Reliability Analysis of Water Distribution Systems Using Graph Decomposition

Jochen Deuerlein*, Andreas Wolters*, Dietmar Roetsch** and Angus R. Simpson***

* 3S Consult GmbH, 30827 Garbsen, Germany,
** Energietechnik & Ingenieursoftware, 06844 Dessau, Germany
*** School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide SA 5005,Australia.

Abstract

Water supply utilities are increasingly being confronted with growing maintenance requirements in order to provide a reliable and cost efficient operation of the water distribution system in order to deliver high standard drinking water. One specific challenge is to find a satisfactory compromise between the contradictory objectives of saving investment cost and meeting the increasing expectations of customers. A valuable tool for the planner is reliability analysis of the system that for instance provides information about the importance of each particular pipe in determining the total reliability of the system. This information can be used to decide which pipes should be replaced next or potentially duplicated to reduce the risk of system isolation. Since structural (topological) reliability only considers the connectivity of the network a new approach for the calculation of structural hydraulic reliability is presented in this paper. The algorithmic implementation is based on a decomposition method of the network graph that greatly enhances both the calculation of pure structural reliability as well as structural hydraulic reliability.

INTRODUCTION

The calculation or estimation of the reliability of a system is a very important issue in all fields of engineering. Reliability and associated key performance parameters are becoming more and more the driving forces for decision making in water distribution network planning and management. There exist different concepts for the definition and calculation of water supply network reliability (Ostfeld, 2004). Xu and Goulter (1998, 1999) use a probabilistic hydraulic simulation model where the nodal demands, tank levels and pipe roughness values are stochastic parameters. The reliability of the system is defined as its capability of satisfying the customer's demands with sufficient pressure availability. Tolson et. al. (2001) distinguished between mechanical and hydraulic failures as well as reliability. Mechanical failures are caused by the components of the system such as pipes and pumps whereas hydraulic failures are the consequence of insufficient pressure conditions.

In this paper the mechanical reliability is called component reliability. It is defined as the probability that a particular component will satisfactorily perform a required function under predefined conditions for a specified period of time. In water supply systems analysis the reliability of the pipelines, pumping stations, control devices such as regulating valves and water treatment plants is of special interest. The reliability of pipes having similar age, material and diameters can be derived for example from defect statistics of such a pipe group. Different stochastic approaches are used for transferring the statistical data into a probability of failure. For a more detailed explanation of the calculation of component reliability the reader is referred to literature (e.g. Mays and Ozger, 2004).

In contrast to the component reliability that is a particular characteristic of a component in the water distribution system, the structural network reliability is a property of the topological composition of the entire distribution system and the reliability of its components. Theoretical network reliability models may be used for the estimation of the probability that the total system performs well under given conditions for a certain period of time. Basically, three types of structural reliability may be distinguished: The *k-out-of-n-reliability* and their special cases for $k = n$ and $k = 2$. The *2-out-of-n-reliability* is also known as source – sink reliability and if $k = n$ the *n-out-of-n-reliability* is called all terminal reliability. Different methods exist for both the exact calculation of the structural (topological) reliability as well as simulation methods. Exact methods include for example full enumeration, minimum-path and minimum-cut-methods, inclusion – exclusion and factorization. A few of these methods will be outlined in this paper. A good overview of different methods can be found in Lucet and Manouvrier (1997) and Kuo (2002).

In the analysis of a hydraulic system like a water distribution network, the structural reliability is applicable only to a limited extent because the connectivity of a system does not guarantee that all customers can be supplied with sufficient pressure. On the other hand hydraulic reliability (sufficient pressures under different load cases) does not consider the topology of the system sufficiently. Consequently in this paper a new concept of structural hydraulic reliability is introduced that is based on the hydraulic supply reliability of the demand nodes. The computationally expensive calculations of structural reliability are simplified by use of decomposition of the network graph. In a general water supply network serial and parallel subsystems can be distinguished from so called complex subsystems. Subsystems are identified by application of a graph decomposition approach that subdivides the total system graph into 1-connected (trees and bridges) components and 2-connected (blocks) components. The structural property of the system describing the interconnectivity of its serial and parallel components is modelled by the so called block graph tree (Deuerlein 2008). Since real water distribution systems usually consist of different components such as main transmission lines (often bridge components), supply and pressure zones (often blocks) and secondary distribution pipes including house connections the calculation time can be dramatically reduced by application of the graph decomposition method.

