Closure to “The Jacobian for solving water distribution system equations with the Darcy-Weisbach head loss model"

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The authors would like to thank Prof Brkic for his interesting and thoughtful comments.

Use of Hazen-Williams versus Darcy-Weisbach

While the authors agree that the Darcy-Weisbach equation is preferred over the Hazen-Williams it must be remembered that many water utilities have been using the Hazen-Williams formulation for a very long time and it is unlikely that they will change to the Darcy-Weisbach model any time soon. As a result, practitioners and students must be familiar with the Hazen-Williams equation and its use.

Balance between complexity and accuracy

The comment by Prof Brkic suggesting the necessity for balance between the computational effort required for accuracy and the complexity is indeed important. In many contexts the effort required is in the implementation of program code and will arguably be worthwhile since it is made just once. Nevertheless, a detailed complexity analysis of the various methods available is well overdue.

Laminar flow friction factor formula implementation

In the commentary surrounding Eq (18) of the discussion by Prof Brkic it is correctly stated that the Darcy-Weisbach friction factor for the laminar flow region can be written f = 64/R. And it does indeed follow that networks in which the flows in all pipes are laminar lead to systems of equations which are linear. However, the fact that the resistance factor for laminar flow is independent of flow is very important for the correct implementation of program codes to solve the systems of equations where zero flows may occur (Elhay & Simpson 2011). To see this consider the case of a pipe in which the head loss is modeled by the Darcy-Weisbach formula. The head loss for this pipe is given, for friction factor f, by

$$h_f = f\frac{LV^2}{2gD} = f\frac{LQ^2}{2gDA^2} = f\frac{8LQ^2}{\pi^2 gD^5} \tag{1}$$

Now, for laminar flow

$$f = \frac{64}{R} = \frac{64v}{VD} = \frac{16\pi vD}{Q} \tag{2}$$

and so, substituting (2) into (1) gives the head loss for laminar flow as

$$h_L = \left(\frac{128v}{\pi g}\right)\frac{L}{D^4}Q = r_L Q \tag{3}$$

with the obvious definition for the laminar flow resistance factor r_L. Since r_L is independent of flow, it is important when writing program code dealing with laminar flows which are very small or zero to use (3) rather than (1). This is because the term for this pipe on the diagonal of the top left block (which itself is a diagonal matrix) of the Jacobian is the differential with respect to Q of the term r_LQ. A zero flow does not affect this term, while using (1) with f replaced by 64/R leads to division by zero when Q = 0.

Replacing the smooth cubic spline transition region approximation of friction factor by a jump discontinuity

It is also suggested by Prof Brkic that for the transition from laminar to turbulent flow friction factors should be modeled by a jump discontinuity rather than by the smooth Dunlop cubic spline approximation used in the paper. This suggestion brings with it some hazards. An assumption underpinning the Newton method is that the function being zeroed is differentiable and therefore continuous. Violating this assumption, as this suggestion does, may lead to unpredictable results and/or divergence which may not otherwise occur. It must be said though, that the shape of the

Dunlop cubic spline could itself misdirect iterates. The Dunlop spline used in the paper is precisely the one used in EPANET and it was used there so that a fair comparison could be made between the formulae proposed in the paper and EPANET.

Alternative approximations to the Colebrook-White formula

Many different approximations to the Colebrook-White formula, which is used in the computation of the Darcy-Weisbach friction factor, exist in the literature. Prof Brkic presents an interesting discussion of some of these, including the Swamee and Jain approximation used in the paper. Perhaps one could choose, from among the more accurate approximations, one for which the formulae in the Jacobian are more comfortable. Once again, the authors used the Swamee and Jain approximation to allow a fair comparison with EPANET.

References

Elhay, S. & Simpson, A. (2011), `Dealing with zero flows in solving the non-linear equations for water distribution systems', Journal of Hydraulic Engineering 137(10), 1216-1224. doi:10.1061/(ASCE)HY.1943-7900.0000411. ISSN: 0733-9429.