points at random. Other parameters of the fractures are generated by their specified probability distributions. Prior distributions for the dip direction, dip angle and rotation angle considered are uniform distributions. For priors for the major and minor axes, lognormal distributions are used.

After the initialization stage, we post-process the fracture map by optimization using the simulated annealing technique. The measure of goodness-of-fit, or objective function for the simulated annealing, consists of two parts: the minimization of the sum of squared distances of the points to the fitted fracture model and the minimization of the number of fractures or the amount of fracturing. The first part is defined as

$$f_d(w) \equiv \sum_{j=1}^{N} d(p_j, H_{k*}) \qquad k* = \arg\min(d(p_j, H_k)) \tag{1}$$

where N is the number of points, j and k stand for the j^{th} point and k^{th} fracture in the network and $d(...)$ represents the orthogonal least squared distance. Minimization of (1) is an essential step to produce an accurate model close enough to the point cloud. The second part of the objective function is expressed as

$$f_{AF} \equiv \sum_{i=1}^{n} \left\{ \frac{(1 + a_i b_i) \times w_p}{1 + N_i \times w_p} \right\} \tag{2}$$

where n is the number of fractures, a_i and b_i are the major and minor axes of i^{th} fracture and w_p is a constant number (here $w_p = 100$).

Two categories of proposals are incorporated in the optimisation process to determine a global optimal solution of the FNM. The first category is related to the parameters of individual fractures, and the second category is used to determine an optimal size of the fracture system (i.e, number of fractures in this case). In the first category, the parameters of individual fractures (the centre, dip direction, dip angle, rotation angle, major and minor axes) are changed, while the aim in the second category is to propose adding new fractures or removing redundant fractures by using the proposals “Split”, “Replacement”, “Joint” or “Removal”.

Based on the consideration of various proposals, the new association between points and fractures is set and the value of the objective function is recalculated by considering the values that are modified by the perturbation. If the change represents a reduction in the objective function, the transformation to the new state is accepted. If it represents an increase in the objective function, the transformation is accepted with a specified probability, p,

$$p = \exp\left(\frac{-f^{new} - f^{old}}{T} \right) \tag{3}$$

where f^{new} and f^{old} are the objective function value in the new and old configurations, T is a control parameter, which corresponds to absolute temperature in the physical process of annealing.

The details of perturbation/modification proposals are as follows:

Centre: The aim is to perturb the centre of a candidate fracture by shifting the location through a direction, d , to one of the associated points of the fracture.

$$(x, y, z) = (x, y, z) + rd$$

where $d \in R^3$ and $r(r \sim U[0,0.1])$ is the step size to determine the amount of the perturbation. Both the candidate fracture and the associated points are chosen at random.

Orientation: The orientation, here, consists of three components, the dip direction, dip angle and rotation angle. The proposal is repeated for each component independently. For a given component, a pair of fractures is selected at random and their corresponding orientation parameters are exchanged.

Major-axis and Minor-axis: Two fractures, H^i and $H^j(i \neq j)$ are chosen at random. The major axes of the selected fractures as well as the minor axes are exchanged independently.

Joint: The steps of the joint proposal are as follows:

- Choose two fractures based on a similarity measure in terms of their parameters (closeness)
- Apply DD-Transform on the associated points of the candidate fractures to find the centre, dip direction and dip angle of a new fracture.
- Generate the rotation angle and major and minor axes from their specific distributions.

Removal: A fracture is chosen from the network based on the lowest density (the ratio of the number of the associated points to the area of the ellipse). The candidate fracture is removed from the network.

Replacement: The steps of the proposal are as follows:

- Choose a set of fractures based on their associated errors, i.e., select fractures with at least one point with the distance to the associated fracture greater than a threshold.
- Apply DD-Transform on the associated points of each fracture selected in previous step to find the "best" orientations of planes passing through all the points concerned.
- Find the highest value in the first column of the matrix. The index of the row corresponding to the highest value represents the candidate fracture and the elements of that row represent the parameters, x, y, z, α and β of the new fracture.
- Generate the other three parameters of the new fracture from their specified distributions.
- Replace the candidate fracture with the new one.

Split: This proposal has similar steps as in the "Replacement" proposal. The key difference is that two new fractures are generated to replace one old fracture. The difference in orientations of the two new fractures should be greater than a threshold in order to represent two distinct fractures.

