# Contents

Abstract \hspace{1cm} xiii

Acknowledgements \hspace{1cm} xvii

1 Introduction \hspace{1cm} 1

1.1 Motivation \hspace{1cm} 2
1.2 Continuum QCD \hspace{1cm} 5
1.3 Gauge Invariance \hspace{1cm} 7
  1.3.1 Fermion Action \hspace{1cm} 7
  1.3.2 Gauge Action \hspace{1cm} 8
1.4 The Path-Integral Formalism \hspace{1cm} 9
  1.4.1 Calculus with Grassmann Variables \hspace{1cm} 9
  1.4.2 The Path Integral \hspace{1cm} 11

2 QCD On the Lattice \hspace{1cm} 15

2.1 The Fermion Action on the Lattice \hspace{1cm} 15
  2.1.1 The Naive Discretisation \hspace{1cm} 16
  2.1.2 Wilson Fermions \hspace{1cm} 20
  2.1.3 Improving the Fermion Action \hspace{1cm} 22
  2.1.4 The Field Strength Tensor on the Lattice \hspace{1cm} 24
  2.1.5 Mean Field Improvement \hspace{1cm} 28
  2.1.6 The FLIC Fermion Action \hspace{1cm} 28
2.2 The Gauge Action on the Lattice \hspace{1cm} 30
  2.2.1 Improving the Gauge Action \hspace{1cm} 32

3 Spectroscopy in Lattice QCD \hspace{1cm} 35

3.1 Correlation Functions at the Baryon Level \hspace{1cm} 35
3.2 Interpolating Fields \hspace{1cm} 39
3.3 Correlation Functions at the Quark Level \hspace{1cm} 42
CONTENTS

3.4 Propagators ........................................... 45

4 Simulation Results ........................................ 55
4.1 The Even-Parity Proton ................................. 56
  4.1.1 Correlation Functions ............................ 56
  4.1.2 Effective Mass Plots .............................. 59
4.2 The Odd-Parity $N^*$ .................................... 62
  4.2.1 Correlation Functions ............................ 62
  4.2.2 Effective Mass Plots .............................. 64
4.3 The Even-Parity $\Delta^{++}$ ............................ 66
  4.3.1 Correlation Functions ............................ 66
  4.3.2 Effective Mass Plots .............................. 68
4.4 The Odd-Parity $\Delta^{++}$ ............................. 70
  4.4.1 Correlation Functions ............................ 70
  4.4.2 Effective Mass Plots .............................. 72
4.5 The Odd-parity $\Lambda$ ................................. 74
  4.5.1 Correlation Functions ............................ 74
  4.5.2 Effective Mass Plots .............................. 77

5 Conclusion ................................................ 81

A The Gell-Mann Matrices .................................. 83

B $\gamma$-Matrices .......................................... 85
  B.1 Dirac Representation ................................. 85
  B.2 Pauli Representation ................................ 87

C Clebsch-Gordan Coefficients ................................ 89

D Wick’s Theorem ............................................ 91

E Transformation Properties of Interpolating Fields ............ 95
  E.1 Lorentz Transformations ............................. 96
    E.1.1 Meson Interpolators ............................ 96
    E.1.2 Baryon Interpolators ........................... 96
    E.1.3 Two-Particle Interpolators ..................... 98
  E.2 Parity ................................................ 98
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The spectrum of two non-interacting particles (with the $1/L$ behaviour) is on the left. The resonance energy is shown by the flat line at 1.4. The right diagram depicts the ALC near the resonance energy. Figure from [19].</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>The masses of the odd-parity, $J^P = 1/2^-$, states of the Λ baryon. The correlation matrix analysis allows the isolation of the three lowest lying states. Figure courtesy of B. Menadue [9].</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>Masses of the positive parity states of the nucleon at various quark masses. The lattice results for the Roper are the red triangles. The black data points to the far left are the physical values obtained from [22]. Figure courtesy of S. Mahbub [21].</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>The smallest possible closed loop on the lattice, the plaquette $U_{\mu\nu}(n)$.</td>
<td>25</td>
</tr>
<tr>
<td>2.2</td>
<td>The terms that contribute to the clover $C_{\mu\nu}(n)$.</td>
<td>27</td>
</tr>
<tr>
<td>3.1</td>
<td>The “fully-connected” (left) and “loop-containing” (right) contributions to the two-point functions given in section (3.3) for the five quark operators in section (3.2). Note the four “types” of propagators we require to evaluate such diagrams.</td>
<td>45</td>
</tr>
<tr>
<td>3.2</td>
<td>An effective mass plot for the pion using the stochastically estimated $S(x,0)$ in the pion correlation function.</td>
<td>52</td>
</tr>
<tr>
<td>3.3</td>
<td>An effective mass plot for the pion using the standard point-to-all propagator $S(x,0)$ in the pion correlation function.</td>
<td>53</td>
</tr>
<tr>
<td>4.1</td>
<td>Correlation function plots for the three-quark nucleon operator with zero momentum and one unit of momentum.</td>
<td>56</td>
</tr>
</tbody>
</table>
4.2 Correlation function plots for the pion and five-quark proton operator. We can observe that the correlator for the five-quark proton operator is very similar to the correlator obtained for the standard three-quark nucleon shown in Figure 4.1 (b). .............................. 57