First, the calculation of structural network reliability is explained using a simple example and the application of graph decomposition is also presented. So far hydraulic issues have not been considered in the analysis. This is done in the second part of the paper where the hydraulic reliability analysis is presented. The nodal supply reliability is introduced followed by an explanation of the so called (*m-0*)-analysis and (*m*-1)-analysis. In the former, the reliability is calculated assuming that all pipes are available. With the latter the so called importance of each pipe of the network can be calculated as the difference between the reliability of the configuration including the particular link and the reliability without that link. In contrast to the definition of hydraulic reliability based on random demands using Monte-Carlo Simulation techniques, the structural hydraulic reliability is calculated for one particular load case.

THEORETICAL FORMULATION OF NETWORK RELIABILITY

General Concepts

The term reliability denotes the probability that a system performs a certain operation at a certain time under predefined conditions. A system usually is composed of several components. Consequently, the reliability of the system is a function of the reliability of its components. There are different kinds of physical systems that can be distinguished by the configuration of their components. There exist parallel and serial systems as well as so called complex systems. The configuration of the components highly affects the complexity of the calculation of system reliability.

Water distribution networks are an example for such complex systems. In terms of reliability modeling the water distribution system is represented by a so called stochastic graph. In a particular state the configuration of the graph depends on the current state of its components. Let us assume that the components have exactly two functioning states and thus failure may be expressed by the Boolean variable x_i with $x_i = 1$, if component i is functioning and $x_i = 0$ if it is not functioning. Let us further assume that only the links of the graph representing connecting components of the network such as pumps, pipes and valves are subject to failure and that the nodes are perfect. Then each state k of the stochastic network graph can be expressed by the vector $\mathbf{x}_k$ of its entire link set. Furthermore let p_j be the probability that a link j is functioning and $q_j = 1 - p_j$ the probability that it is not. The probability of a state k of the graph G can then be calculated (Lucet and Manouvrier, 1997) by

$$\Pr(G_k) = \prod_{j=1}^{m} \left[[\mathbf{x}_k]_j \, p_j + \left(1 - [\mathbf{x}_k]_j\right) q_j \right] \tag{1}$$

The reliability of the system is the probability that it fulfills a certain expectation. In general three kinds of network reliability are distinguished: The *2-out-of-n-reliability*, the *k-out-of-n-reliability* and the *n-out-of-n-reliability*. The *2-out-of-n-reliability* is also known as source-sink-problem and for the calculation of the all terminal reliability, the system is classified as functioning if there exists at least one spanning tree of the network graph. The different reliabilities can be calculated as the sum of the probabilities of system states where the network graph meets its specific requirements (J: set of all those states):

$$R(G) = \sum_{k \in J} \Pr\left(G_k\right) \tag{2}$$

Theoretically, the exact calculation of network reliability could be carried out by full enumeration of all possible states of the system. However, in reality, full enumeration is impractical for real existing systems with hundreds of thousands of links due to the extremely large number of combinations that would need to be enumerated. The total number of possible states of the system is 2^m. For only 100 network links this number is 1.2677e+30. The network reliability problem belongs to the class of NP-hard problems (see for example Ramesh et. al. 1987).

Serial Systems

Since a serial system functions only if all its components are functioning, its reliability depends heavily on the reliability of its weakest component. Consequently, a serial system in total is always only as good as its weakest component. The calculation of the reliability of the serial system S can be derived from Eq. (1) and Eq. (2) and results in the simple expression:

$$R(S) = \prod_{j=1}^{m} p_j \tag{3}$$

Parallel Systems

In contrast to serial systems, a parallel system is intact if at least one of its components works properly. In other words, a parallel system only fails if all its components fail. The reliability of a parallel system P is 1 minus the probability that the system fails. Using again equations (1) and (2) the formula for the calculation of the reliability of the parallel system is:

$$R(P) = 1 - \prod_{j=1}^{m} q_j = 1 - \prod_{i=j}^{m} \left(1 - p_j\right) \tag{4}$$

Complex Systems

Systems that do not fall under the category of serial or parallel systems are denoted as complex systems. As mentioned above the reliability of those systems theoretically can be calculated by a full enumeration as the sum of probabilities of all feasible states (see Eq. (1) and Eq. (2)). Due to the great amount of calculation time required by this method the reliability of complex networks is usually determined by other approaches. Methods include path enumeration where all possible paths between the terminals are investigated and the cut enumeration where all minimal cuts leading to an infeasible system state are considered.

