# Trajectory Design for a Very-Low-Thrust Lunar Mission

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# Contents

# Nomenclature

## Notation

Bold text represents a vector. A hat (for example $\hat{\mathbf{r}}$) represents a unit vector. A quantity that is normally a vector that is not in bold (for example $r$) represents the magnitude of that vector. Parameters are relative to the central body of that phase, except where identified with an astronomical symbol.

| | |
|---|---|
| $\odot$ | Astronomical symbol for the Sun |
| $\oplus$ | Astronomical symbol for the Earth |
| ☾ | Astronomical symbol for the Moon |
| ♀ | Astronomical symbol for Venus |
| ♂ | Astronomical symbol for Mars |
| ♃ | Astronomical symbol for Jupiter |

**Chapter 3**

| | |
|---|---|
| $t_0$ | Start of the phase (symbolic) |
| $t_f$ | End of the phase (symbolic) |
| $\mathbf{p}$ | Set of optimisable parameters |
| $\mathbf{x}$ | Set of state parameters |
| $\mathbf{u}$ | Set of control variables |
| $F$ | Cost function |
| $\sigma$ | Cost function weighting factor (-) |
| $\mathfrak{L}$ | Lagrangian (see Section 3.3.3) (symbolic) |
| $\lambda_i$ | Equality Lagrangian/KKT multipliers (-) |

| | |
|---|---|
| $\mu_i$ | Inequality Lagrangian/KKT multipliers (-) |
| $\alpha$ | Optimisation step size (-) |

**Chapter 4**

| | |
|---|---|
| $\epsilon$ | Specific orbital energy ($m^2s^{-2}$) |
| $\epsilon_k$ | Specific orbital kinetic energy ($m^2s^{-2}$) |
| $\epsilon_p$ | Specific orbital potential energy ($m^2s^{-2}$) |
| $\mathbf{v}$ | Velocity of spacecraft ($ms^{-1}$) |
| $\mu$ | Gravitational constant of central body ($m^3s^{-2}$) |
| $\mathbf{r}$ | Distance of spacecraft from central body (m) |
| $I$ | Impulse ($ms^{-1}$) |
| $\mathbf{p}$ | Momentum ($kgms^{-1}$) |
| $I_{sp}$ | Specific impulse (s, see Section 4.8.1) |
| $g_0$ | Standard Earth gravity (9.80665 $ms^{-2}$, Bureau International des Poids et Mesures 1901) |
| $g(r)$ | Classic gravity relative to the primary body at $r$ metres from its centre ($ms^{-2}$) |
| $m_{exhaust}$ | Mass of exhaust (kg) |
| $v_{exhaust}$ | Exhaust velocity ($ms^{-1}$) |
| $\Delta v$ | Delta-v ($ms^{-1}$, see Section 4.8.2) |
| $m$ | Mass of spacecraft (kg) |
| $\mathbf{T}$ | Applied thrust (N) |
| $\mathbf{D}$ | Aerodynamic drag (N) |
| $\gamma$ | Velocity vector angle (°, see Figure 4.9) |
| $\alpha$ | Body axis angle (°, see Figure 4.9) |
| $\varepsilon$ | Thrust angle (°, see Figure 4.9) |
| $r_{SOI}$ | Radius of sphere of influence (m) |
| $a_s$ | Semimajor axis of the secondary body's orbit about the primary body (m) |
| $m_s$ | Mass of the secondary body (kg) |
| $m_p$ | Mass of the primary body (kg) |
| | |
| $\mathbf{r}$ | Position of spacecraft relative to primary body (m) |

| | |
|---|---|
| $\mathbf{v}$ | Velocity of spacecraft relative to primary body ($ms^{-1}$) |
| $a$ | Keplerian element semimajor axis (m) |
| $e$ | Keplerian element eccentricity (-) |
| $i$ | Keplerian element inclination (°) |
| $\omega$ | Keplerian element argument of periapsis (°) |
| $\Omega$ | Keplerian element longitude of the ascending node (°) |
| $\nu$ | Keplerian element true anomaly (°) |
| $p$ | Modified equinoctial element semilatus rectum (m) |
| $f$ | Modified equinoctial element f (-) |
| $g$ | Modified equinoctial element g (-) |
| $h$ | Modified equinoctial element h (-) |
| $k$ | Modified equinoctial element k (-) |
| $L$ | Modified equinoctial element true longitude (°) |
| $\hat{\mathbf{i}}_r$ | Unit vector in radial direction |
| $\hat{\mathbf{i}}_\theta$ | Unit vector tangential to primary body |
| $\hat{\mathbf{i}}_h$ | Unit vector in direction of orbital momentum |
| $\Delta_r$ | Total force acting on spacecraft in the $\hat{\mathbf{i}}_r$ direction (N) |
| $\Delta_\theta$ | Total force acting on spacecraft in the $\hat{\mathbf{i}}_\theta$ direction (N) |
| $\Delta_h$ | Total force acting on spacecraft in the $\hat{\mathbf{i}}_h$ direction (N) |
| $\boldsymbol{\Delta}_\mathbf{q}$ | Total force on spacecraft due to third bodies (N) |
| $\mathbf{d}_j$ | Position of third body $j$ relative to spacecraft (m) |
| $\mathbf{s}_j$ | Position of third body $j$ relative to primary body (m) |
| $\boldsymbol{\Delta}_\mathbf{g}$ | Total force on spacecraft due to primary body oblateness (N) |
| $J_2$ | Second zonal harmonic coefficient of Earth |
| $J_3$ | Third zonal harmonic coefficient of Earth |
| $J_4$ | Fourth zonal harmonic coefficient of Earth |