4.3 The correlation function plots for the loop-containing and fully-connected pieces of the five-quark proton operator. We observe that the loop-containing piece of the five-quark proton operator in (a) is virtually indistinguishable from the total five-quark operator correlation function in Figure 4.2 (b), indicating that this piece is the dominant contribution. .............................. 57

4.4 A comparison of the loop-containing and fully-connected pieces of the five-quark proton operator correlation function. Here we observe the fully-connected piece has a greater slope indicating this piece is associated with more massive contributions. .............................. 58

4.5 Effective Mass plots for the nucleon and pion with one unit of momentum each. ......................................................... 59

4.6 An effective mass plot for the standard three-quark nucleon operator at zero momentum. ......................................................... 59

4.7 A mass plot for the five-quark proton operator compared to the extracted three-quark nucleon result at zero momentum shown in Figure 4.6. ......................................................... 60

4.8 A comparison of the effective mass plots for the fully-connected and loop-containing pieces. Recall that the total correlation function is almost entirely dominated by the disconnected piece, and as such they are virtually indistinguishable. ......................................................... 60

4.9 Correlation function plots for the three-quark and five-quark proton interpolators. ......................................................... 62

4.10 Correlation function plots for the loop-containing and fully-connected pieces of the five-quark proton interpolator. ......................... 62

4.11 A comparison of the loop-containing and fully-connected pieces of the five-quark proton correlation functions. ......................... 63

4.12 Effective Mass plots for the three-quark proton operator and the pion at $\vec{p} = 0$. ......................................................... 64

4.13 An effective mass plot for the five-quark proton operator. ........... 64

4.14 An effective mass plot comparing the loop-containing and fully-connected pieces of the five-quark proton. ......................... 65

4.15 A correlation function plot for the five-quark $\Delta^{++}$ interpolator. ... 66
4.16 Correlation function plots for the loop-containing and fully-connected pieces of the five-quark $\Delta^{++}$. ........................................... 66
4.17 A comparison of the loop-containing and fully-connected pieces of the five-quark $\Delta^{++}$ correlation functions. ......................... 67
4.18 An effective mass plot for the five-quark $\Delta^{++}$ operator. .... 68
4.19 An effective mass plot comparing the loop-containing and fully-connected pieces of the five-quark $\Delta^{++}$. ......................... 68
4.20 A correlation function plot for the five-quark $\Delta^{++}$ interpolator. 70
4.21 Correlation function plots for the loop-containing and fully-connected pieces of the five-quark $\Delta^{++}$. ................................. 70
4.22 A comparison of the loop-containing and fully-connected pieces of the five-quark $\Delta^{++}$ correlation functions. ......................... 71
4.23 An effective mass plot for the five-quark $\Delta^{++}$ operator. .... 72
4.24 An effective mass plot comparing the loop-containing and fully-connected pieces of the five-quark $\Delta^{++}$. ......................... 72
4.25 Effective mass plots for the flavour-singlet and “common” three-quark $\Lambda$ interpolators. ................................. 74
4.26 Correlation function plots of the three-quark octet interpolator and five-quark operator for the $\Lambda$. ................................. 75
4.27 Plots of the loop-containing and fully-connected pieces for the five-quark operator $\Lambda$. ................................. 75
4.28 A comparison of the fully-connected and loop-containing pieces of the five-quark $\Lambda$ interpolator. As was the case with the proton and $\Delta^{++}$, we observe the fully-connected piece possessing a steeper slope and therefore is associated with more massive parts of the spectrum. ......................... 76
4.29 Effective Mass plots for various three-quark $\Lambda$ interpolators. 77
4.30 An effective mass plot for the five-quark $\Lambda$ interpolator. .... 78
4.31 A comparison of the fully-connected and loop-containing pieces of the five-quark $\Lambda$ interpolator. The nucleon mass is presented in Section 4.1. (Recall that as we are at the $SU(3)$ flavour limit the kaon is identical to the pion - see Appendix F.) ................................. 78