Theoretical Concepts of Component Importance

The importance of a network component is a measure for the change in structural reliability of a general system (graph G) if the component either is removed or fails. There exist qualitative and quantitative methods. One example is the so called reliability or Birnbaum importance $I_{B,j}$ (B-importance) which is defined by the equation

$$I_{B,j} = \frac{\partial R(G)}{\partial p_j} \tag{5}$$

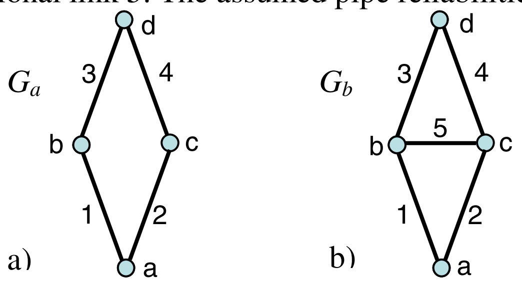
The B-Importance is a measure for the change of system reliability as a result of a change of reliability of component j (Kuo, 2002). In general the analytical solution of Eq. (5) is very difficult for large systems. Another method of estimating the component importance is derived from the so called (m-1)-analysis. The reliability calculation as described above is called (m-0)-reliability since all network links are considered. Accordingly, the (m-1)-reliability is the reliability of the reduced system where one link is removed. The (m-1)-reliability of the reduced system can be calculated for each link with the same methods. Using the results of the (m-0)- and (m-1)-analysis the reliability importance $I_{B,j}$ of the network link j of graph G can be simply approximated by the difference of the two reliabilities for the system with and without component j:

$$I_{B,j} = R(G)_{(m-0)} - R(G)_{(m-1)_j} \tag{6}$$

A complete (m-1)-reliability analysis requires the calculation of m system configurations and is very time consuming. After the example in the following section the application of the decomposition of a general network graph as described in Appendix I to a more efficient calculation of structural network reliability will be outlined.

Example

Figure 1 shows the graphs of two simple networks. Network a) consists of four links, four nodes and one loop. In network b) the nodes b and c are connected by an additional link 5. The assumed pipe reliabilities range from 0.6 for link 4 to 0.9 for link 1.



Link	p
1	0.9
2	0.8
3	0.7
4	0.6
5	0.85

Figure 1: Example networks: a) rhombus, b) bridge system

The two-terminal reliability of the network is to be calculated. Node a is the source node and node d is the sink node. From the section above it follows that in case a) the network can be divided into serial and parallel components. The paths a-1-b-3-d and a-2-c-4-d are alternative paths from node a (source) to node d (sink). The two paths consist each of a serial system of two links. Consequently, the two-terminal-reliability of the system can be calculated using Eq. (3) and Eq. (4):

$$R(G_a) = 1-(1-p_1 p_3)(1-p_2 p_4) = 1-(1-0.63)(1-0.48) = 0.8076$$

The same result is obtained by use of the general formulations of Eq. (1) and Eq. (2).

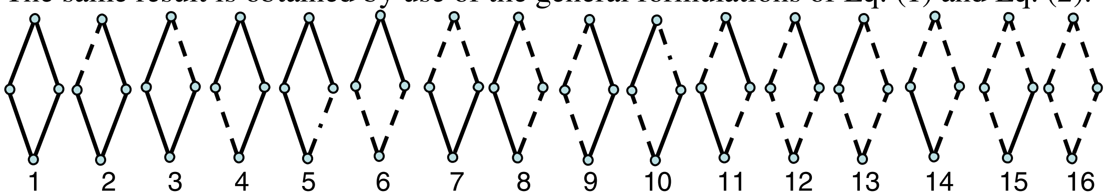


Figure 2: Full enumeration of possible system states of network G_a

A full enumeration of feasible states of the system (see Figure 2) shows that only the states 1, 2, 3, 4, 5, 8, 9 are feasible since in all other cases node a and node d are disconnected. Let J={1, 2, 3, 4, 5, 8, 9} be the set of feasible states. The reliability calculation using Eq. (1) and Eq. (2) can then be written as

