THE UNIVERSITY OF ADELAIDE

Constrained Parameter Estimation in Multiple View Geometry

by

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A thesis submitted in partial fulfillment for the
degree of Doctor of Philosophy

in the
Faculty of Engineering, Computer and Mathematical Sciences
School of Computer Science

July 2013

Declaration of Authorship

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. The author acknowledges that copyright of published works contained within this thesis resides with the copyright holder(s) of those works. I also give permission for the digital version of my thesis to be made available on the web, via the Universitys digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

In carrying out the research that underlies this thesis the following papers were published or are currently under review:

1. Chojnacki W., Szpak Z.L., Brooks, M.J. and van den Hengel A., “Multiple Homography Estimation with Full Consistency Constraints”, in *Proc. International Conference on Digital Image Computing: Techniques and Applications*, Sydney, Australia, December 2010.
2. Szpak Z.L., Chojnacki W., and van den Hengel A., “Guaranteed ellipse fitting with the Sampson distance”, in *Proc. 12th European Conference Computer Vision*, Florence, Italy, October 2012.
3. Szpak Z.L., Chojnacki W., and van den Hengel A., “A comparison of ellipse fitting methods and implications for multiple-view geometry estimation ”, in *Proc. International Conference on Digital Image Computing: Techniques and Applications*, Freemantle, Western Australia, December 2012.
4. Chojnacki W., Szpak Z.L., Brooks, M.J. and van den Hengel A., “Multiple Homography Estimation with Latent Variables”, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, manuscript submitted for review.

5. Szpak Z.L., Chojnacki W., Eriksson, A. and van den Hengel A., "Sampson Distance Based Joint Estimation of Multiple Homographies", *Proc. IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2013)*, manuscript submitted for review.

The authors' contributions in each of the aforementioned papers are as follows:

Zygmunt L. Szpak: Literature review; development and implementation of all algorithms; collection of data; conceptualisation, design and analysis of experiments; testing and mathematical modelling of algorithms and writing the manuscript.

Wojciech Chojnacki: Conceptualisation and mathematical modelling of algorithms; critical feedback and discussion on ideas and concepts; interpretation of experimental results and writing the manuscript.

Anders Eriksson: Early discussion of ideas; evaluating and editing the manuscript.

Michael J. Brooks: Early discussion of ideas; evaluating and editing the manuscript.

Anton van den Hengel: Supervising development of work; evaluating and editing the manuscript.

By signing the Declaration of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

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Anton van den Hengel Signed: Date:

“The more we study the less we know, because we see new avenues of knowledge down which we might travel for a lifetime.”

Fulton J. Sheen

THE UNIVERSITY OF ADELAIDE

Abstract

Faculty of Engineering, Computer and Mathematical Sciences
School of Computer Science

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Multiple view geometry is a branch of computer vision devoted entirely to the study of the relationship between images generated from a fixed three-dimensional scene. Thanks to the body of knowledge generated in this domain some of the most exciting developments in navigation have recently been realised. Google's release of Street-view maps is the most remarkable example. Currently there is a growing demand for new insight and knowledge originating from multiple view geometry, as two of the most popular technological companies, Google and Apple, embark on a mission to generate three-dimensional maps. The research conducted in this thesis makes a direct contribution to two specific problems that arise frequently in the context of multiple view geometry: homography estimation and ellipse fitting. A homography is used to establish a relationship between two images of a scene, whenever the scene consists of a flat surface. If the scene consists of several flat surfaces, such as walls of buildings in urban environments, then multiple homographies are required to adequately represent the relationship between a pair of images. But when multiple homographies are required, computer vision practitioners typically estimate homographies separately. This thesis demonstrates that multiple homographies must not be estimated separately, because additional inter-homography constraints need to be satisfied in order for a collection of homographies to accurately reflect the three-dimensional geometry of the scene. This thesis offers a comprehensive account of a variety of subtleties that arise in the estimation of multiple homographies, and presents detailed novel algorithms for fulfilling the estimation task. A central contribution is the development of a new framework for jointly estimating multiple homographies. The new framework leads to considerably more accurate homography estimates than previous approaches. The second major contribution of this thesis relates to another frequently encountered task in multiple view geometry: ellipse fitting. Recently many new cost functions promising unbiasedness, consistency or hyperaccuracy have been reported to improve the state-of-the-art in fitting ellipses to data.

