Analytical and Numerical Analyses for Rock Slope Stability Using the Generalized Hoek-Brown Criterion

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A thesis submitted for the degree of

Doctor of Philosophy

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July 2013
To my parents

Meihua and Binying
Statement of Originality

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Acknowledgments

I gratefully acknowledge my supervisors Dr. Murat Karakus, Associate Prof. Chaoshui Xu and Prof. Stephen Priest, for their invaluable guidance, encouragement and constructive criticism during my candidature period. Without their great contribution this thesis would not be possible. I am particularly grateful to Dr. Murat Karakus for not only gives research supports but also offers encouragement and emotional supports in the past three years.

I would like to sincerely thank the China Scholarship Council and the University of Adelaide for providing the joint PhD scholarship.

I would like to express my gratitude to Mrs Barbara Brougham for editing the submitted and published journal papers which are composed of the thesis.

Many thanks go to Associate Prof. Rafael Jimenez for supervision and cooperation on the rock failure criterion research topic when I was a visiting scholar in the E.T.S.I Caminos, Canales y Puertos at the Universidad Politécnica de Madrid, Spain.

Many thanks go to Prof. Jian Zhao for hosting me as a visiting scholar at Laboratory for Rock Mechanics at the Swiss Federal Institute of Technology Lausanne, Switzerland.

Many thanks go to all the staff of the School of Civil, Environmental and Mining Engineering for their individual help and support. I am especially grateful to Dr. Stephen Carr for helping to install research softwares and for providing IT supports.

Finally, I would also like to thank my parents, for supporting me emotionally during my PhD research.

VIII
Introduction

Design of rock slope is one of the major challenges at every stage of open pit mining operations. Providing an optimal excavation design based on a robust analysis in terms of safety, ore recovery and profit is the ultimate goal of any slope design. The rock slope stability is predominantly controlled by the strength and deformation of the rock mass which characteristically consists of intact rock materials and discontinuities. Initially, movement of the slope occurs due to stress relaxation as a result of removal of rocks which used to provide confinement. This behavior of slope can be attributed to linear elastic deformation. In addition to this, sliding along discontinuity surfaces and dilation in consequence of formation of cracks can occur. Ultimately all these instabilities lead to failure of the slopes. Therefore, formulation of slope designs plays critical role in the process of slope stability. In conventional approaches for assessing the stability of a homogeneous slope, such as the limit equilibrium method (LEM) and shear strength reduction (SSR) method, rock mass strength is usually expressed by the linear Mohr-Coulomb (MC) criterion. However, rock mass strength is a non-linear stress function. Therefore, the linear MC criterion generally do not agree with the rock mass failure envelope, especially for slope stability problems where the rock mass is in a state of low confining stresses that make the nonlinearity more dominant.

With the aim of better understanding the fundamental rock slope failure mechanisms and improving the accuracy of the rock slope stability results, this research focuses on the application of the Hoek-Brown (HB) criterion, which can ideally represent the non-linear behavior of a rock mass, on the rock slope stability analysis.

There, three major sections are available in the thesis. The first section, from Chapters 1 to 4, proposes new methods for estimating the intact rock and rock mass properties, which will be
used for slope stability analysis. In the second section studied in Chapter 5, a new non-linear shear strength reduction technique is proposed for the analysis of three-dimensional (3D) slope modeling. In section three (Chapter 6), novel stability charts are proposed, which have the merit of estimating factor of safety (FOS) for a given slope directly from the HB parameters and rock mass properties. These charts can provide a quick and reliable assessment of rock slope stability.

The major research contributions and outcomes of the overall researches are presented in six journal publications which are forming the thesis. The titles of Chapters 1 through 6 reflect the titles of the journal papers.

