

Resonance in a Finite Volume
in Quantum Chromodynamics

Anthony C. -P. Hsu

Supervisors:

Anthony W. Thomas

Ross D. Young

Department of Physics

The University of Adelaide

Abstract

Quantum chromodynamics (QCD) is a theory which describes the strong interactions of the quarks. When the energy is lower than about 1 GeV, the QCD coupling become so large that we can not study it perturbatively. Lattice QCD can be used to study QCD non-perturbatively and is suitable for the low energy region.

Much remains to be studied in QCD, such as resonances in scattering processes. Lüscher's formula can relate the scattering process in finite volume lattices with phase shifts in the infinite volume scattering in the real world. In this study, we will construct a model for $\pi - \pi$ scattering on a lattice. We will use this model to investigate Lüscher's approach. $\rho \rightarrow \pi\pi$, and $\rho \rightarrow \pi\omega$ channels in the $J = 1$ isovector $\pi - \pi$ scattering will be considered.

Statement of Originality

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