Unfortunately, the new cost functions have not been substantiated with thorough experimental comparisons. This thesis offers an extensive evaluation of both new and old ellipse fitting methods with the aid of comprehensive simulations. The findings suggest that there is not much difference between the newer and more established estimators. There is, however, a significant difference between the sole estimator that guarantees an ellipse fit, and other estimators which are prone to occasionally producing hyperbolas. The estimator that guarantees an ellipse fit is significantly less accurate. To remedy this undesirable discovery, a new ellipse estimator is proposed that shares a similar statistical accuracy to the unbiased, consistent or hyper-accurate estimators, but unlike all of these, still guarantees an ellipse fit.

Acknowledgements

I owe sincere and earnest thankfulness to Prof. Wojciech Chojnacki, without whom the research conducted in this thesis would never have been carried out. He took me under his wing, and introduced me to the wonderful and mesmerising world of projective geometry and parameter estimation. Virtually all that I have learned during my candidature in Adelaide, from mathematical modelling, presenting research and writing a paper, to professional conduct and general research attitude, I have learned by observing and imitating him. I am especially grateful for his assistance in the derivation of homography matrix covariances that appear in the Appendix of this thesis. It has been a joy and blessing to work with him and to learn from him. Thank you for many shared meals, innumerable captivating whiteboard discussions and the occasional serving of delicious polish food!

I would also like to extend my deep gratitude to Prof. Ernest Fokoue, with whom I shared countless uplifting and technical discussions. Despite residing in America, he stayed awake many late nights in order for us to talk over Skype. His advice and encouragement throughout my candidature was priceless. He breathed new life into my work at a time when I needed it most.

My first encounter with image processing and computer vision was thanks to Prof. Jules R. Tapamo. I would never have left South Africa to pursue a PhD and an academic career if it were not for his encouragement and belief in my abilities. It is no overstatement to say that when he arrived at the School of Computer Science at the University of KwaZulu-Natal, he lifted the standard of research and sparked the imagination of many young students. I am first and foremost a disciple of the Tapamo clan! *Mzansi fo sho*!

Thanks must also go to the School of Computer Science here in Adelaide for providing me with excellent facilities to pursue my PhD, and ensuring that I have a stable and state-of-the-art work environment. I would also like to thank my co-supervisors, Dr. Anthony Dick and Dr. Anders Eriksson, for all of their valuable time, and for always being available to answer any questions that I brought. I am also very grateful to my principal supervisor, Prof. Anton van den Hengel, for taking me on as his student and making me part of the ACVT family. Thank you for offering me the scholarship, for giving me the freedom to pursue my research ideas and for always making sure that I had everything I needed to carry out my research successfully.

In the pursuit of a PhD one inevitably encounters a large bureaucracy that requires various paper-work to be filled out and filed. I would like to thank Pru Carter for taking care of countless bureaucratic issues that arose during my candidature. Thank you for your cheerful disposition, and for taking care of everything from conference travel to misunderstandings with the graduate centre, with absolute professionalism. Without your critical interventions, this thesis would not have been completed in the allotted time.

My close friends in the School of Computer Science deserve a special mention. Thank you to Dr. John Bastian, Lachlan Fleming, Yanzhi Chen, Liang Li, and Dr. Javen Shi for many shared experiences and memories. You made my journey a happy one! Thanks also to my good friend abroad, Dr. Jean Vincent Fonou-Dombeu, for his backing and confidence.

Lastly, I would like to thank my family for all of their support and encouragement. I would especially like to thank my wife Ancret Szpak, with whom I shared all the highs and lows of the PhD experience. Your constant love and belief in me has been the source of my energy and enthusiasm throughout my studies. *Serdecznie dziękuje Ci, Kochanie*!

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To my academic parents: Wojciech, Ernest and Jules.