$$R(G_a) = \sum_{i \in J} \prod_{j=1}^{m} \left[[\mathbf{x}_i]_j p_j + (1-[\mathbf{x}_i]_j) q_j \right] = p_1 p_2 p_3 p_4 + p_1 p_2 q_3 p_4 + p_1 p_2 p_3 q_4 +$$

$$q_1 p_2 p_3 p_4 + p_1 q_2 p_3 p_4 + p_1 q_2 p_3 q_4 + q_1 p_2 q_3 p_4 = 0.8076,$$

which agrees with the value computed previously. Now let us consider system b) with the additional link 5. In this case the system can not be broken down into serial and parallel components. The number of possible system states is $2^5 = 32$, twice the number as in case a. Its two-terminal-reliability can be calculated for example by a full enumeration (result: $R(G_b) = 0.85418$).

Calculation of structural reliability using graph decomposition

It has been shown before that the calculation of structural reliability of serial and parallel systems is straight forward in contrast to that of complex systems. A serial representation of a complex system is its block graph tree (see Appendix I). From the right system in Figure 4 in Appendix I it is clear that if the reliabilities of the virtual block links or the connection nodes were known then the supply reliabilities of the block graph tree nodes could be calculated easily. For example if we know the reliability of the root node of tree 1 the supply reliability of the tree can be calculated as the product of the root node reliability and all tree link reliabilities. However, in general the blocks are complex systems that can not be reduced to combinations of serial and parallel systems and other methods need to be used.

Block reduction

The advantages of the application of the graph decomposition can be demonstrated by the comparison of the number of system states that have to be considered using a full enumeration of possible states of the system in Figure 4. Without application of the graph decomposition the total number of possible states is: $2^{19} = 524{,}288$. Using the knowledge of the block graph tree structure the calculation of the reliability by full enumeration is only required for the complex systems of block 1 and block 2. Following the example of Figure 1b) the number of system states to be calculated is 32+32=64. However for real networks the number of pipes within a block often prevents the application of a full enumeration even for separate blocks.

Path element reduction

For the simplification of the block reliability calculation the nodes of the block are distinguished according to their nodal degree (number of connected links). A node with degree two is called an inner path node, a node with degree greater or equal three is denoted as path node. The link sequences without bifurcation between two path nodes are called path elements. Figure 3 shows a looped block subgraph and its path representation. The reliability calculation of a path element can be performed using the formula for serial systems. In the original system the full enumeration of the remaining complex system consists of $2^{17}=131{,}072$ different states whereas in the reduced system only $2^{11}= 2048$ states have to be calculated.

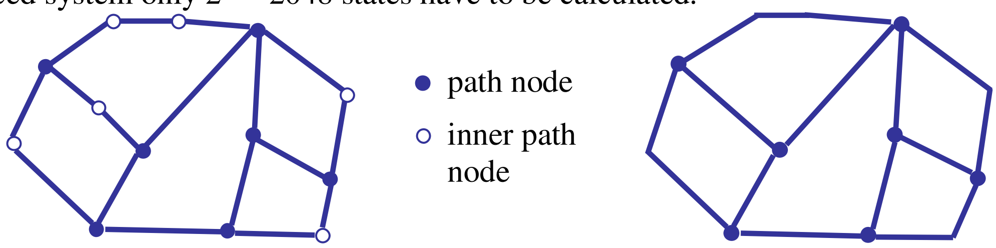


Figure 3: Path reduction of a looped block.

RELIABILITY CALCULATION OF WATER SUPPLY NETWORKS

General assumptions

The structural reliability calculation as explained above is suitable only to a limited extent for hydraulic systems analysis. In order to supply the customers with a sufficient amount of water it is not only sufficient that all nodes of the system are connected to a water source (topological criterion). In addition, the pressure heads at all demand nodes must be above a certain minimum allowable pressure. A full enumeration approach of hydraulic systems reliability would include the consideration of all 2^m possible states of the network similar to the structural reliability calculation above followed by hydraulic steady-state calculations or even extended period simulations of topologically feasible system states for different loading conditions. Since an exact calculation that considers all the different aspects is not possible for real existing water distribution systems assumptions and simplifications of the problem are needed.

In this paper a new concept for the supply reliability of a water distribution system is introduced that combines a hydraulic steady-state calculation with structural reliability analysis. In order to get an idea about the importance of each pipe of the system a so called (m-1)-analysis is carried out. Each (m-1) case includes the calculation of the hydraulic steady-state of the system followed by a topological (or structural) analysis of the system based on the particular flow distribution (as a result of the (m-1)-topology and the particular load case). In particular the (m-1)-calculations require the consideration of pressure dependency of nodal demands.

