

Design of bi-orthogonal rational Discrete Wavelet Transform and the associated applications

by

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Abstract

Time-frequency analysis has long been a very useful tool in the field of signal processing, especially in dealing with non-stationary signals. Wavelet transform is amongst many time-frequency analysis techniques whose attributes have been well exploited in many classic applications such as de-noising and compression. In recent years, representation sparsity, a measure of the representation's ability to condense signals' energy into few coefficients, has raised much interest from researchers in many fields such as signal processing, information theory and applied mathematics due to its wide range of use. Thus, many classes of time-frequency representations have recently been developed from the conventional ones in maximising the representation sparsity recently. Rational discrete wavelet transform (RADWT), an extended class of the conventional wavelet family, is among those representations. This thesis discusses the design of bi-orthogonal rational discrete wavelet transform which is constructed from finite impulse response (FIR) two-channel rational rate filter banks and the associated potential applications. Techniques for designing the bi-orthogonal rational filter bank are proposed, their advantages and disadvantages are discussed and compared with the existing designs in literature. Experimental examples are provided to illustrate the use of the novel bi-orthogonal RADWT in application such as signal separation. The experiments show sparser signal representations with RADWTs over conventional dyadic discrete wavelet transforms (DWTs). This is then exploited in applications such as de-noising and signal separation based on basis pursuit.

Statement of Originality

I certify that this work contains no material that has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint award of this degree.

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Nguyen Nguyen Si Tran

Thesis Conventions

The following conventions have been adopted in this Thesis:

1. **Spelling.** Australian English spelling conventions have been used, as defined in the Macquarie English Dictionary, A. Delbridge (Ed.), Macquarie Library, North Ryde, NSW, Australia, 2001.
2. **Typesetting.** This document was compiled using \LaTeX 2e. TeXnicCenter was used as text editor interfaced to \LaTeX 2e. Adobe Illustrator CS2 and Inkscape was used to produce schematic diagrams and other drawings.
3. **Mathematics.** MATLAB code was written using MATLAB Version R2007b/R2008a; URL: <http://www.mathworks.com>.
4. **Referencing.** The Harvard style has been adopted for referencing.

Publications

Journal articles

- NGUYEN-S. T. N., AND NG-B.-W.-H. (2012). Design of two-band critically sampled rational rate filter banks with multiple regularity orders and associated discrete wavelet transforms, *Signal Processing, IEEE Transactions on*, **60**(7), pp. 3863–3868.
- S. TRAN NGUYEN NGUYEN, AND B.W.-H.NG (2013). Bi-orthogonal rational discrete wavelet transform with multiple regularity orders and application experiments, *Signal Processing (2013)*, <http://dx.doi.org/10.1016/j.sigpro.2013.04.001i>.

Conference articles

- NGUYEN-S., AND NG-B.-H. (2010). Critically sampled discrete wavelet transforms with rational dilation factor of $3/2$, *Signal Processing (ICSP), 2010 IEEE 10th International Conference on*, pp. 199–202.

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