Habanero reservoir dataset

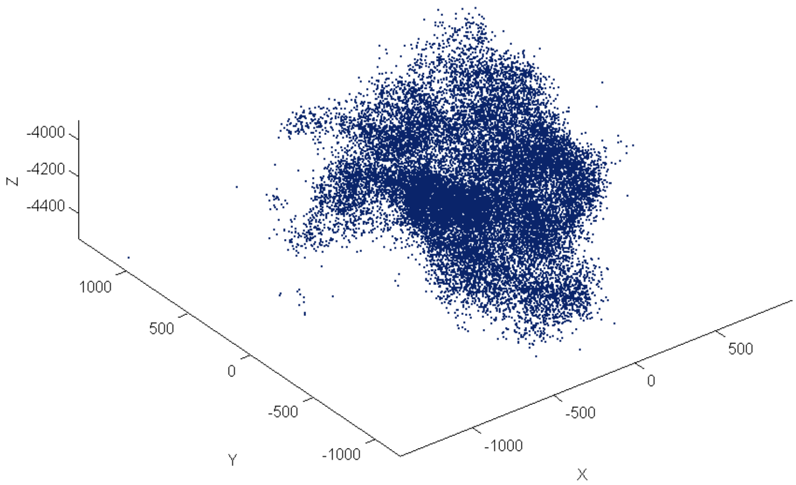

Figure 1: *Hypocentre locations of the seismic events*

The Habanero wells are the key components of Geodynamics' HDR geothermal project in the Cooper Basin, South Australia. These wells have been drilled to depths of about 4400m below the surface or about 700m into the bedrock where temperatures reach 250 °C (Baisch et al., 2006). The dataset used in this study is the Q-Con dataset which contains a total of 23,232 seismic events covering an approximate area of 2.5 km^2. The absolute hypocentre locations of these events are shown in Figure 1.

The resulting fractures after optimization (30000 iterations of simulated annealing) are shown in Figure 2 which represents an optimal realization to the given point cloud. The number of fractures after optimization is 521 compared to 613 fractures in the work of Baisch et al., 2006, and Xu et al., 2011.

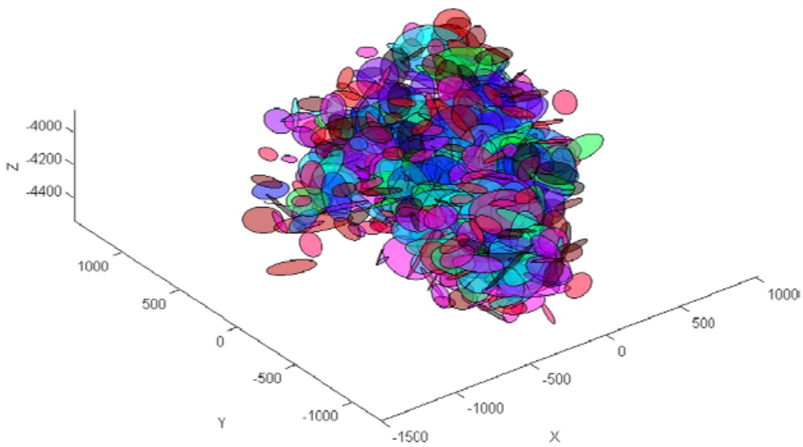

Figure 2: *Habanero fracture model after optimization*

Figure 3 demonstrates the distributions of major and minor axes of the detected fractures which follow a lognormal distribution. The distribution of the point associations is demonstrated in Figure 4. Figure 5 shows the distribution of error, in which the horizontal axis is the error of the points to the nearest fractures.