The calculation of the reliability of water supply networks in this paper is based on the following assumptions.

1.) For the definition of the stochastic network graph only the pipe states are subject to uncertainty. The nodes are assumed to be perfect and also pipe failures are assumed to be stochastically independent.
2.) The probability of failure of water treatment plants and storage tanks is considered as the reliability of the virtual links.
3.) It is assumed that both structural as well as hydraulic reliability of the virtual ground nodes is 1.

In the following measures for the reliability of the system and the supply reliability of a network node are introduced.

Nodal supply reliability

A very important piece of information in hydraulic systems analysis is the probability that a certain node is supplied with adequate amount of water under sufficient pressure conditions. This probability is denoted as nodal supply reliability $NSR_i \in [0;1]$. Its corresponding value is the probability of insufficient supply $NFP_i = 1 - NSR_i$. It can be visualized as the time fraction for which the node is not sufficiently supplied (e. g. a value of 0.00228311 means 20 hours/year). The nodal supply reliability $NSR_{i,(m-0)}$ of node i of the full system is distinguished from $NSR_{i,(m-1)}$ of the reduced system which is called (m-1)-nodal supply reliability (with $R_{s,i}$: structural 2-out-of-n-reliability, D_i, (Q_i): demand, (withdrawal) of node i):

$$a)\quad NSR_{i,(m-0)} = R_{s,i}\,\frac{Q_i}{D_i} \qquad b)\quad NSR_{i,(m-1)} = \frac{1}{m}\sum_{j=1}^{m} R_{s,i,(m-1)_j}\,\frac{Q_{i,j}}{D_i} \qquad \textbf{(7)}$$

Network reliability definition

The definition of network reliability in this paper is a combination of both hydraulic as well as structural reliability (Index HS) based on nodal supply reliabilities. The reliability of the system is defined as the mean value of all nodal supply reliabilities.

$$a)\quad R_{HS,(m-0)}(G) = \frac{1}{n}\sum_{i=1}^{n} NSR_{i,(m-0)} \quad and \quad b)\quad R_{HS,(m-1)}(G) = \frac{1}{n}\sum_{I=1}^{n} NSR_{i,(m-1)} \qquad (8)$$

The information that can be gained from this reliability formulation is more significant to hydraulic systems analysis than the pure structural reliability since it takes also into account the hydraulic properties of the system. An additional benefit is that the nodal supply reliabilities can be weighted, for example, by the demand and further scale factors for important customers like hospitals, etc.

Hydraulic pipe importance

The definition of the hydraulic importance of a network pipe is based on the (m-1)-hydraulic steady-state calculation with pressure dependent demands.

$$I_{H,j} = 1 - \frac{Q_{total}(n-1)_j}{Q_{\text{total}}(n-0)} \qquad (9)$$

$Q_{total}(n-0)$ denotes the total withdrawal of the system in the (m-0) case and $Q_{total}(n-1)_j$ is the maximal withdrawal with respect to the pressure dependency of the nodal demands for the network without pipe j. The hydraulic importance of a pipe can take values between 0 and 1. If as a result of the (m-1)-calculation all nodes of the system can be supplied with sufficient pressure then the value for the hydraulic importance of the corresponding pipe is zero. In contrary, if all the nodes of the system are disconnected by a pipe break, for example of a transmission line, then the hydraulic importance of this pipe is one. The hydraulic simulation algorithm used for the hydraulic importance calculations must be capable of simulating the pressure dependent behavior of the nodal demands realistically.

Algorithmic calculation of nodal supply reliability

As explained before the (m-1)-analysis consists of the calculation of structural reliability for a particular load case with one link removed from the network graph. A full (m-1)-reliability calculation including the calculation of the hydraulic importance of all pipes of the network requires the number of pipes independent steady-state calculations. Since nowadays the usage of 1:1 network models has become more and more common practice in hydraulic system analysis the number of network pipes can often exceed 100,000. Even with respect to the increasing speed of modern computers a full (m-1)-analysis would result in unacceptable calculation time. Therefore it is very important to use as much topological information as possible for simplifications of the water distribution system. In particular the (m-1)-calculations can take advantage of the knowledge of the different network components and their interconnectivity.