| | |
|---|---|
| $W$ | Orbital energy (J) |
| $\Phi$ | Energy due to angular momentum of orbit (J) |
| $V$ | Gravitational potential energy of orbit (J) |
| $\bar{P}_{nm}(\sin\phi')$ | Normalised associated Legendre polynomials |
| $C_{n,m}$ | Normalised gravitational coefficient |
| $S_{n,m}$ | Normalised gravitational coefficient |
| $r_{peri}$ | Periapsis of the orbit (m) |
| | |
| $\mathbf{\Delta_{\odot}}$ | Total force on spacecraft due to solar radiation (N) |
| $\beta$ | Optical reflection constant (-) |
| $A_{eff}$ | Effective cross-sectional area of spacecraft ($m^2$) |
| $r_{\odot}$ | Distance of satellite from centre of Sun (m) |
| | |
| $\mathbf{\Delta_T}$ | Total force on spacecraft due to thrust (N) |
| $\hat{\mathbf{u}}$ | Unit control vector governing thrust direction |

**Chapter 5**

| | |
|---|---|
| $E$ | Energy level in the batteries (J) |
| $P$ | Net power generation or consumption (W) |
| $Ln$ | Normalised longitude (-) |

**Chapter 6**

| | |
|---|---|
| $\eta$ | Power efficiency |
| $\alpha_u$ | Half-angle of umbral cone (°) |
| $\alpha_p$ | Half-angle of penumbral cone (°) |
| $R_{\odot}$ | Radius of the Sun (m) |
| $R_{\oplus}$ | Radius of the Earth (m) |
| $\mathbf{r}_{\oplus}$ | Position of the Earth from the Sun (m) |
| $\mathbf{r}_{☾}$ | Position of the Moon from the Sun (m) |
| $Q$ | Solar energy flux ($Wm^{-2}$) |
| $\eta_a$ | Area efficiency of solar cells (-) |
| $\eta_c$ | Power efficiency of solar cells (-) |
| $\eta_{DC}$ | Power efficiency of voltage regulator (-) |

| | |
|---|---|
| $\Psi_\odot$ | Angle of Sun on solar panels (°) |
| $\mathfrak{R}$ | Power degradation of solar cells (-) |
| $\mathfrak{F}$ | Equivalent fluence of solar cells (-) |

# Acronyms

| | |
|---|---|
| AOCS | Attitude & Orbit Control System |
| ASTOS | Aerospace Trajectory Optimisation Software |
| BFGS | Broyden-Fletcher-Goldfarb-Shanno |
| CAD | Computer Aided Design |
| CAMTOS | Collocation and Multiple Shooting Trajectory Optimisation Software |
| CGA | Constrained Genetic Algorithm |
| COTS | Commercial Off-The-Shelf |
| CNES | Centre National d'Études Spatiales |
| DLR | Deutsches Zentrum für Luft- und Raumfahrt |
| DSN | Deep Space Network |
| EADS | European Aeronautic Defence and Space Company |
| ECI | Earth Centred Inertial |
| ECR | Electron Cyclotron Resonance |
| EML | Earth-Moon Lagrange point |
| ESA | European Space Agency |
| ESOC | European Space Operations Centre |
| ESTEC | European Space Research and Technology Centre |
| ET | Ephemeris Time |
| GCR | Galactic Cosmic Ray |
| GESOP | Graphical Environment for Simulation and Optimisation |
| GEO | Geostationary (Earth) Orbit |
| GSLV | Geosynchronous Satellite Launch Vehicle |
| GTO | Geosynchronous Transfer Orbit |
| HEO | High Earth Orbit |
| HLO | High Lunar Orbit |

| | |
|---|---|
| IAU | International Astronomical Union |
| ICRF | International Celestial Reference Frame |
| IEEE | Institute of Electrical & Electronic Engineers |
| IERS | International Earth Rotation Service |
| IFR | Institut für Flugmekanik und Flugregelung |
| IRS | Institut für Raumfahrtsysteme |
| ISRO | Indian Space Research Organisation |
| ITRF | International Terrestrial Reference Frame |
| JAXA | Japanese Aerospace Exploration Agency |
| JD | Julian Date |
| JGM3 | Joint Gravity Model 3 |
| JPL | Jet Propulsion Laboratory |
| KKT | Karush-Kuhn-Tucker |
| LEO | Low Earth Orbit |
| LLO | Low Lunar Orbit |
| LP165 | Lunar Prospector Gravity Model, degree and order 165 |
| NASA | National Aeronautics & Space Administration |
| NIMA | National Imagery & Mapping Agency |
| NLP | Non-Linear Programming |
| ODE | Ordinary Differential Equation |
| PPT | Pulsed Plasma Thruster |
| PROMIS | Parameterised tRajectory Optimisation by direct MultIple Shooting |
| PTFE | Polytetrafluoroethylene (Teflon™) |
| SEL | Sun-Earth Lagrange point |
| SEPTOP | Solar Electric Propulsion Trajectory Optimization Program |
| SIMPLEX | Stuttgart Impulsing MagnetoPlasmadynamic thruster for Lunar EXploration |
| SNOPT | Sparse Nonlinear OPTimiser |
| SOCS | Sparse Optimal Control Software |
| SOI | Sphere of Influence |
| SPE | Solar Particle Event |
| SQP | Sequential Quadratic Programming |