C.1 Clebsch-Gordan coefficients for the case $I' = 1/2, I'' = 1/2$. Recall there is an implicit square root sign over the positive part of each table entry. ................................. 89
C.2 Clebsch-Gordan coefficients for the case \( I' = 1, I'' = 1/2 \). Recall there
is an implicit square root sign over the positive part of each table entry. 90
## List of Tables

3.1 The classification of the various particles relevant to this work and their corresponding interpolating fields. ........................................... 39

3.2 The values of various traces for the stochastically estimated propagator with full spin, colour and time dilution. The average is taken over all space-time points and gauge configurations. One noise vector has been used. ........................................... 49

3.3 The values of various traces for the stochastically estimated propagator with full spin and colour dilution and interleaved 4-time dilution. The average is taken over all space-time points and gauge configurations. One noise vector has been used. ........................................... 49

3.4 The values of various traces for the standard point-to-all propagator, with the “all” $x$ set to the “point value”, to make a loop. The average is taken over gauge configurations. ........................................... 50

3.5 The values of various traces for the stochastically estimated propagator with full spin and colour dilution and interleaved 4-time dilution. Two noise vectors are averaged over in addition to the averaging over all space-time points and gauge configurations. ........................................... 51

3.6 The values of various traces for the stochastically estimated propagator with full spin, colour and time dilution. Two noise vectors are averaged over in addition to the averaging over all space-time points and gauge configurations. ........................................... 51
Quantum Chromodynamics (QCD) is widely accepted as the theory that describes the strongest force in Nature (by coupling constant), aptly named the strong nuclear force. The challenge is to understand the phenomena that emerge from this fundamental quantum field theory. Hadronic spectroscopic calculations can be performed utilising the formalism of lattice QCD by discretising space-time onto a hypercube. This is the only known non-perturbative ab-initio approach for studying QCD. Equipped with a tractable formalism, we consider some recent work done extracting resonances, in particular the Roper and the Λ(1405) resonances studied at the CSSM in Adelaide. These studies are done with three quark interpolators, and as such we expect to be extracting resonances having strong overlap three-quark states. In order to rule out the possibility of contamination from more exotic five-quark states, and to extract multi-particle states in their own right, the use of five-quark interpolators is of considerable interest. We first construct five-quark interpolating fields for the $p$, Λ and Δ$^{++}$. The corresponding correlation functions are calculated which can be of considerable size. Relevant elements of the all-to-all propagator (the so-called loop propagator), are calculated using stochastic estimation techniques. Dilution in spin, colour and time are implemented as a means of variance reduction. We conclude by presenting effective mass plots for the five-quark interpolators, the relevant contributions from fully connected and loop containing pieces, and comparing them to the masses extracted from standard three-quark operators.
Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Adrian Leigh Kiratidis and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

I also give permission for the digital version of my thesis to be made available on the web, via the University’s digital research repository, the Library catalogue, the Australasian Digital Theses Program (ADTP) and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

Adrian Leigh Kiratidis
Acknowledgements

First and foremost, I’d like to thank my supervisors Derek Leinweber and Waseem Kamleh for their exceptional guidance, encouragement and patience throughout the last two years of my M.Phil. During this time they have constantly made time for me in their busy schedules, and have always provided clear explanations. Without them none of this would have been possible, so to that end I owe them greatly.

The Adelaide CSSM lattice collaboration’s cola and genef were used extensively in this research, and I’d therefore like to acknowledge the authors Waseem Kamleh and Peter Moran.

To the guys in the office over the past two years, Daniel, Ben O., Nathan, Sam, Ben M., Phiala and Sophie, thanks for all the informative and thought provoking discussions, both on physics and non-physics related topics. They were of great help both aiding my physics understanding and making the journey enjoyable. In particular, I owe a special thanks to Daniel and Ben O. for the many useful algorithm discussions. The M.Phil experience has also given me the opportunity to discuss with many other great people, Dale, Selim, Ross, James, Lewis, Manuel and Mariusz, just to name a few.

Finally I’d like to thank my family and friends for their support, encouragement and understanding. In particular I owe a special thanks to my parents who not only have listened to my desperate “preserving the universe’s order” arguments with respect to the state of my room and tolerated my occasionally nocturnal inclinations, but have also taken essentially all financial and logistical issues out of my life for which I am greatly thankful.