In Chapter 1, laboratory tests conducted on Hawkesbury sandstone obtained from New South Wales are carried out to investigate the relationship between the HB constant $m_i$ and uniaxial compressive strength (UCS) of intact rock. Based on the analysis of the laboratory tests and the existing database, a new method that can estimate the HB constant $m_i$ values from UCS and rock types is proposed. The proposed method can reliably be used in the HB criterion for intact rock strength estimation when the triaxial tests are not available.

In Chapter 2, an analytical solution for estimating the instantaneous MC shear strength from the HB failure criterion for highly fractured rock mass is presented. The proposed solution is based on the assumption that the HB parameter, $s$ is equal to zero. The proposed solution has the merit of producing very accurate shear strength for highly fractured rock mass where the Geological Strength Index (GSI) is less than 40.

In Chapter 3, an analytical solution, which can calculate the shear strength of rock masses accurately for the whole range GSI values, is proposed as an extension to the work in Chapter 2. The proposed approach is based on a symbolic regression analysis performed by genetic programming (GP). The proposed solution not only can be implemented into the LEM to
calculate the instantaneous shear strength of each slice of a failure surface under a specified normal stress, but also can be implemented into finite element method performed by SSR approach to calculate the instantaneous shear strength of each element under different stress state of a slope.

**In Chapter 4,** as a part of estimating rock mass strength and elastic properties in the first section, the most widely used empirical equations for the estimation of deformation modulus of rock masses ($E_m$) are reviewed. Two simplified empirical equations for estimating of $E_m$ are also presented. The proposed empirical equations use the Rock Mass Rating classification system and the deformation modulus of intact rock ($E_i$) as input parameters. These equations can be used in the numerical modelling for slope stability analysis, which is conducted in Chapter 5.

**In Chapter 5,** a new non-linear shear strength reduction technique is proposed to analysis the stability of 3D rock slopes satisfying the HB failure criterion. The method for estimating the instantaneous MC shear strength from the HB criterion described in Chapters 2 and 3 are used to estimate shear strength of elements in FLAC$^3$D model. The proposed 3D slope model is used to analyse the influence of boundary condition on the calculation of FOS using 21 real open pit cases where the values of $m_i$ and $E_m$ values are calculated from the methods introduced in Chapters 1 and 4, respectively. Results show that the values of FOS for a given slope will be significantly influenced by the boundary condition, especially the case where the slope angle is less than 50\(^\circ\).

**In Chapter 6,** extensive slope stability analyses using LEM are carried out. The calculation of FOS is based on estimating the instantaneous MC shear strength of slices of a slip surface from the HB criterion. Based on the analysis results, novel stability charts are proposed. The proposed charts are able to estimate the FOS for a given slope directly from the HB parameters,
slope geometry and rock mass properties. It is suggested that the proposed chats can be used as useful tools for the preliminary rock slope stability assessment.
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List of Symbols

\(a\) Hoek-Brown input parameter for the rock mass
\(c\) Cohesion
\(D\) Disturbance factor
\(E_i\) Deformation modulus of the intact rock
\(E_m\) Deformation modulus of the rock mass
\(f_B\) Boundary weighting factor
\(f_D\) Disturbance weighting factor
\(f_\beta\) Slope angle weighting factor
\(H\) Slope height
\(m_b\) Hoek-Brown input parameter for the rock mass
\(m_i\) Hoek-Brown constant for the intact rock
\(m_{in}\) Normalized \(m_i\) for the Hoek-Brown criterion
\(m_c\) Constant for calculating \(m_{in}\)
\(m_d\) Constant for calculating \(m_{in}\)
\(s\) Hoek-Brown input parameter for the rock mass
\(\beta\) Slope angle
\(\phi\) Angle of friction
\(\gamma\) Unit weight of the rock mass
\(\nu\) Poisson’s ratio
\(\sigma_1\) Major principal stress
\(\sigma_3\) Minor principal stress
\(\sigma_{ci}\) Uniaxial compressive strength of the intact rock
$\sigma_n$ Normal stress  
$\sigma_i$ Tensile strength of the intact rock  
$\tau$ Shear stress