| | |
|---|---|
| SSO | Sun Synchronous Orbit |
| STK | Satellite Tool Kit |
| TALOS | Thermal Arcjet for Lunar Orbiting Satellite |
| TLI | Trans-lunar Injection |
| TROPIC | Trajectory OPtimisation by dIrect Collocation |
| TT | Terrestrial Time |
| UTC | Universal Coordinate Time |

# Abstract

The University of Stuttgart is conducting a research program to build a succession of small satellites. The ultimate goal of this program is to build and launch a craft named *Lunar Mission BW-1* (after the federal state that Stuttgart is situated in, Baden-Württemberg) into lunar orbit, for eventual impact with the Moon. As with the majority of space missions, launch cost is a severely limiting factor so it is necessary to carefully plan the trajectory before launch, to ensure lunar capture and minimise the amount of fuel needed by the spacecraft.

This thesis outlines work conducted to find a robust fuel-optimal trajectory for *Lunar Mission BW-1* to reach the Moon. Several unique aspects of this craft require a novel approach to that optimisation. Firstly, the spacecraft uses a new low-cost propulsion system, severely limiting maneouvrability and accessibility of transfer trajectories. Secondly, to reduce the mass and complexity of moving parts, the solar panels are fixed to the body; consequently, the craft must rotate itself to point its solar panels towards the Sun to recharge. No thrusting can occur during this time. This magnifies the effect of the third design decision, which is to restrict the dry mass of the craft by giving it very little on-board power storage. After approximately an hour of accelerating it is expected to need to coast for several hours to recharge its batteries, resulting in a relatively high frequency stop-go-stop thrust profile.

Due to these constraints, the trajectory optimisation is one of the most complex ever attempted. Since the craft will be built and launched, many simplifications made in purely theoretical studies could not be utilised, such as neglecting the weaker forces acting on the spacecraft in cis-lunar space.

The very low thrust results in very long transfer times, during which even small magnitude forces acting on the spacecraft can significantly perturb its trajectory. However, including these forces creates non-linearities in the equations of motion associated with spacecraft trajectories, limiting the optimisation methods that could be used, and increasing computational complexity.

Optimisation methods for low-thrust spacecraft trajectories have been the subject of much research, but most studies conclude that knowledge is still lacking in this area. Furthermore, many optimisation methods investigated in existing literature are incompatible with the intermittent thrust profile required by the *Lunar Mission BW-1* thrusters. For this reason it was necessary to thoroughly review available optimisation methods and determine which may be adapted to this scenario. The resulting optimisation method was applied to the *Lunar Mission BW-1* scenario to determine an efficient thrusting profile that will get the craft to the Moon.

It was found that very few established optimisation algorithms can support the number of variables required for such a complex, long duration trajectory. The Sparse Optimal Control Software (SOCS) marketed by The Boeing Corporation was used via an interface developed at the University of Stuttgart called the the Graphical Environment for Simulation and Optimisation (GESOP). Due to unknown constraints such as launch date, the phases defined by the mission architecture were modelled and optimised independently. This approach allows mission planning flexibility while still providing reliable estimates for optimal fuel use, mission duration and power limitations.

A trajectory is presented for each of the phases, ascending from the intial geosynchronous transfer orbit (GTO) to the eventual low lunar orbit (LLO). The resulting science phase is propagated forward in time to ensure orbital lifetime meets the mission requirements. Recommendations are subsequently made for the continuing development of the mission architecture.

The primary outcome of this study is a procedure for developing an operational trajectory for *Lunar Mission BW-1* after launch details are

known. Given the current mission architecture and assumed launch details, the thermal arcjet requires 1205 hours (50.2 days) of operation while consuming 93 kg of ammonia propellant, and the pulsed plasma thrusters require 29177 hours (3.3 years) of operation while consuming 19 kg propellant. Power constraints were not found to be mission limiting for the current spacecraft configuration. Consequently, although the laboratory testing burden on the PPTs is already quite heavy, it is recommended that the mission architecture be adjusted to shorten arcjet phases and lengthen PPT phases. Furthermore, this project found that the optimisation package SOCS was the best commercially available option for low-thrust trajectory optimisation, but that it would benefit greatly by adaptation to a parallel shooting algorithm that may be distributed amongst multiple computer processors.

# Statement of Originality

I, Rogan Shimmin, certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Rogan Shimmin

Date

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