For the sake of simplicity it is assumed in the following description that under normal (m-0) - conditions all demands at the network nodes can be satisfied with sufficient pressure.

Algorithmic steps:

1.) (m-0)-calculation of hydraulic steady-state and nodal supply reliabilities of the total system using graph decomposition as described above and in Deuerlein (2008). Calculation of system reliability by Equation (8 a).

2.) (m-j)-calculations of hydraulic steady-state and nodal supply reliabilities for all pipes $j = 1,\ldots,m$ by using the following subdivision of cases:
Case 1: Pipe j is part of the 1-connected subgraph (Serial systems bridges and trees):
The network graph is subdivided into two separate components $C_{1,j}$ and $C_{2,j}$. The component $C_{2,j}$ that is not connected to the source node is disconnected from any supply node. In this case the hydraulic importance (see Eq. (9)) of the corresponding pipe can be stated simply as the ratio of the sum of demands of the disconnected components to the total network demand, without the need for a hydraulic steady-state calculation. In this case the nodal supply reliabilities of the disconnected nodes are zero as well as the topological all terminal reliability. The decrease of the total system demand has no effect on the reliabilities of the nodes of the supplied component since it was assumed that in the (m-0) case all nodes are fully supplied and the decrease in demand results in higher pressures in component $C_{i,j}$.
Case 2: Pipe j is part of the 2-connected subgraph (complex system block):
The (m-1)-calculation is carried out only for the block that pipe j belongs to. After the calculation it is checked if all the demand nodes of the block and all subsequent components (other blocks, bridges or tress) are above the minimum allowable pressure. If not, a pressure driven calculation of these components is carried out. The preliminary blocks are not negatively affected (only a rise of pressure is possible there). The topological reliabilities of the preliminary blocks are not affected at all; the ones of subsequent blocks can be easily updated by multiplication with the new cut node reliabilities. The reliability calculation within the block takes advantage of the path elimination (Figure 3). The algorithm processes only path elements and path nodes. The path reliabilities can be easily calculated from the pipe reliabilities by using the formula for serial systems. However, for real existing distribution networks with a large number of pipes and nodes this process may be still a very time consuming task.

3.) Calculation of the (m-1)-nodal supply reliabilities and (m-1)-system reliability: After all the $(m\text{-}1)_j$-cases are calculated the (m-1)-nodal supply reliability and the (m-1)-reliability of the total system follow from Eq. (7b) and Eq. (8b), respectively.

SUMMARY AND CONCLUSION

A new approach for the estimation of the water distribution system reliability has been presented in this paper. The main difference in comparison to other methods is that it is based upon both structural reliability as well as network hydraulics. The consideration of pressure dependent demands requires a hydraulic solver that is capable of performing correct pressure driven analysis in hydraulic steady-state calculations. Although the graph decomposition approach is a promising tool for simplification of the analysis the structural reliability calculations are still a very time consuming task for large distribution networks having multiple redundancies. The graph decomposition supports the enhancement of the algorithm, yet more research needs to be done for the development of more sophisticated methods for a simplified and approximate calculation of the block reliability.

APPENDIX I: Graph Decomposition

In Deuerlein (2008) the decomposition of a general water supply network graph is described. An example is given in Figure 4. Firstly, a virtual ground node (node “r” in Figure 4) is introduced that is connected with all input nodes such as reservoirs and tanks (nodes “m”, “n”) by so called virtual links (links “16”, “17”). The first step of the graph decomposition consists of the identification of connected components that are further subdivided into 1-connected and 2-connected components. The 2-connected subgraph includes the looped blocks of the network graph. The 1-connected subgraph can be further subdivided into bridge components (link “4”) and the forest that is composed of several trees. Whereas the link sets of the graph components are distinct, the nodal sets are overlapping in the so called connection nodes.

Three different types of connection nodes are distinguished: The root nodes connecting a tree with a block (nodes “k”, “e”) or a bridge, the bridge terminal nodes that a bridge and the 2-connected blocks have in common (nodes “c”, “d”) and the articulation nodes that directly connect two looped blocks (node “l”). A very useful simplification of the network graph is the block graph tree where all looped blocks are substituted by a single block node that is connected with all cut nodes of the block. The application of the decomposition of the network graph and the block graph tree representation will be used for a more efficient calculation of network reliability of water supply networks. Figure 4 shows the decomposition of a simple network with three blocks B_1, B_2, B_3, one bridge and two Trees T_1 and T_2.