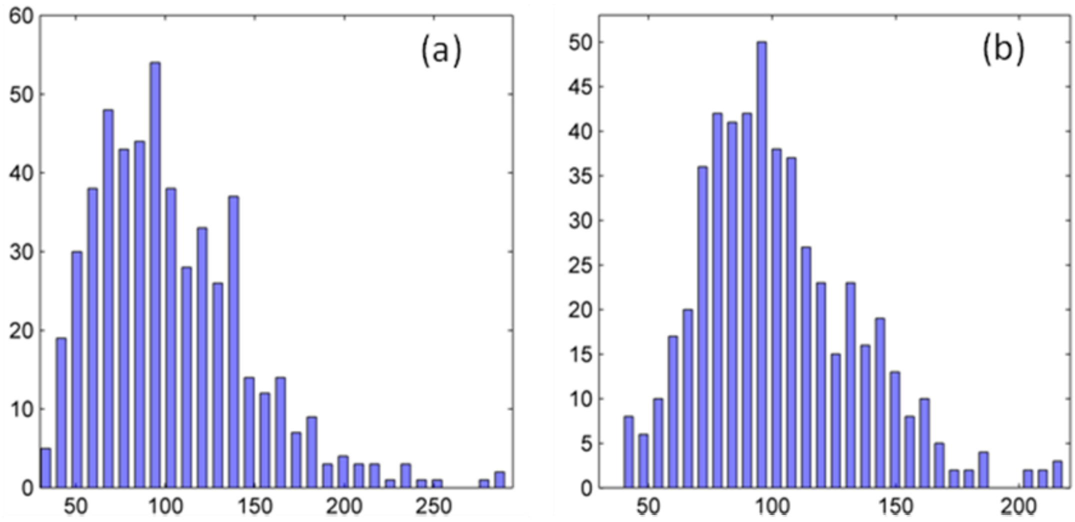

Figure 3: *Distribution of size (parameters a and b) of obtained fractures*

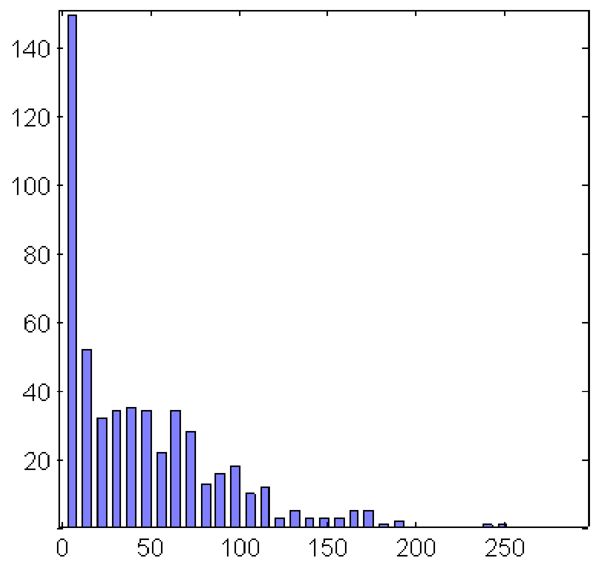

Figure 4: *Distribution of the point association*

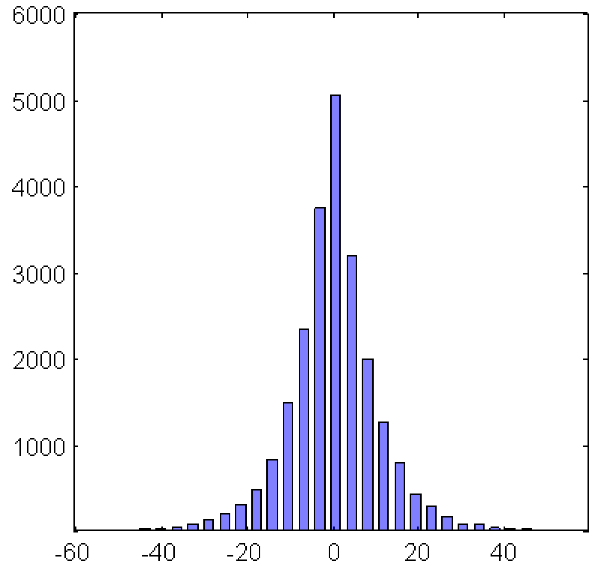

Figure 5: *Distribution of the distances of the points to the fractures*

Conclusions

A global optimization approach has been proposed to generate a remarkably good fit of a fracture map to seismic points recorded during fracture stimulation of the geothermal reservoir. A constructive contribution of the approach is that it addresses two main issues in the optimisation of a fracture model, i.e., a suitable objective function and a model modification system, which still remain as the two major challenging aspects of any global optimization technique. A new method, called DD-Transform, is developed to initialize the parameters of the new generated fractures.

The work demonstrates the effectiveness of the optimization approach for fracture modelling conditional on seismic events. By increasing the number of iterations of simulated annealing, it is expected to get more accurate solution. Work is now proceeding to deal with the time domain feature of the events to get a more realistic and reliable fracture model for the EGS system.

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