Oka Theory of Riemann Surfaces

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Abstract

In his 1993 paper, J. Winkelmann determined the precise pairs of Riemann surfaces for which every continuous map between them can be deformed to a holomorphic map. In particular, it is true for all maps from non-compact Riemann surfaces into $\mathbb{C}$, $\mathbb{C}^*$, the Riemann sphere or complex tori. This is a result of M. Gromov's seminal paper in 1989, where he introduced elliptic manifolds and showed that every continuous map from a Stein manifold into an elliptic manifold can be deformed to a holomorphic map. The elliptic Riemann surfaces are $\mathbb{C}$, $\mathbb{C}^*$, the Riemann sphere and complex tori. Gromov incorporated versions of the Weierstrass and Runge approximation theorems into the deformation to get stronger Oka properties, known as BOPAI and BOPAJI in the literature. It has since been shown, using deep, higher dimensional techniques, that maps from Stein manifolds into elliptic manifolds satisfy BOPAI and BOPAJI. In this thesis we strengthen Winkelmann's results to find the precise pairs of Riemann surfaces that satisfy the stronger Oka properties of BOPAI and BOPAJI. We rely on Riemann surface theory, Morse theory and algebraic topology, rather than techniques from higher dimensional complex analysis.

Signed Statement

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