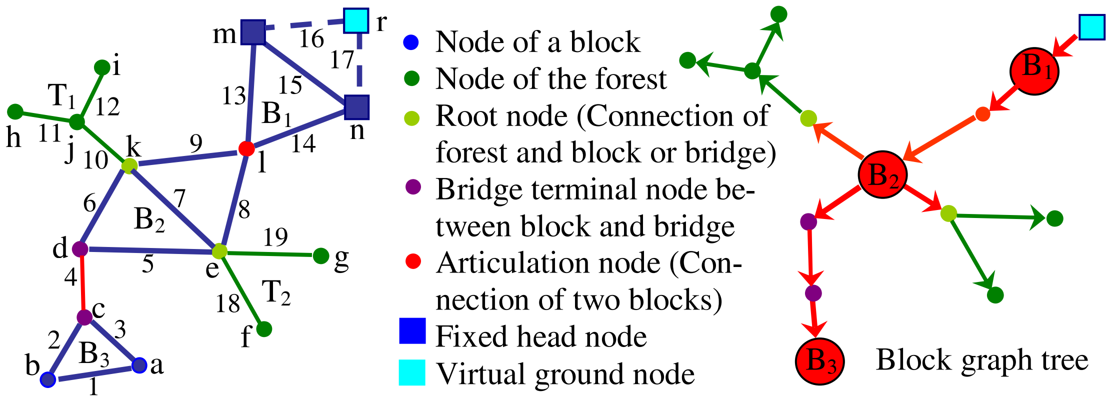


Figure 4: Graph Decomposition of an example network and its block graph tree

APPENDIX II: Notation and Abbreviations

n:	Number of nodes
m:	Number of pipes
D_i:	Demand at node i
Q_i:	Calculated outflow at node i
$NSR_{i,(m-0)}$ (or NSR_i):	(m-0)-reliability of node i (total system including all pipes)
$NSR_{i,(m-1)}$:	(m-j)-reliability of node i for the system without pipe j
$NSR_{i,(m-1)_j}$:	(m-1)-reliability of node i: mean value of all $NSR_{i,(m-j)}$
$R(G)_{(m-k)}$	(m-k)-structural reliability of G (k=0, 1)
$R_{HS}(G)_{(m-k)}$	(m-k)-$h$ydraulic supply reliability *of* G (k=0, 1)
$I_{B,j}$:	Reliability (Birnbaum) importance of pipe j
$I_{H,j}$:	Hydraulic importance of pipe j

REFERENCES

Deuerlein, J. (2008): "Decomposition model of a general water supply network graph". *Journal of Hydraulic Engineering*, 134(6), 822-832.

Kuo, W. (2002). *Optimal reliability modeling : Principles and applications*. Hoboken, NJ, John Wiley & Sons, Inc.

Lucet, C. and J. Manouvrier (1997). *Exact methods to compute network reliability*. In MMR'97, 1st International Conference on Mathematical Methods in Reliability, Bucharest, Romania.

Mays, L. and Ozger, S. (2004): "*Optimal location of isolation valves in water distribution systems: A reliability/optimization approach*." URL: http://www. public.asu.edu/~lwmays/Ch07_Mays_144381-9.pdf (January, 16th, 2009).

Ostfeld, A. (2004): "Reliability analysis of water distribution systems." *Journal of Hydroinformatics*, 6(4), 281 – 294.

Ramesh, A.; Ball, M. O.; Colbourn, C. J. (1987): Bounds for all terminal-reliability in planar networks. *Annals of Discrete Mathematics*, 33, 261-273.

Tolson, B. A.; Maier, H. R.; Simpson, A. R.: (2001) : **"***Water distribution network reliability estimation using the First-Order Reliability Method.*" World Water and Environmental Resources Congress 2001 Don Phelps, Gerald Sehlke, Editors Orlando, Florida, USA, May 20–24.

Xu, C and Goulter, I. C (1998): "Probabilistic model for water distribution reliability." *Journal of Water Resources Planning and Management*, 124(4), 218–228.

Xu, C and Goulter, I. C (1999): "Reliability-based optimal design of water distribution networks." *Journal of Water Resources Planning and Management*, 125(6), 352–362.