DC Anisotropic Resistivity Sensitivity and Inversion

presented by

Timothy C. Wiese
B.Sc. Hons. (Physics)

THE UNIVERSITY of ADELAIDE

The University of Adelaide
School of Chemistry and Physics
Discipline of Physics

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Contents

Abstract

Many rocks and layered/fractured sequences have a clearly expressed electrical anisotropy, although it is rare in practice to incorporate anisotropy into the interpretation of electrical resistivity survey data. This thesis comprises a series of journal papers directed at increasing understanding of electrical anisotropy in resistivity investigations. Particular attention is given to how anisotropy affects both forward modelling and the resistivity inverse problem.

Chapter 2 derives the analytic solutions for the electric potential, current density and Fréchet derivatives inside a 3D tilted transversely isotropic medium. The solutions hold for a surface current source above an anisotropic but otherwise homogeneous medium. Profiles for potential, current density and sensitivity are presented for various strike and dip orientations of the axis of symmetry. Equipotentials exhibit an elliptical pattern and are not orthogonal to the current density vectors. Sensitivity patterns are strongly asymmetric compared to the isotropic case, with strong dependence on the axis of symmetry direction.

Chapter 3 presents a general numerical formulation for calculating the electric potential and Fréchet derivatives in an arbitrary 3D anisotropic heterogeneous medium. It is based on a new Gaussian quadrature grid formulation for calculating the 3D Green's functions. Explicit expressions for the Green's functions and their gradients are developed. A critical factor in the equations is the derivative of the conductivity tensor with respect to the principal conductivity values and angles defining the axes of symmetry. Special cases such as an isotropic earth and tilted transversely isotropic media emerge from the general solutions.

Chapter 4 makes use of the given analytic and numerical sensitivity formulations to examine the various sensitivity patterns which emerge for different uniform anisotropic media and for various surface electrode array configurations. Pole-pole, pole-dipole, Wenner and square arrays produce distinctive patterns, valuable in assessing resolution. It was found that sensitivity patterns vary greatly for different anisotropic model parameters in terms of strength and shape, depending on the nature of the anisotropy.

Chapter 5 presents a reformulation for the description of anisotropic media in terms of the coefficient of anisotropy λ and the geometric mean (average) conductivity σ_m. Sensitivity functions are plotted and described for these parameters. Also comparison of sensitivity patterns for isotropic, homogeneous models with those for equivalent transversely isotropic medium parameters is given by plotting the ratios of the Fréchet derivatives of the anisotropic to the isotropic values. Prominent differences in both sign and magnitude are observed, especially for steep dips and strong anisotropy. The plots highlight the dangers of an isotropic assumption. Even for mildly anisotropic rock ($\lambda < 1.2$), there is potential for error in interpretation.

Chapter 6 presents 2.5D synthetic inversion experiments for various electrode configurations and anisotropic models. The experiments compare image reconstructions obtained using the correct anisotropic inversion code and those obtained using the false but widely used isotropic assumption. Superior reconstruction in terms of reduced data misfit, true anomaly shape and position is obtained when the correct anisotropic assumption is employed. When an isotropic inversion algorithm is used to invert anisotropic data, the images are dominated by patterns of banded artefacts and high data misfits. Various data sets were investigated and evaluated for the accuracy of the inversion result, the corresponding eigenspectra (information content) of the pseudo Hessian matrix and the relative resolution plots. An effective data selection strategy based on high sensitivity measurements is presented. It drastically reduces the

number of data to be inverted but still produces comparable results to that obtained from the comprehensive data set. Inversion was carried out using transversely isotropic model parameters described in two different co-ordinate frames for the conductivity tensor: Cartesian versus natural or eigenframe. The Cartesian frame provided a more stable inversion product. This can be explained by the differing magnitudes of the eigenspectra of the pseudo-Hessian matrix for the two model descriptions.

Originality Statement

I, Timothy Wiese certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

Timothy Wiese
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*** Publications Arising out of this Research:**

1. Greenhalgh S.A., Marescot L., Zhou B., Greenhalgh M., Wiese T.C., 2009. Electric potential and Fréchet derivatives for a uniform anisotropic medium with a tilted axis of symmetry. *Pure and Applied Geophysics* **166**, 673 – 699.

2. Greenhalgh S.A., Zhou B., Greenhalgh M., Marescot L., Wiese T.C., 2009. Explicit expressions for the Fréchet derivatives in 3D anisotropic resistivity inversion. *Geophysics* **74**, 31 – 43.

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5. Wiese T., Greenhalgh S.A., Zhou B., 2011. Resistivity inversion in 2D anisotropic media: numerical experiments. *Geophysical Prospecting*, (accepted subject to moderate revision, 2011).

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