Surface Shape Regions
as
Manifestations of a Socio-economic Phenomenon

a solution to the choropleth mapping problem

by

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dedicated to my wife, partner and friend ...

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ABSTRACT

A choropleth map is a cartographic document. It shows a geographic study area tessellated by a set of polygons that differ in shape and size. Each polygon is depicted by a uniform symbol representing the manifestation of some phenomenon. This thesis focuses on socio-economic phenomena. We want to delineate a set of socio-economic regions within a study area. These regions are used for decision making about the delivery of specific goods and services and/or the provision of specific community infrastructure. However, we have identified three fundamental weaknesses associated with the use of choropleth maps for socio-economic regionalisation. Therefore, as an alternative to the choropleth map if we think explicitly in $\mathbb{R}^3$, then the best representation of the spatial distribution of a socio-economic phenomenon is a smooth surface.

The socio-economic data we use are collected during a national census of population and are summarised for areas, i.e., polygons. To accommodate these data we have developed and applied a method for gridding and smoothing — termed regularisation — in order to build a smooth surface. We apply Green's theorem and use path integrals with much simplification to compute a smoothed datum for each intersection of a, say, 100 by 100 grid that describes a surface. Mathematically, surface shape is interpreted through the comparison of curvatures. Surface shape analysis involves the measurement of the Gaussian and mean curvatures at the internal intersections of the grid.

Curvature measurement requires at least a twice differentiable function. We have invented such a function based on Lagrange interpolation. It is called a Lagrange polynomial in $x\,y$. Each internal intersection of the grid is the $(2, 2)$ element of a 3×3 matrix extracted from the grid. We compute a Lagrange polynomial in $x\,y$ for each 3×3 matrix. Then we use this polynomial to measure the curvatures and classify the shape. Contiguous grid intersections of the same shape class comprise a shape neighbourhoods region interpreted as a specific manifestation of a socio-economic phenomenon. Hence, we have the basis for describing the spatial distribution of the phenomenon.

Three investigations into the construction of quadratic polynomials as alternative functions are described. Two of these quadratic polynomials are called 'exact fit' in the sense that the polynomial returns the exact z-datum associated with each xy-pair used in its construction. Construction of a 'best fit' quadratic polynomial based on least squares interpolation comprises the third investigation. We compare the four different types of polynomials and of these we choose the Lagrange polynomial in $x\,y$ as most appropriate. Given a relatively high density grid, e.g., 250 by 250, regardless of the polynomial used the resulting maps of shape neighbourhoods regions are virtually identical. This surprising convergence in $\mathbb{R}^2$ is explained.

Is a map of shape neighbourhoods regions an accurate description of the spatial distribution of a socio-economic phenomenon? We effect an indirect evaluation of a known phenomenon represented by the spatial distribution of $f(x, y) = \sin x \sin y$. We compute the true map of shape neighbourhoods regions of this phenomenon. An approximate map of shape neighbourhoods regions is computed by sampling with 100 randomly generated polygons. Comparison implies that the approximate map is an accurate representation of the true map. This conclusion is supported strongly by the results of a study of a nonperiodic–nonrandom known phenomenon, based on a combination of exponential functions in x and y. This has a surface similar to that of a socio-economic phenomenon.

We review selected geographic studies in which mathematical tools have been used for analytical purposes. Mathematical analysis is gaining broader acceptance in geography. The innovative, high quality *Surpop* work of British geographers is described, and we comment on the strongly complementary nature of the research presented in this thesis to the *Surpop* work. We describe 18 future research directions and themes; suggestions are made on how each may be undertaken. Next, we summarise each of the ten results of the research presented in this thesis. The thesis concludes with a statement of the medium-term research directions of the researcher and his acknowledgements.

CONTENTS

STATEMENT OF ORIGINALITY AND CONSENT

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