Modelling environmental turbulent fluids and multiscale modelling couples patches of wave-like system

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Abstract

Many environmental flows of water have large lateral extent compared to the thickness, such as rivers, floods, tides and tsunamis. This dissertation firstly develops a 2D model to more appropriately model large scale simulations, derived from a 3D turbulence model based on the Smagorinski large eddy closure. We explore the implications of changing the theoretical base from depth-averaging to a slow manifold of the turbulent Smagorinski large eddy closure. Centre manifold theory suggests the existence of slow manifold in the system. Embedding the physical problem into a family of problems, computer algebra constructs the slow manifold of the flow in terms of fluid depth and depth-averaged lateral velocities. The model includes the effects and interactions of inertia, advection, bed drag, gravitational forcing and turbulent dissipation with minimal assumptions. Numerical simulations, implemented on staggered grids in space, of channel flows show that the model is reasonable and reliable to describe the dynamics of large scale environmental turbulent fluids.

Sediment transport is important in the environment. Then the dissertation adapts the turbulent modelling and dynamics to include the suspended sediment transport. A slow manifold exists in the system. The evolution of the depth-averaged concentration on the slow manifold governs the dynamics of the suspended sediment in the turbulent fluid flows. The sediment model includes the effects of sediment erosion, advection, dispersion, and also the interactions between the sediment and the turbulent flow. The applications of the suspended sediment model on concentration distributions in channel flows and under large waves indicate that this model is reasonable.

The dissertation secondly develops a gap-tooth scheme to significantly reduce the expensive numerical simulations of complicated waves over large spatial domains. We aim to develop and explore the methodologies for wave dominant dynamics. The gap-tooth scheme is built from given microscale simulations of complicated physical processes such as sea ice or turbulent shallow water. Our long term aim is to enable macroscale simulations obtained by coupling small patches of simulations together over large physical distances.

A staggered grid is used for the microscale simulation of the fields of depth h and velocity u in the wave-like systems. We introduce a macroscale staggered grid to couple the microscale patches. Linear or cubic or quintic interpolation provides boundary conditions on the field in each patch. Linear analysis of the whole coupled multiscale system establishes that the resultant macroscale dynamics is appropriate. Numerical simulations support the linear analysis.

Eigenvalue analysis suggests that the gap-tooth scheme empowers feasible computation of large scale simulations of wave-like dynamics with complicated underlying physics. As an pilot study, the dissertation implements numerical simulations of dam-breaking waves by the gap-tooth scheme. Comparison among the gap-tooth simulation, the microscale simulation over the whole domain and the published experimental data shows the gap-tooth scheme is feasible to compute large scale wave-like dynamics.

Viscous thin fluid flow has long wave dynamics. The dissertation primarily attempts to use the gap-tooth scheme to explore viscous flow of a layer of fluid. An outstanding issue is the need to create microscale details for each patch appropriate to the macroscale information. The dissertation develops a two-layer model for this viscous layer of fluid, which will have more microscale modes than classic one-layer models, but without the full complexity of fully resolved vertical structures. The two layers are artificial and have no distinguishing physical feature. Linear analysis indicates that an unphysical instability appears for high wavenumber. We introduce a regularising operator to stabilise the model.

The gap-tooth scheme is used to model the viscous layer of fluid with the microscale simulator of the developed two-layer model. To create microscale details for each patch appropriate to the macroscale information, computer algebra leads to the classic one-layer model from the developed two-layer model. The coupling conditions of the gap-tooth scheme are developed by interpolating the macroscale value at the centre of a patch to the boundaries of each neighbouring patch. Numerical simulations of the viscous layer of fluid are successfully implemented by such gap-tooth scheme. Results show that the developed gap-tooth scheme is feasible to model the viscous layer of fluid.

Keywords: Navier–Stokes equation, Smagorinski, suspended sediment, channel flows, gap-tooth scheme, dam-breaking waves, thin fluid flow, two-layer model

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